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RELAY SYSTEMS
THEORY AND APPLICATION

RELAY SYSTEMS

THEORY AND APPLICATION

BY

I. T. MONSETH

*General Engineer, Westinghouse Electric &
Manufacturing Company*

AND

P. H. ROBINSON

Engineer, Houston Lighting and Power Company

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PREFACE

This volume has been developed from a series of articles in the *Electric Journal* written by the authors in 1930, 1931, and 1932. The original presentation has been completely rewritten, expanded, and rearranged to include a complete treatise on the theory and application of relay systems.

Believing that no real progress can be made in the development and application of relay systems without basing it upon a thorough quantitative analysis of the magnitude and relations of currents and voltages, the authors have attempted to give a practical system of fault calculations for all arrangements of system connections and all types of short circuits. Six chapters have been devoted to a complete analysis of fault calculations. For unbalanced fault calculations, the fundamental system of symmetrical components is presented. Numerous examples have been given which it is hoped will be found useful not only to practicing engineers but also to students and teachers in engineering schools and colleges.

Relay systems and relay-application data are much more complete than originally outlined and include the recent developments in high-speed relaying of the distance, balance, differential, pilot-wire, and carrier-current types. A discussion of instrument-transformer characteristics, vector analysis, wave form, and testing, is included.

Many works of an excellent character relating to fault calculations, machine characteristics, and relay application have appeared in the *Transactions* of the American Institute of Electrical Engineers, the *Electrical World*, the *Electric Journal*, the *General Electric Review*, and the National Electric Light Association "Relay Handbook." A few publications on general fault calculations and symmetrical components are available and have been referred to in this volume. In addition to the above references, extensive use has been made of published data of the Westinghouse Electric & Manufacturing Company and the General Electric Company.

The editors of the *Electric Journal* were of material assistance in arranging the original series of articles in form for publication. The authors are indebted to the engineers of the Westinghouse Electric & Manufacturing Company for their assistance in preparing the original *Electric Journal* series and for the data they have supplied for this book.

The authors have drawn freely from the works of others, and references and acknowledgments are given in each chapter. They especially acknowledge their indebtedness to the following: C. L. Fortescue, C. A. Nickel, H. P. Sleeper, H. A. Travers, R. H. Park, B. L. Robertson, W. A. Lewis, O. C. Traver, W. W. Edson, H. D. Braley, C. F. Wagner, R. D. Evans, S. L. Goldsborough, E. H. Klemmer, and Edith Clarke.

I. T. MONSETH.

P. H. ROBINSON.

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CHAPTER I

THE RELAYING PROBLEM

The function of protective relays in modern power systems is to initiate the operation of devices to isolate transmission circuits and apparatus when trouble develops. The nature of the trouble is generally in the form of short circuits resulting from equipment failures due to extraneous effects. The majority of the faults in many areas develop from insulation failures occasioned by abnormal voltages caused by lightning or other high-voltage phenomena. The origin of faults can be classified in two groups: (1) inherent defects; (2) external effects. Inherent defects generally involve inadequate design, application of incorrect design principles, improper manufacture, or installation work. In the classification of external effects are included such elements as lightning, switching surges, overvoltage due to overspeeding of synchronous apparatus, insulation failure due to corona or foreign substances such as moisture, mechanical failures, vibration, sleet, wind, and trouble caused by the public, birds, or animals.

It would not be practical or economical to attempt to build fault-proof equipment; however, it is possible to obtain an economic balance of the various factors which influence successful system design and operation. Well-insulated equipment with exceptionally high mechanical and electrical strength may be used to reduce the possibility of faults developing, or duplicate equipment may be used and protected in such a manner that the loss of certain units will not cause interruption of service. Generally, some compromise combination of these two methods, with adequate protection, will give an economical system design. The determining factor in any case is, of course, the importance of service continuity. Certain classes of loads, such as residential lighting, may be interrupted without damage, while industrial loads, involving manufacturing processes, may suffer severe damage from loss of service.

Adequate system design will preclude the necessity of providing protective relays for normal overloads except in special cases. If adequate equipment is available, the system operators should be capable of operating the equipment in such a manner that loads higher than the apparatus rating are not carried. In certain special cases, such as isolated automatic stations, such protection may be necessary. In general, these cases may be handled by the use of some type of thermal relay having a heating characteristic similar to the characteristic of the apparatus protected. If the system load control can be cared for by proper system design and operation, the function of protective relays is reduced to one operation; the clearing of the minimum equipment necessary to isolate and correct abnormal conditions arising from system faults.

The protective relays must be so designed and applied that the presence of a fault is detected, the location determined, and the operation of initiating the isolation executed. To fulfill an ideal principle, the relay should be influenced only by abnormal quantities existing during a fault and the operation should be instantaneous. Actually, certain relays and certain connections can be devised which give a close approach to the ideal, but here, again, the question of economics enters the problem and, in most cases, a compromise short of the ideal is the only practical solution.

There are four general independent types of faults and a fifth combination type which may occur and for which protective relay systems must be designed to isolate. The five types of faults are as follows:

1. Single line to ground.
2. Double line to ground.
3. Line to line.
4. Three phase or three phase to ground.
5. Simultaneous faults at different locations involving any combinations of types 1 to 4, inclusive.

The first four types are of most particular concern. The fifth, or combination type, will be treated as a special case. Average values for the distribution of the different types of faults, using the total number of short circuits per year on average systems, expressed as a percentage of the total, is as follows:

1. Single line to ground.....	65%
2. Double line to ground.....	20%
3. Line to line.....	10%
4. Three phase.....	5%

It should be taken into consideration that these values are average and may vary considerably for some particular system. The distribution of the types of faults is influenced greatly by the general design features, such as in transmission circuits, the conductor arrangement and the relative clearances between phases and between phases and ground and, also, the origin of the disturbing influence initiating the fault. Double line-to-ground and three-phase faults frequently originate as single line-to-ground, and the arc is carried by air currents to the other phases. The fifth type of fault cannot be given an average evaluation because of the very random manner of occurrence and because of the difficulty of determining definitely that such a fault did occur in a particular case. Some systems are more susceptible to simultaneous faults than others, owing to the method of operation. When a single-line-to-ground fault occurs at some point on a system, with isolated neutral or neutral grounded through a high impedance, the two unfaulted phases may be subjected to a maximum of 1.73 times the normal phase voltage to ground. If the insulation is weak at some point, this over-voltage may cause a breakdown at that point, thus producing simultaneous faults. Arcing grounds, occurring on such systems, may cause much higher voltages.

Probably the most frequent cause of simultaneous faults at different locations is lightning. The results of observation and camera studies indicate that the lightning stroke, as it approaches the ground, branches out into many streamers. These streamers frequently cover a very large area, sometimes a circle of 1000 ft. in diameter. Figure 1 is a photograph taken with a fixed camera showing streamer formation near the ground.¹ These streamers may strike the line at different locations, such as on either side of a substation, causing simultaneous flashover of the insulation. In Fig. 2 is shown a photograph of a lightning stroke taken with a moving camera. This photograph shows that the stroke may be recurrent. In this particular case the period of time over which the stroke lasted was about two seconds and approxi-

¹ Acknowledgment is made to J. J. Torok for Figs. 1 and 2.

mately 13 strokes occurred over the same path during this period. The distinct lines of the arc after each discharge can be seen. This is caused by the cloud discharging. After the arc is extinguished, the discharged section of the cloud again builds up rapidly owing to migration of charge from other parts of the cloud. The path of the first discharge is not completely deion-

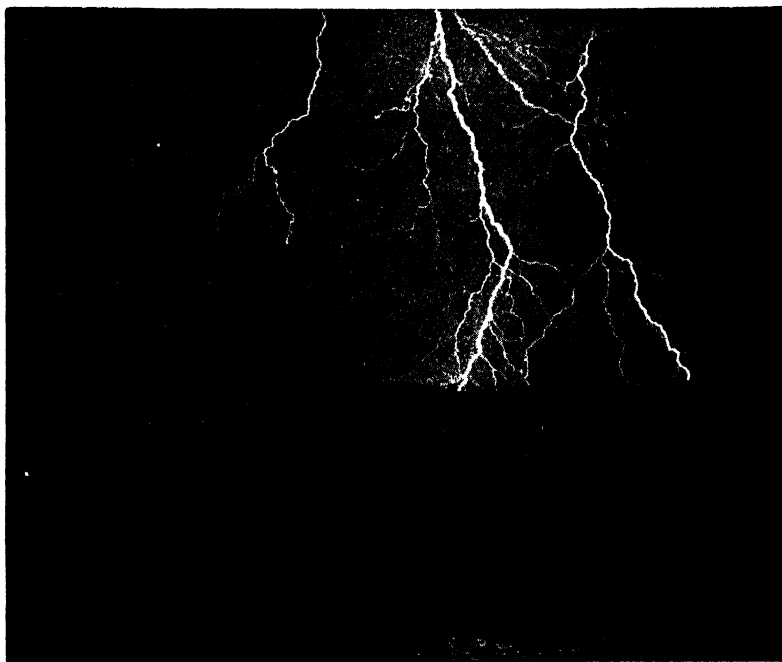


FIG. 1.—Photograph of lightning showing streamer formation near the ground.

ized, hence its breakdown is much lower than any other paths. Usually the side streamers are not reestablished after the first discharge.

Impedance may be present in any of the types of faults enumerated, thus producing lower currents and higher voltages than would exist in its absence. Fault impedance is generally in the nature of an arc, the impedance of which is practically all resistance. It is evident that the magnitude of fault currents and voltages may vary within wide limits. In some cases the fault currents may be less than the system load currents. Very

severe operating requirements are placed upon relay systems which must detect fault currents that are less than load currents.

The problem of relaying is concerned with the protection of electrical equipment which may be classified in two groups: (1) apparatus protection; (2) circuit protection. Group 1 includes all apparatus not classed as transmission, such as



FIG. 2.—Photograph of lightning stroke taken with a moving camera.

generators, transformers, motors, converters, and auxiliary equipment. Group 2 includes all transmission and distribution circuits. The same fundamental principles of protection are applicable to both groups. The nature of the equipment in the two groups requires the use of slightly different methods of accomplishing certain results, and specific principles have practical application to one group and not to the other, but the general

principles are the same. The differences will be readily recognized as the study progresses.

Before proceeding with a consideration of the component quantities in a system or particular piece of apparatus, which may be compared by a relay to determine the existence of a fault,

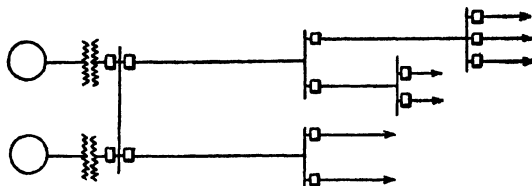


FIG. 3.—Radial system.

it is well to review briefly the general types of systems encountered. All systems may be classified in three groups, as follows:

1. Radial.
2. Loop or ring.
3. General network.

The radial system, in its simplest form, consists of a generating station supplying feeders connected to the generator bus, each feeder, in turn, being subdivided into a number of smaller feeders, such as is shown in Fig. 3. The radial system is not restricted to single circuits and any number of circuits may extend between bussing points. This general type of system is used for any voltage.

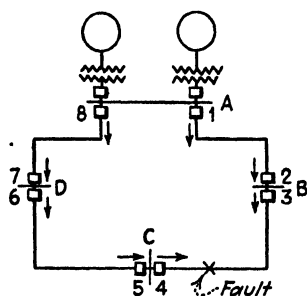


FIG. 4.—Loop system.

The loop system is not restricted to single circuits and any number of circuits may extend between bussing points. This general type of system is used for any voltage.

The loop or ring system, in its simplest form, consists of a single generating station and a single-circuit transmission line. The loop is formed by starting at the generating station and carrying the transmission circuit through one or more substations, terminating at the same generating station, as is shown in Fig. 4. The equivalent of duplicate feed to each station is thus obtained. The loop system is not restricted to a single circuit line but may consist of any number of parallel lines between stations.

The general network is, in reality, a combination of the radial and loop systems. The network usually originates from the

The general network is, in reality, a combination of the radial and loop systems. The network usually originates from the

addition of tie lines between substations and several sources of power as the system load builds up. The possible combinations of circuits, generating stations, and substations are almost unlimited. A simple network system is shown in Fig. 5.

The three types of system plans may be classified into two groups as to method of operation:

1. Isolated neutral.
2. Grounded neutral.

On the isolated neutral system, the line-to-line and three-phase faults are the same as on a grounded neutral system, but a fault to ground does not produce a short circuit, as in the grounded system. The current which will flow, due to an accidental ground, is a charging current through the electrostatic capacity of the conductors to earth.

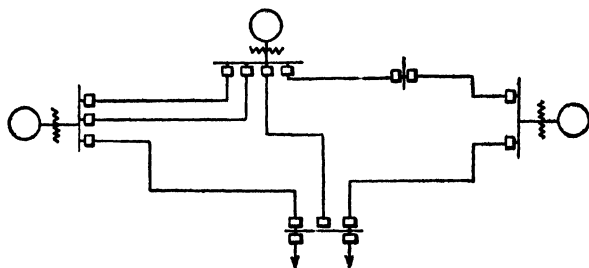


FIG. 5.—Network system.

The neutral of the grounded neutral system may be grounded solidly or through an impedance. The impedance may be ordinary resistance, reactance, resistance and reactance, or tuned reactance. The number of grounding points and the type of grounding, whether through impedance or solid, bear a direct influence upon the characteristics of the short-circuit currents and voltages.

It has been pointed out that protective relays must detect and determine the location of faults. Consideration must now be given to the methods available for accomplishing these results. Every section of line, in any type of power system, falls in one of two classifications:

1. Power source at one end of section.
2. Power sources at both ends of section.

It will later be seen that this has an important bearing on the type of protection necessary. The presence of a fault may be detected by the following general methods:

1. Magnitude of electrical quantities:
 - a. Over current.
 - b. Combination overcurrent and undervoltage.
2. Comparison of electrical quantities:
 - a. Series balance.
 - b. Parallel balance.
 - c. Balance of dissimilar quantities.
3. Relations of sequence components.

In the detection and location of faults by method 1a, the element of time is necessarily associated with the operation; furthermore, for lines having a power source at both ends, the additional element of direction of power flow must be introduced. For example, in Fig. 4, suppose a fault occurs on the section of line between stations B and C. Overcurrent will exist on the system due to the short circuit, but it is obvious that the overcurrent will exist on all of the loop lines; therefore, the relays at all stations will detect the fault by the magnitude and, obviously, will all operate. By introducing the element *direction*, the relays at any station can be made to operate only when the flow of current is away from the bus, as shown in Fig. 4. It will be seen, by an inspection of this figure, that the current is in a direction to operate only relays 1, 3, 4, 6, and 8; however, if the element time is not introduced, all of these relays will operate when operation is desired only of relays 3 and 4. The introduction of the element time allows the selective timing of the relays so that, upon the occurrence of a short circuit in any section, only the relays protecting that section operate, the fault detection and location being affected by means of current magnitude, current direction, and time. It is obvious that the element direction does not enter the problem if there is a power source at only one end of each section of line, such as in Fig. 3.

Method 1b applies to such principles as operation by overcurrent with voltage restraint; *i.e.*, the *distance effect* in which the time of operation is determined by the distance of the fault from the relaying point. The elements magnitude, direction, and time enter into the operation. A distinction between 1b and 2c is marked by the element time.

The protection of systems where a problem of stability is involved requires the clearing of short circuits in a very few cycles to prevent loss of synchronism between stations. In general, it is desirable to clear faults in the minimum time in order to reduce the system disturbance, prevent excessive damage due to spreading of the trouble, and minimize the dropping of load due to low voltage. The methods of relay operation in group 1, involving the element *time*, are, therefore, objectionable. It is readily seen that, in the application of these principles to the system of Fig. 4, successive relays require longer time settings as the generating station is approached. Relays 2, 4, 6 and 8 (Fig. 4) are timed selectively and also 7, 5, 3 and 1. The fault-current and system shock are greatest for faults close to the generating station where the relay time is longest.

The methods of detection and selection in group 2 inherently eliminate the element *time*. The series-balance method compares the instantaneous magnitudes of the currents at the ends of the section protected. The currents are normally equal. All pilot wire and differential schemes are in the series-balance classification and, also, the carrier-current pilot wire and other pilot-wire systems which compare the relative direction of the currents at the ends of the sections protected.

The parallel-balance method consists of the comparison between currents in two parallel lines which, normally, bear a constant ratio. When a fault occurs on one line, the normal balance is upset. Split-conductor cable schemes and conventional parallel-balance schemes fall in this class.

In the method of balance of dissimilar quantities, advantage may be taken of the fact that the relations of voltage and current, under normal and fault conditions, are distinctly different. Under fault conditions, the ratio of the voltage to the current is a measure of the impedance and, also, distance, neglecting the fault component. Since the impedance of the protected section is known, a relay may be designed to operate only when the fault is within the protected section.

Method 3, the relations of sequence quantities, has no precise distinction from 1 and 2, and certain cases may fall in either or both classes. The magnitude and relations of the sequence quantities, under normal and fault conditions, are entirely dif-

ferent. Sequence components may, therefore, be compared for determining the existence of faults.

The fundamental principles outlined are applicable to practically all present-day alternating-current systems and not only to circuits but to rotating and static apparatus, such as generators, motors, and transformers.

It is obvious that a thorough understanding of the theory and application of protective relay systems depends, primarily, upon a knowledge of the fundamentals of fault detection and system characteristics. The fundamentals cannot be properly presented in a non-mathematical manner. The application of the principles of relaying to a particular system requires, as a prerequisite, a thorough quantitative knowledge of the magnitude and relations of currents and voltages during short circuits. The determination of these relations and characteristics requires a sound mathematical foundation. Thus far, there has been only one method devised which is practical for the calculation of unbalanced short circuits. This is the method of symmetrical components. In the succeeding chapters an analysis and description are given for the calculation of any type of fault on any type of three-phase system. A thorough knowledge of the theory and mechanics of calculations bears stressing, as no real progress can be made in the protective art without this knowledge.

CHAPTER II

SYMMETRICAL COMPONENTS

All problems in alternating-current circuits involve vector quantities. Vectors may be referred to rectangular coordinate axes and expressed in terms of the components along the two axes, or they may be expressed in terms of polar coordinates. In Fig. 1 is shown a vector $\tilde{E}$ which has an angle θ with respect to the X axis. In the notation used, a $\sim$ placed above a symbol

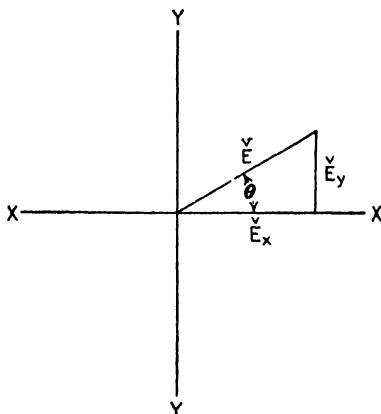


FIG. 1.—Vector and components.

indicates the vector or complex form. $\tilde{E}$ may be resolved into horizontal and quadrature components $\tilde{E}_x$ and $\tilde{E}_y$, respectively. The absolute value of the vector is $E = \sqrt{E_x^2 + E_y^2}$, where E_x and E_y are the magnitudes of the two components.

The vector $\tilde{E}$ may be completely described by the complex expression

$$\tilde{E} = \tilde{E}_x + j\tilde{E}_y \quad (1)$$

The operator j is so chosen that it rotates any vector to which it is attached through an angle of 90° in a positive or counterclockwise direction; therefore, if any vector is operated upon twice by j , i.e.,

j^2 , the vector will simply be reversed and the result is the same as if the original vector were multiplied by -1 ; thus $j^2 = -1$ and therefore $j = \sqrt{-1}$.

The operator $j = \sqrt{-1}$ is an imaginary quantity, but when used as an operator it does not make the vector, upon which it operates mathematically, imaginary. However, the Y axis is usually termed the *axis of imaginaries*, because the j , or quadrature component of the vector of Eq. (1), is parallel to this axis, although this component of the resultant vector is real.

Equation (1) may be expressed in polar form:

$$\tilde{E}_x = E \cos \theta \quad \tilde{E}_y = E \sin \theta$$

Therefore

$$\tilde{E}_x + j\tilde{E}_y = E(\cos \theta + j \sin \theta) = \tilde{E} \quad (2)$$

and

$$\tilde{E}_x - j\tilde{E}_y = E(\cos \theta - j \sin \theta) = \tilde{E}' \quad (3)$$

The expression $(\cos \theta + j \sin \theta)$ is an operator which, applied to any vector, will rotate it through an angle θ in a positive direction, and the operator $(\cos \theta - j \sin \theta)$ will produce rotation θ degrees in a negative direction. If an operator

$$K = \cos \alpha + j \sin \alpha$$

is applied to the vector $\tilde{E}$, the magnitude of the vector will not be changed, but it will be rotated through a positive angle α , thus

$$K\tilde{E} = \tilde{E}(\cos \theta + j \sin \theta)(\cos \alpha + j \sin \alpha) = \tilde{E}[\cos(\theta + \alpha) + j \sin(\theta + \alpha)].$$

Successive applications of operators produce a rotation equal to the sum of the rotations produced by the individual operators.

When vectors are to be added or subtracted, the complex or polar form of expression is convenient, but for multiplication and division of vectors the exponential form involving $e^{j\theta}$ is more convenient since the processes involve only the addition and subtraction of exponents and the multiplication and division of scalar quantities.

The operator $e^{j\theta}$ may be expanded into an infinite series by Maclaurin's theorem

$$\begin{aligned}
 e^{j\theta} &= 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \dots \\
 &= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \right) \\
 &\quad + j \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \right) \quad (4)
 \end{aligned}$$

Similarly, for the sine and cosine of θ ,

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \quad (5)$$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \quad (6)$$

Therefore

$$e^{j\theta} = \cos \theta + j \sin \theta \quad (7)$$

It can be shown in like manner that

$$e^{-j\theta} = \cos \theta - j \sin \theta \quad (8)$$

The magnitude of $\cos \theta + j \sin \theta = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$. The magnitude of $e^{j\theta}$ and $e^{-j\theta}$ is, therefore, unity. $e^{j\theta}$ and $e^{-j\theta}$ may be represented as unit vectors, as shown in Fig. 2. It can be seen that when θ increases, $e^{j\theta}$ rotates in a positive direction and $e^{-j\theta}$ in a negative direction.

The operators $e^{j\theta}$ and $e^{-j\theta}$ produce the same effect when applied to a vector as $(\cos \theta + j \sin \theta)$ and $(\cos \theta - j \sin \theta)$ respectively. The operator $e^{j\theta}$ rotates a vector, to which it is applied, through a positive angle θ , and $e^{-j\theta}$ rotates a vector through a negative angle θ .

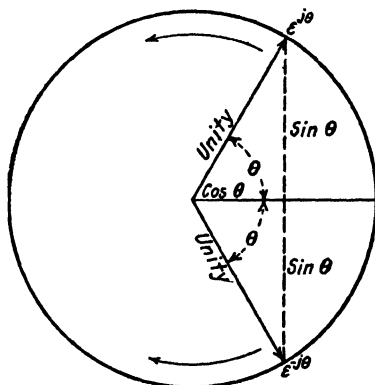


FIG. 2.—Unit vectors.

From Eqs. (2), (3), (7), and (8) we may write

$$\tilde{E} = \tilde{E}_x + j\tilde{E}_y = E e^{j\theta} = E(\cos \theta + j \sin \theta) \quad (9)$$

$$\tilde{E}' = \tilde{E}_x - j\tilde{E}_y = E e^{-j\theta} = E(\cos \theta - j \sin \theta) \quad (10)$$

The real components of these vectors are equal and have the same sign, and the imaginary components are equal but have

opposite signs. The vectors are said to be conjugates of each other. A vector is indicated by a $\sim$ placed above the symbol representing the vector, and its conjugate is represented by an inverted $\sim$. Thus, if

$$\tilde{E} = \tilde{E}_x + j\tilde{E}_y = Ee^{j\theta}$$

the conjugate is

$$\hat{E} = \tilde{E}_x - j\tilde{E}_y = Ee^{-j\theta}$$

The conjugate of any vector quantity is obtained by changing the sign of the j term. The same laws may be applied to the exponents of the exponential operator as to the exponents of ordinary numbers.

$$\begin{aligned} e^{j\theta}e^{j\alpha} &= e^{j(\theta+\alpha)} \\ e^{j\theta}e^{-j\alpha} &= e^{j(\theta-\alpha)} \end{aligned}$$

The product of the exponential expressions of two vectors can be obtained in a similar manner. Let

$$\begin{aligned} \tilde{E} &= Ee^{j\theta} \\ \tilde{I} &= Ie^{j\alpha} \end{aligned}$$

then

$$\tilde{E}\tilde{I} = EIe^{j(\theta+\alpha)} \quad (11)$$

This shows that the product of two vector quantities is a new vector having an absolute value equal to the product of the absolute values of the two vectors and an angular position which is the algebraic sum of the two angles. It follows that the square of a vector quantity is a new vector having a magnitude the square of the original and twice its angle.

$$(\tilde{E})^2 = (Ee^{j\theta})^2 = E^2e^{j2\theta} \quad (12)$$

If a complex number is multiplied by its conjugate the result is a real quantity equal to the square of the absolute value of the complex number.

$$\begin{aligned} \tilde{E} &= Ee^{j\theta} \\ \hat{E} &= Ee^{-j\theta} \\ \tilde{E}\hat{E} &= E^2e^{j(\theta-\theta)} = E^2 \end{aligned} \quad (13)$$

If a vector is multiplied by the conjugate of another, a new vector is obtained having an angular position, which is the difference of the angles of the original vectors.

$$\begin{aligned}\check{E} &= E e^{j\theta} \\ \check{I} &= I e^{-j\alpha} \\ \check{E}\check{I} &= EI e^{j(\theta-\alpha)} = EI [\cos(\theta-\alpha) + j \sin(\theta-\alpha)]\end{aligned}\quad (14)$$

When one complex quantity is divided by another, a new vector is obtained having a magnitude which is the quotient of the original magnitudes and an angle with the reference axis equal to the difference of the angles of the original vectors.

$$\frac{\check{E}}{\check{I}} = \frac{E e^{j\theta}}{I e^{-j\alpha}} = \frac{E}{I} e^{j(\theta+\alpha)} \quad (15)$$

The reciprocal of a vector is a new vector having a magnitude equal to the reciprocal of the original vector and an angle of the same magnitude but of opposite sign as the original.

$$\frac{1}{\check{E}} = \frac{1}{E e^{j\theta}} = \frac{1}{E} e^{-j\theta} \quad (16)$$

The determination of roots and powers of complex numbers is readily accomplished when expressed in the exponential form.

A vector is not changed in magnitude or position by multiplication with an operator which produces rotation through 2π radians or any whole number of 2π radians.

Let $\check{E} = E e^{j\theta}$ be a vector having an angle θ with respect to the reference axis. This vector will not be changed by the operator $e^{j2\pi q}$ where q is any integer.

$$\check{E} = E e^{j\theta} e^{j2\pi q} = E e^{j(\theta+2\pi q)}$$

By the laws of exponents,

$$\sqrt[n]{\check{E}} = \sqrt[n]{E e^{j(\frac{\theta+2\pi q}{n})}} = \sqrt[n]{E} \left[\cos\left(\frac{\theta+2\pi q}{n}\right) + j \sin\left(\frac{\theta+2\pi q}{n}\right) \right]$$

This expression will have n roots. Consider the simple case of extraction of the square root of the vector $\check{E}$, $n = 2$

$$\sqrt{\check{E}} = \sqrt{E} e^{j(\frac{\theta}{2} + \pi q)}$$

There are two different roots of the vector, consisting of the two roots of $\sqrt{E}$, one plus and one minus, and the two roots of the exponential operator. The roots of the operator occur for $q = 0$ and $q = 1$.

When $q = 0$,

$$\sqrt{\epsilon^{j\theta}} = \epsilon^{j\left(\frac{\theta}{2}\right)}$$

when $q = 1$,

$$\sqrt{\epsilon^{j\theta}} = \epsilon^{j\left(\frac{\theta}{2} + \pi\right)} = -\epsilon^{j\left(\frac{\theta}{2}\right)}$$

since the addition of π to the exponent of an operator simply reverses its sign.

For higher values of q the roots repeat. Usually the negative root of a complex number has no significance and is consequently not considered.

The expression for the n th power of a vector is

$$(E)^n = (E\epsilon^{j\theta})^n = E^n \epsilon^{jn\theta} \quad (17)$$

In the solution of alternating-current problems, so long as the system is symmetrical and loading conditions balanced on all phases, it is only necessary to obtain the solution for a single phase. The solution for the two other phases is obtained from symmetry. When the electrical quantities, voltage and current, become unbalanced, the resulting unbalanced coupling between phases makes the problem difficult to solve. This is particularly true when dealing with rotating apparatus. It should be readily appreciated that to obtain a solution for the voltages and currents in all parts of a complex power system by ordinary methods involving simultaneous equations would lead to such cumbersome results as to be impractical for use.

A method devised by C. L. Fortescue¹ consists of breaking down the unbalanced system into several balanced symmetrical systems which permits a practical solution for any unsymmetrical system.

Polyphase Vectors.—A symmetrical three-phase system of voltages or currents can be represented by three vectors spaced 120° or $2\pi/3$ radians apart, as shown in Fig. 3. Suppose that the order in which the three voltages reach their maximum is a, b, c , with positive rotation. In this discussion the direction of rotation of all vectors is assumed to be counterclockwise. If

¹ FORTESCUE, CHARLES L., Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks, *Trans. A.I.E.E.*, vol. 37, p. 1027, 1918.

the voltages of Fig. 3 are symmetrical, they may be represented in terms of only one, *i.e.*,

$$E_a = E_a, \quad E_b = E_a \epsilon^{\frac{j4\pi}{3}}, \quad E_c = E_a \epsilon^{\frac{j2\pi}{3}}$$

If we let a represent the vector $\epsilon^{\frac{j2\pi}{3}}$, then $a = \epsilon^{\frac{j2\pi}{3}}$; $a^2 = \epsilon^{\frac{j4\pi}{3}}$; and $a^3 = \epsilon^{j2\pi} = 1$. The three-phase vector of Fig. 3 then becomes

$$\left. \begin{aligned} E_a &= E_a \\ E_b &= a^2 E_a \\ E_c &= a E_a \end{aligned} \right\} \quad (18)$$

E_b is equal to E_a rotated 240° or $4\pi/3$ radians in a positive direction, and E_c is equal to E_a rotated 120° or $2\pi/3$ radians in the same direction.

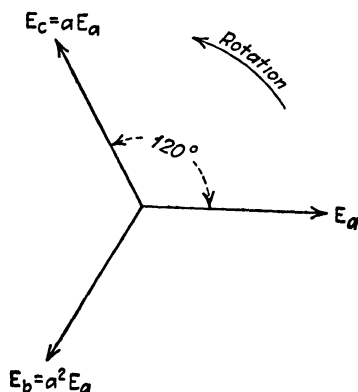


FIG. 3.—Symmetrical system of vectors.

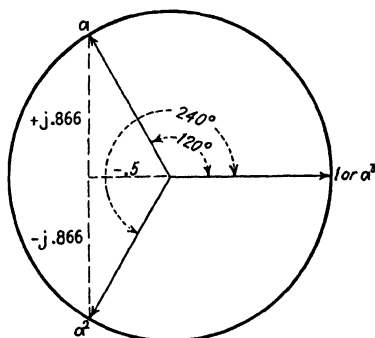


FIG. 4.—Properties of unit operators 1, a , a^2 .

The properties of the unit operators 1, a , and a^2 are shown in Fig. 4. It can be seen that higher powers of a repeat. Operating upon a vector with a rotates it 120° , and, when starting with a vector and operating with a three times, *i.e.*, a^3 , the vector is returned to the starting point.

Properties of the Vector a

$$\begin{aligned} a &= -0.5 + j0.866 = \epsilon^{\frac{j2\pi}{3}} & 1 + a &= -a^2 \\ a^2 &= -0.5 - j0.866 = \epsilon^{\frac{j4\pi}{3}} & 1 + a^2 &= -a \\ a^3 &= 1 & &= \epsilon^{j2\pi} & a + a^2 &= -1 \end{aligned}$$

$$\begin{aligned}
 a^4 &= a \\
 a^5 &= a^2 \\
 1 + a + a^2 &= 0 \\
 1 - a &= j\sqrt{3}a^2 & \epsilon^{j0^\circ} &= 1 \\
 1 - a^2 &= -j\sqrt{3}a & \epsilon^{j90^\circ} &= j = \frac{a - a^2}{\sqrt{3}} \\
 a - a^2 &= j\sqrt{3} & \epsilon^{j120^\circ} &= a \\
 a - 1 &= -j\sqrt{3}a^2 & \epsilon^{j180^\circ} &= -1 \\
 a^2 - 1 &= j\sqrt{3}a & \epsilon^{j270^\circ} &= -j = \frac{a^2 - a}{\sqrt{3}} \\
 a^2 - a &= -j\sqrt{3}
 \end{aligned}$$

Any system of three arbitrary vectors may be reduced to three symmetrical three-phase systems: one of positive-phase sequence a, b, c ; one of negative sequence a, c, b ; and one of zero-phase

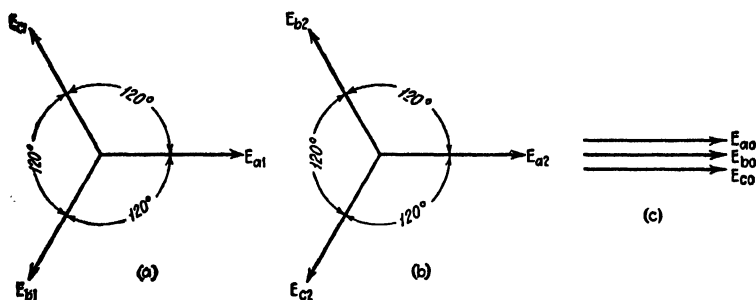


FIG. 5.—(a) Positive-sequence system; (b) negative-sequence system; (c) zero-sequence system.

sequence consisting of three equal vectors, all in phase. The positive-sequence system of vectors will be designated by the subscript 1, the negative-phase sequence by 2, and the zero-phase sequence by 0.

In Fig. 5a is shown the symmetrical positive-sequence three-phase system of voltages. This system is termed *positive* sequence since E_{a1} leads E_{b1} by 120° and E_{b1} leads E_{c1} by 120° . The phase order is a, b, c , counterclockwise rotation. The system is symmetrical, therefore, the following relations exist:

$$\left. \begin{aligned} E_{a1} &= E_{a1} \\ E_{b1} &= a^2 E_{a1} \\ E_{c1} &= a E_{a1} \end{aligned} \right\} \quad (19)$$

If a three-phase system of voltages or currents is balanced, as in Fig. 3, $E_{a1} = E_a$, $E_{b1} = E_b$ and $E_{c1} = E_c$. Therefore, a balanced three-phase system is made up only of positive-sequence quantities. Since the positive-phase sequence system is symmetrical,

$$E_{a1} + E_{b1} + E_{c1} = 0 \quad (20)$$

The symmetrical negative-sequence system of voltages is shown in Fig. 5b. The direction of rotation of this system is the same as the positive, but the phase order is reversed. The vectors reach their maximum values in the order a, c, b . In the negative-sequence system the following relations exist:

$$\left. \begin{aligned} E_{a2} &= E_{a2} \\ E_{b2} &= aE_{a2} \\ E_{c2} &= a^2E_{a2} \end{aligned} \right\} \quad (21)$$

Also, since the system is symmetrical,

$$E_{a2} + E_{b2} + E_{c2} = 0$$

It will be observed that the direction of rotation of the positive and negative sequences is the same, but the phase order is opposite. Physically this means that, if positive-sequence voltages a, b, c and negative-sequence voltages a, c, b are applied to the terminals a, b, c , respectively, of a three-phase machine, they will produce rotating magnetic fields which revolve in opposite directions. If the positive-sequence voltages are applied to the a, b, c terminals of the machine in the order a, c, b , respectively, the rotation of the resulting magnetic field will be the same as for the negative-sequence system. This is true, since reversing the position of two phases of the positive-sequence system converts it to a negative-sequence system.

The zero-sequence system is shown in Fig. 5c. This system consists of three equal vectors, all in phase:

$$\left. \begin{aligned} E_{a0} &= E_{a0} \\ E_{b0} &= E_{a0} \\ E_{c0} &= E_{a0} \end{aligned} \right\} \quad (22)$$

Also

$$E_{a0} + E_{b0} + E_{c0} = 3E_{a0} \quad (23)$$

The three systems in Eqs. (19), (21), and (22) form nine vector components distributed in the three phases of the original system as follows;

Phase *a*

$$E_{a1} = E_{a1} \quad E_{a2} = E_{a2} \quad E_{a0} = E_{a0} \quad (24)$$

Phase *b*

$$E_{b1} = a^2 E_{a1} \quad E_{b2} = a E_{a2} \quad E_{b0} = E_{a0} \quad (25)$$

Phase *c*

$$E_{c1} = a E_{a1} \quad E_{c2} = a^2 E_{a2} \quad E_{c0} = E_{a0} \quad (26)$$

The original arbitrary system of vectors is thus resolved into three systems and each phase of the original system is represented by the three components, one of zero-, one of positive-, and one of negative-phase sequence. The three original vectors may be equated.

$$\left. \begin{aligned} E_a &= E_{a0} + E_{a1} + E_{a2} \\ E_b &= E_{b0} + E_{b1} + E_{b2} \\ E_c &= E_{c0} + E_{c1} + E_{c2} \end{aligned} \right\} \quad (27)$$

Substituting the equivalent values from Eqs. (19), (21), and (22), to obtain expressions for the three phases in terms of phase *a*:

$$E_a = E_{a0} + E_{a1} + E_{a2} \quad (28)$$

$$E_b = E_{a0} + a^2 E_{a1} + a E_{a2} \quad (29)$$

$$E_c = E_{a0} + a E_{a1} + a^2 E_{a2} \quad (30)$$

An examination of these equations will show the vectors to be arranged vertically in the following order:

Vector positions of E_{a0} are 1, 1, 1 (zero sequence)

Vector positions of E_{a1} are 1, a^2 , a (positive sequence)

Vector positions of E_{a2} are 1, a , a^2 (negative sequence)

Each of three sets of vectors, zero, positive, and negative sequence, must be treated as a unit. In the application of the *j* operator, it is applied as a unit to a single phase. In the application of three-phase complex algebra, a unit or sequence involves three vectors.

The individual sequence components can be determined in terms of the original vectors, since three independent equations are given

The zero-sequence component is obtained directly by adding Eqs. (28), (29), and (30):

$$E_a + E_b + E_c = 3E_{a0} + (1 + a^2 + a)E_{a1} + (1 + a + a^2)E_{a2} = 3E_{a0}$$

$$1 + a + a^2 = 0$$

$$E_{a0} = \frac{E_a + E_b + E_c}{3} \quad (31)$$

Multiplying Eq. (29) by a , Eq. (30) by a^2 , and combining with Eq. (28), in order to eliminate the zero and negative sequences, gives the positive-sequence component:

$$E_a = E_{a0} + E_{a1} + E_{a2}$$

$$aE_b = aE_{a0} + E_{a1} + a^2E_{a2}$$

$$a^2E_c = a^2E_{a0} + E_{a1} + aE_{a2}$$

Adding

$$E_a + aE_b + a^2E_c = (1 + a + a^2)E_{a0} + 3E_{a1} + (1 + a^2 + a)E_{a2} = 3E_{a1}$$

$$E_{a1} = \frac{E_a + aE_b + a^2E_c}{3} \quad (32)$$

In a similar manner, multiplying Eq. (29) by a^2 and (30) by a ,

$$E_{a2} = \frac{E_a + a^2E_b + aE_c}{3} \quad (33)$$

If the original system of vectors E_a , E_b , and E_c is equal in magnitude and symmetrically spaced,

$$E_a = E_a$$

$$E_b = a^2E_a$$

$$E_c = aE_a$$

Substituting these values in (31), (32), and (33),

$$E_{a0} = \frac{(1 + a^2 + a)}{3} E_a = 0$$

$$E_{a1} = \frac{(1 + a^3 + a^3)}{3} E_a = E_a$$

$$E_{a2} = \frac{(1 + a^4 + a^2)}{3} E_a = 0$$

Therefore, if the original system of vectors is balanced, the system is composed of positive-phase sequence components only.

The graphical determination of the sequence components of an arbitrary three-phase system of vectors can readily be illustrated.

Consider the unsymmetrical vectors shown in Fig. 6a. The complex expressions for the vectors are

$$\begin{aligned} E_a &= 40 + j0 \\ E_b &= -20 - j50 \\ E_c &= -30 + j80 \end{aligned}$$

In Fig. 6b are shown the zero-sequence components obtained by adding vectorially the three vectors E_a , E_b , and E_c in accordance

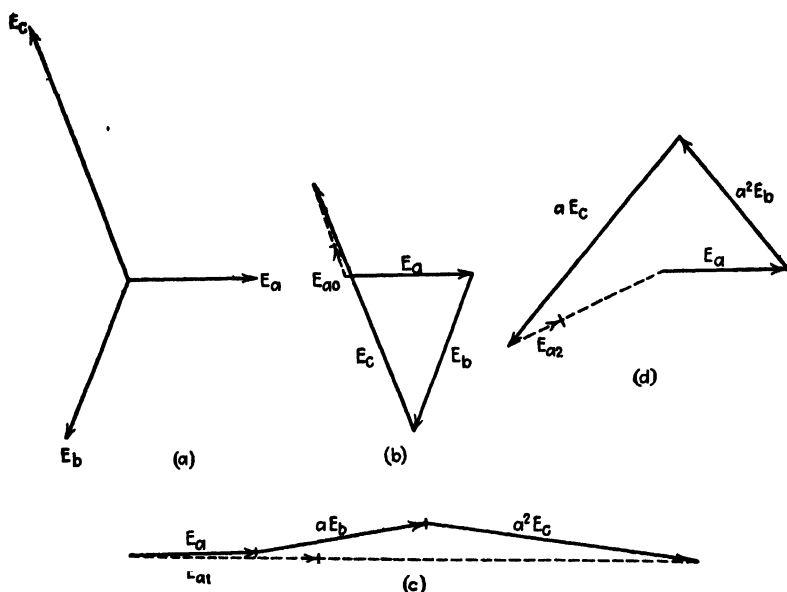


FIG. 6.—Graphical determination of sequence components. (a) Original vector system; (b) zero sequence; (c) positive sequence; (d) negative sequence.

with Eq. (31). One-third of the vector sum gives the magnitude and phase position of the zero-sequence system. The positive-sequence set [Eq. (32)] is obtained by rotating E_b by 120° and E_c by 240° , both in a counterclockwise direction. One-third of the vector sum of these three vectors gives the magnitude and phase position of the positive-sequence component in phase a . The negative-sequence component E_{a2} is obtained by rotating E_b clockwise 120° , E_c counterclockwise 120° and taking one-third their sum, in accordance with Eq. (33).

The symmetrical components may be evaluated mathematically from the complex expressions for E_a , E_b , and E_c , using Eqs. (31), (32), and (33).

$$\begin{aligned} E_{a0} &= \frac{1}{3}[(40 + j0) + (-20 - j50) + (-30 + j80)] \\ &= \frac{1}{3}(-10 + j30) = -3.33 + j10 \\ E_{a1} &= \frac{1}{3}[(40 + j0) + (-0.5 + j0.866)(-20 - j50) + \\ &\quad (-0.5 - j0.866)(-30 + j80)] \\ &= \frac{1}{3}(177.8 - j6.3) = 59.3 - j2.1 \\ E_{a2} &= \frac{1}{3}[(40 + j0) + (-0.5 - j0.866)(-20 - j50) + (0.5 + \\ &\quad j0.866)(-30 + j80)] \\ &= \frac{1}{3}(-47.8 - j23.7) = -15.9 - j7.9 \end{aligned}$$

A comparison of the magnitude and phase position of E_{a0} , E_{a1} , and E_{a2} , as obtained mathematically and graphically, will show them to be identical.

Substituting the values of the symmetrical components obtained above in Eqs. (28), (29), and (30), the original vectors are obtained, thus proving the correctness of the calculations:

$$\begin{aligned} E_a &= E_{a0} + E_{a1} + E_{a2} \\ &= (-3.33 + j10) + (59.3 - j2.1) + (-15.9 - j7.9) \\ &= 40 + j0 \\ E_b &= E_{a0} + a^2 E_{a1} + a E_{a2} \\ &= (-3.33 + j10) + (-0.5 - j0.866)(59.3 - j2.1) \\ &\quad + (-0.5 + j0.866)(-15.9 - j7.9) \\ &= -20 - j50 \\ E_c &= E_{a0} + a E_{a1} + a^2 E_{a2} \\ &= (-3.33 + j10) + (-0.5 + j0.866)(59.3 - j2.1) \\ &\quad + (-0.5 - j0.866)(-15.9 - j7.9) \\ &= -30 + j80 \end{aligned}$$

Sequence Operators.—In dealing with symmetrical components, the operations in the three sequences may be indicated by a sequence operator¹ which is analogous to the j operator in complex algebra of simple alternating currents.² The sequence operator is restricted to operate only on polyphase systems. The operator is useful in circuits involving unsymmetrical impedances in the phases.

¹ FORTESCUE, CHARLES L., *Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks*, A.I.E.E. vol. 37, p. 1027, 1918.

² WAGNER, C. F., and R. D. EVANS, *Symmetrical Components*, *Elec. Jour.*, April, 1928, p. 194.

Let $S^1(E_{a1})$ represent the three vectors E_{a1} , a^2E_{a1} , and aE_{a1} , forming a positive-sequence system. The symbol S^1 is defined as an operator which, when written next to a polyphase vector, changes it into a polyphase-vector system of positive-phase sequence. The quantities which S^1 affect are considered as polyphase vectors. Thus, in $S^1(E_{a1})$, the symbol E_{a1} stands for three vectors, each equal to E_{a1} . Operating on E_{a1} with S^1 means that the three vectors E_{a1} , E_{a1} , E_{a1} are to be multiplied by 1, a^2 , and a , respectively. The first vector is unchanged, the second rotated 240° and the third 120° in a positive direction. S^1 alone has no meaning,¹ unless it is understood to be followed by 1. In this case it stands for a unit-polyphase vector of positive-phase sequence obtained by operating with S^1 on the three vectors (1, 1, 1).

The expression $S^2(E_{a2})$ represents the system of vectors of negative sequence E_{a2} , aE_{a2} , and a^2E_{a2} . S^2 operating on E_{a1} means that the three vectors E_{a2} , E_{a2} , and E_{a2} are to be multiplied by 1, a , and a^2 , respectively.

The expression $S^0(E_{a0})$ represents the system of vectors E_{a0} , E_{a0} , and E_{a0} . Operating on E_{a0} with S^0 means multiplying the three vectors E_{a0} , E_{a0} and E_{a0} by unity. The zero-sequence operator S^0 is a unit operator and does not change a polyphase vector to which it is applied.

The following summary may be written:

$$S^0(E_{a0}) = (1, 1, 1)E_{a0} \quad (34)$$

$$S^1(E_{a1}) = (1, a^2, a)E_{a1} \quad (35)$$

$$S^2(E_{a2}) = (1, a, a^2)E_{a2} \quad (36)$$

The system of three vectors E_a , E_b , and E_c may be completely represented by the single expression

$$S(E_a) = S^0(E_{a0}) + S^1(E_{a1}) + S^2(E_{a2}) \quad (37)$$

The expression $S(E_a)$ completely specifies the system E_a , E_b , and E_c . The equation states that the three vectors $S(E_a)$ are equal to the sum of three systems of vectors, the first being of zero sequence S^0 , the second of positive sequence S^1 , and the third of negative sequence S^2 . The system of positive sequence is obtained by multiplying the vectors E_{a1} , E_{a1} , E_{a1} by 1, a^2 ,

¹ WAGNER, C. F. and R. D. EVANS, *op. cit.*, March, 1928, p. 156.

and a , respectively. In a similar manner, the zero- and negative-sequence systems are found, as previously explained.

The meaning assigned to the sequence operators allows the exponents to be treated by the ordinary laws of exponents. Thus, in the expression $S^1(S^1E_{a1})$, the first vector of the system E_{a1} , E_{a1} , E_{a1} remains unchanged, the second is multiplied by a^2 and again by a^2 , the total change being equal to a alone, and the third is multiplied by a and again by a , the total change being equal to a^2 . Since $S^2(E_{a1})$ equals $(1, a, a^2)E_{a1}$, it is obvious that for $S^1(S^1E_{a1})$ the same result would have been obtained by the expression $S^2(E_{a1})$.

The sum of two vectors, operated on individually by the same sequence operator, is equal to the sum of the vectors operated on by the sequence operator; or, stated in other words, the sum of two vector systems of the same sequence is a new vector system of that sequence, the component vectors of which are the sum of the component vectors of the original systems. Thus the expressions may be written

$$\left. \begin{aligned} S^0(E_a) + S^0(E_b) &= S^0(E_a + E_b) \\ S^1(E_a) + S^1(E_b) &= S^1(E_a + E_b) \\ S^2(E_a) + S^2(E_b) &= S^2(E_a + E_b) \end{aligned} \right\} \quad (38)$$

In obtaining the product of two different vector systems $S^1(E_{a1})$ and $S^1(I_{a1})$, the multiplication must be performed symmetrically. $S^1(E_{a1})$ and $S^1(I_{a1})$, each consist of three terms, so the first terms, the second terms, and the third terms, respectively, must be multiplied. There is no cross-multiplication since each term represents a separate vector. It can be shown that

$$S^1(E_{a1}) \times S^1(I_{a1}) = S^2(E_{a1}I_{a1})$$

as follows:

$$S^1(E_{a1}) \times S^1(I_{a1}) = (E_{a1}, a^2E_{a1}, aE_{a1})(I_{a1}, a^2I_{a1}, aI_{a1})$$

Multiplying corresponding terms of the right-hand member.

First terms $E_{a1}I_{a1}$

Second terms, $a^2E_{a1} \times a^2I_{a1} = a^4E_{a1}I_{a1} = aE_{a1}I_{a1}$

Third terms, $a^2E_{a1}I_{a1}$

Therefore

$$\begin{aligned} S^1(E_{a1}) \times S^1(I_{a1}) &= (E_{a1}I_{a1}, aE_{a1}I_{a1}, a^2E_{a1}I_{a1}) \\ &= (1, a, a^2)E_{a1}I_{a1} \\ &= S^2(E_{a1}I_{a1}) \end{aligned} \quad (39)$$

The product of a positive-sequence and a negative-sequence operator produces a zero-sequence operator

$$S^1 \times S^2 = S^0$$

In the discussion, phase *a* has been used as the reference phase and all components expressed in terms of this phase. Phase *b* and *c* might just as well have been used as the reference. It should be understood that each of the following expressions also completely defines the system of vectors E_a, E_b, E_c with phase *b* and *c*, respectively, the reference.

$$S(E_b) = S^0(E_{b0}) + S^1(E_{b1}) + S^2(E_{b2}) \quad (40)$$

$$S(E_c) = S^0(E_{c0}) + S^1(E_{c1}) + S^2(E_{c2}) \quad (41)$$

For the general case of unsymmetrical currents flowing through

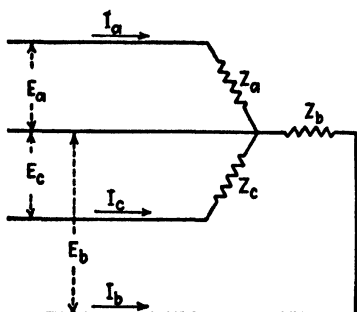


FIG. 7.—Unsymmetrical star-connected impedances with neutral return.

unsymmetrical impedances, the voltage drops can now be determined in terms of the symmetrical components, using the principles previously developed. Consider the three unequal star-connected impedances (Fig. 7) with neutral return. Three currents are assumed to flow in the three impedances and the problem is to determine the voltage drops E_a, E_b , and E_c . The expressions for the voltage

drops of the individual phases are

$$E_a = I_a Z_a \quad (42)$$

$$E_b = I_b Z_b \quad (43)$$

$$E_c = I_c Z_c \quad (44)$$

Expressing these relations by use of the sequence operators

$$\begin{aligned} S(Z_a I_a) &= S(Z_a) S(I_a) \\ &= [S^0(Z_{a0}) + S^1(Z_{a1}) + S^2(Z_{a2})][S^0(I_{a0}) + S^1(I_{a1}) \\ &\quad + S^2(I_{a2})] \end{aligned}$$

Expanding and applying the laws of exponents

$$\begin{aligned} S(Z_a I_a) &= S^0(Z_{a0} I_{a0} + Z_{a2} I_{a1} + Z_{a1} I_{a2}) \\ &\quad + S^1(Z_{a1} I_{a0} + Z_{a0} I_{a1} + Z_{a2} I_{a2}) \\ &\quad + S^2(Z_{a2} I_{a0} + Z_{a1} I_{a1} + Z_{a0} I_{a2}) \quad (45) \end{aligned}$$

This expression completely specifies the system of the three voltages E_a , E_b , and E_c .

In Eq. (45) Z_{a0} , Z_{a1} , and Z_{a2} have the following values:

$$Z_{a0} = \frac{1}{3}(Z_a + Z_b + Z_c) \quad (46)$$

$$Z_{a1} = \frac{1}{3}(Z_a + aZ_b + a^2Z_c) \quad (47)$$

$$Z_{a2} = \frac{1}{3}(Z_a + a^2Z_b + aZ_c) \quad (48)$$

These terms are impedances since they are the ratio of the impressed voltage and resulting current. The combination impedances Z_{a0} , Z_{a1} , and Z_{a2} are termed zero, positive and negative sequence respectively. These impedances are not zero, positive and negative in the sense that each should be used only with quantities of corresponding sequence but because they are evaluated from the unbalanced impedances by exactly the same process as used in determining the symmetrical components of a system of unbalanced voltages or currents.

An examination of Eq. (45) will show that each symmetrical component of current gives rise to symmetrical voltages of all three sequences. It is further observed that there is a certain sequence in the arrangement of terms producing the voltage drops.

Equation (45) can also be derived by substituting I_a , I_b , and I_c in terms of their symmetrical components in Eqs. (42), (43), and (44) and then finding the symmetrical components of the voltages E_a , E_b , and E_c by substituting the values of the voltages in Eqs. (31), (32), and (33).

A clearer conception of the physical significance of this expression can be obtained by assuming in Fig. 7 that only currents of zero sequence flow.¹ For this case the voltage drops are

$$S^0(Z_{a0}I_{a0}) + S^1(Z_{a1}I_{a0}) + S^2(Z_{a2}I_{a0})$$

If only currents of positive sequence flow, the drops are

$$S^0(Z_{a2}I_{a1}) + S^1(Z_{a0}I_{a1}) + S^2(Z_{a1}I_{a1})$$

If only currents of negative sequence flow, the drops are

$$S^0(Z_{a1}I_{a2}) + S^1(Z_{a2}I_{a2}) + S^2(Z_{a0}I_{a2})$$

An examination of these three expressions shows that with a flow of current of any sequence, there arises a voltage of the same sequence caused by the zero-sequence impedance. A

¹ WAGNER, C. F., and R. D. EVANS, *op. cit.*

current of positive sequence flowing through an impedance of positive sequence gives rise to a voltage of negative sequence, and a current of positive sequence flowing through an impedance of negative sequence gives rise to a voltage of zero sequence. It will be observed that the order of the voltage arising from a particular sequence current and impedance may be determined by applying the laws of exponents to the subscripts of the current and impedance.

In case the three impedances Z_a , Z_b , and Z_c are equal, a great simplification results:

$$\left. \begin{aligned} Z_a &= Z_b = Z_c = 0 \\ Z_{a0} &= Z \\ Z_{a1} &= 0 \\ Z_{a2} &= 0 \end{aligned} \right\} \quad (49)$$

Substituting Eq. (49) in Eq. (45) gives

$$S(Z_a I_a) = S^0(ZI_{a0}) + S^1(ZI_{a1}) + S^2(ZI_{a2})$$

Thus, it is evident that when the three phases are symmetrical, zero-sequence voltages are produced only by zero-sequence currents, positive-sequence voltages by positive-sequence currents and negative-sequence voltages by negative-sequence currents.

STAR AND DELTA CURRENTS AND VOLTAGES¹

The relations between star and delta currents and voltages may be found by using the relations developed in the preceding

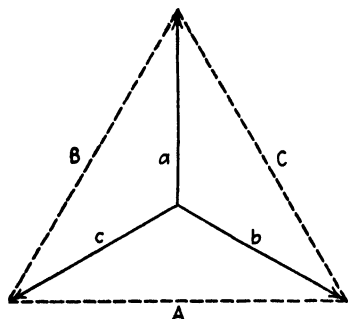


FIG. 8.—Relations between star and delta quantities.

paragraphs. Frequently, problems occur involving apparatus and circuits, such as delta-connected transformers, wherein both delta and star currents exist in different parts of the circuit and it is necessary for simplicity of analysis to convert from delta to star or vice versa. Before considering the conversions, it is well to specify the relations and nomenclature to be followed in designating star and delta systems.

A convenient graphical representation is shown in Fig. 8 and this will be adhered to in further

¹ WAGNER, C. F. and R. D. EVANS, *op. cit.*, March, 1928, p. 156.

discussion. The full lines indicate star values and the dotted lines delta values. It will be noted that a star component a and a delta component A are at right angles to each other. In what follows, small subscript letters indicate star and capital subscript letters delta quantities.

In Fig. 9 is shown the notation which will be used to represent the currents in a delta-connected system. The choice in

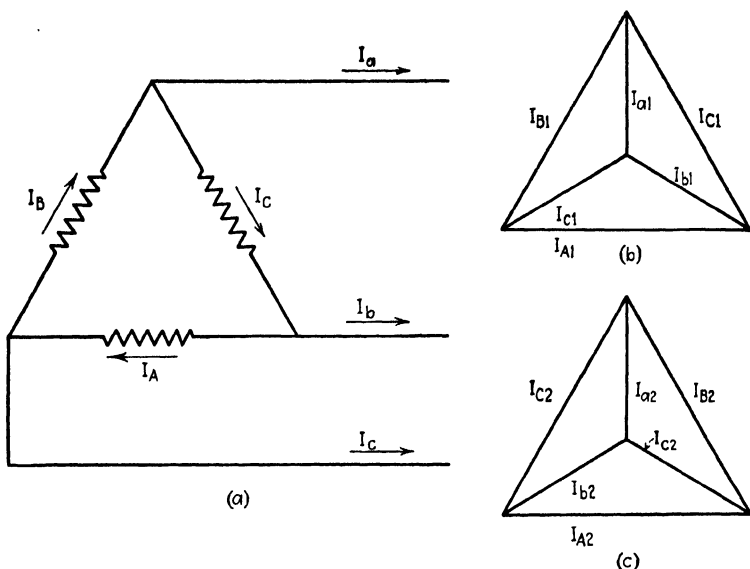


FIG. 9.—(a) Currents in a delta-connected system; (b) positive-sequence current relations; (c) negative-sequence current relations.

the direction of current flow is arbitrary. I_A , I_B , and I_C are the currents inside the delta and I_a , I_b , and I_c the currents in the lines. The system of line and delta currents may be expressed in terms of their sequence components in the usual manner:

$$S(I_a) = S^0(I_{a0}) + S^1(I_{a1}) + S^2(I_{a2}) \quad (50)$$

$$S(I_A) = S^0(I_{A0}) + S^1(I_{A1}) + S^2(I_{A2}) \quad (51)$$

It is evident that the following relations exist at the junction points in Fig. 9:

$$I_a = I_B - I_C$$

$$I_b = I_C - I_A$$

$$I_c = I_A - I_B$$

Or the system of currents I_a, I_b, I_c may be expressed:

$$S(I_a) = S(I_b) - S(I_c) \quad (52)$$

The two terms on the right of Eq. (52) may be expressed, using I_A as the first term, by means of the relations

$$S(I_b) = S^0(I_{a0}) + S^1(a^2 I_{a1}) + S^2(a I_{a2}) \quad (53)$$

$$S(I_c) = S^0(I_{a0}) + S^1(a I_{a1}) + S^2(a^2 I_{a2}) \quad (54)$$

Substituting the value of $S(I_a)$ from Eq. (50) into (52), we obtain

$$\begin{aligned} S^0(I_{a0}) + S^1(I_{a1}) + S^2(I_{a2}) &= S^0(I_{A0}) + S^1(a^2 I_{A1}) \\ &\quad + S^2(a I_{A2}) - S^0(I_{A0}) - S^1(a I_{A1}) - S^2(a^2 I_{A2}) \end{aligned}$$

Collecting the terms having the same sequence operator:

$$S^0(I_{a0}) = S^0(I_{A0} - I_{A0}) = 0$$

$$S^1(I_{a1}) = S^1(a^2 I_{A1} - a I_{A1}) = S^1(-j\sqrt{3} I_{A1})$$

$$S^2(I_{a2}) = S^2(a I_{A2} - a^2 I_{A2}) = S^2(+j\sqrt{3} I_{A2})$$

The conversion may now be written:

$$\left\{ \begin{array}{ll} I_{a0} = 0 & I_{A0} = \text{indeterminate} \\ I_{a1} = -j\sqrt{3} I_{A1} & I_{A1} = +\frac{j}{\sqrt{3}} I_{a1} \\ I_{a2} = +j\sqrt{3} I_{A2} & I_{A2} = -\frac{j}{\sqrt{3}} I_{a2} \end{array} \right\} \quad (55)$$

The graphical representation of the relations of Eq. (55) is shown in Fig. 9b and c.

In a similar manner the relations between star and delta voltages may be obtained. The notation used to represent the voltages in a star-connected system is shown in Fig. 10.

The star and delta systems are given by the relations

$$S(E_a) = S^0(E_{a0}) + S^1(E_{a1}) + S^2(E_{a2}) \quad (56)$$

$$S(E_A) = S^0(E_{A0}) + S^1(E_{A1}) + S^2(E_{A2}) \quad (57)$$

The directions of the voltage vectors are purely arbitrary. We shall choose the relations between the star and delta voltages as follows:

$$E_A = E_c - E_b$$

$$E_B = E_a - E_c$$

$$E_C = E_b - E_a$$

or

$$S(E_A) = S(E_c) - S(E_b) \quad (58)$$

The two terms on the right of Eq. (58) may be expressed using E_a as the first term from the relations in (53) and (54) and substituting the value of $S(E_A)$ from (57) gives

$$\begin{aligned} S^0(E_{A0}) + S^1(E_{A1}) + S^2(E_{A2}) &= S^0(E_{a0}) + S^1(aE_{a1}) + S^2(a^2E_{a2}) \\ &\quad - S^0(E_{a0}) - S^1(a^2E_{a1}) - S^2(aE_{a2}) \end{aligned}$$

Collecting terms having the same sequence operator:

$$\left\{ \begin{array}{ll} E_{a0} = \text{indeterminate} & E_{A0} = 0 \\ E_{a1} = -\frac{j}{\sqrt{3}}E_{A1} & E_{A1} = +j\sqrt{3}E_{a1} \\ E_{a2} = +\frac{j}{\sqrt{3}}E_{A2} & E_{A2} = -j\sqrt{3}E_{a2} \end{array} \right\} \quad (59)$$

The graphical representation of the relations given in Eq. (59) is shown in Fig. 10b and c.

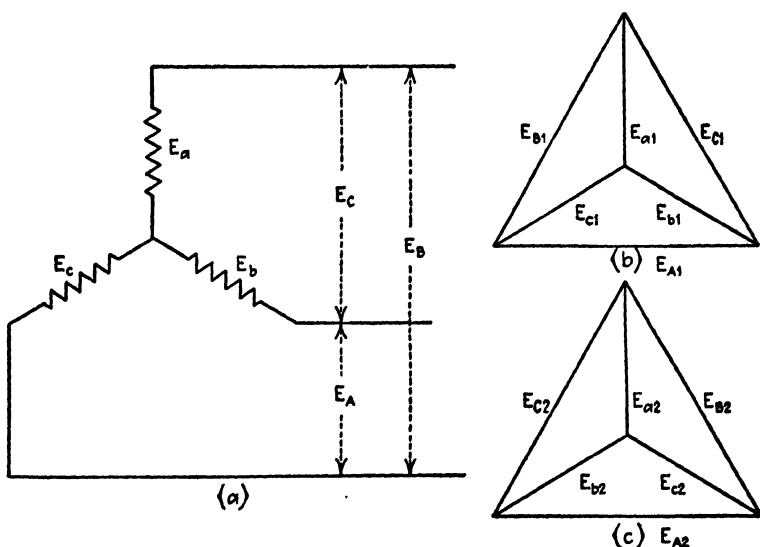


FIG. 10.—(a) Voltages in a star-connected system; (b) positive-sequence voltage relations; (c) negative-sequence voltage relations.

From an inspection of the relations expressed in Eq. (55), it is evident that no currents of zero sequence can flow in lines

connected to a delta. Furthermore, there can be no zero-sequence currents flowing in the lines to a star connection without a neutral connection since the vector sum of the currents must always be zero. There may be currents of zero sequence circulating inside the delta, and, therefore, the zero-sequence currents in the delta cannot be determined from the star currents; however, the delta currents may be determined from other known conditions.

An examination of the relations in Eq. (59) will show that no voltages of zero sequence can exist in the delta voltages, since the vector sum must always be zero. Voltages of zero sequence may exist in the star connection, but it is clear that the zero-sequence voltages in a star cannot be determined from the delta voltages.

Current and Voltage Relations in Unsymmetrical Circuits.—

In a given circuit two equations may be written for the voltage of any phase at any particular point in the circuit; for example, at the terminals of the load. For this point in the circuit, one of the equations would be for the voltage drop from the source of potential to the load; the other would be the equation for the voltage drop in the load. If the circuit is unsymmetrical, the voltage of each phase, at the point considered, may be resolved into its symmetrical components. There will thus be obtained three equations, one for each phase sequence, corresponding to each of the two above mentioned equations. Since, in a general unsymmetrical circuit, each phase-sequence voltage drop is composed of drops due to all three sequence currents, the equation for each phase-sequence voltage will be in terms of the three unknown sequence currents. Two of the unknown sequence currents may be eliminated from any one of the voltage equations. This gives two equations for each phase-sequence voltage, one for the drop on each side of the point considered, each in terms of drops due only to its corresponding sequence current. Equating the two expressions for the voltage at the point considered gives an equation involving only one sequence current, the corresponding sequence component of applied voltage which is known, and the sequence components of the circuit impedances, also known. This equation may be solved for the sequence current. Having found one phase-sequence current, the other two are easily

obtained by substituting in equations which were developed in obtaining the first component.

In order to illustrate the application of the relations which have been developed, a solution will be obtained for the currents in the unsymmetrical circuit shown in Fig. 11 in terms of the circuit constants and bus voltage.¹ An unsymmetrical star load is connected to the secondary of an unsymmetrical bank of three single-phase delta-delta connected transformers through lines of negligible impedance. The impedances of the three

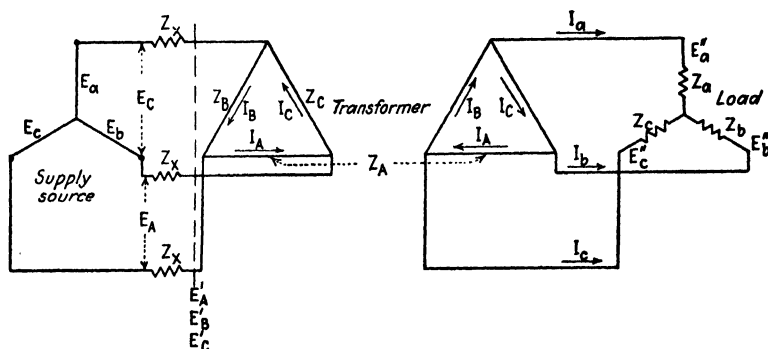


FIG. 11.—Unsymmetrical star-connected impedance load supplied from an unsymmetrical bank of transformers.

transformers Z_A , Z_B , and Z_C are unequal, and likewise the load impedances Z_a , Z_b , and Z_c are unequal. For simplicity, a one-to-one ratio of transformation is assumed. Current is supplied to the primaries of the transformers from an infinite bus through lines having symmetrical impedances Z_x per phase. At the bus, the voltages E_a , E_b , E_c are sustained and balanced.

The positive-sequence voltage at the terminals of the load is equal to the positive-sequence voltage minus the positive-sequence voltage drop from the bus to the load terminals. Since the impedance Z_x in each phase is symmetrical, a voltage drop of a particular sequence will be produced only by a current of that sequence flowing through Z_x . Therefore, the positive-sequence drop through Z_x is

$$-Z_x I_{a1}$$

¹ Discussion of Fortescue, *op. cit.*, by J. Slepian, *Trans. A.I.E.E.*, vol. 37, p. 1160.

Converting this from a star to a delta voltage drop by means of Eqs. (55) and (59) gives for the delta drop $-3Z_z I_{A1}$. The impedance $3Z_z$ is the delta equivalent of the star impedance Z_z . We may write the equations for the positive- and negative-sequence voltages at the terminals of the load in terms of the drops from the bus to the load.

$$E''_{A1} = E_{A1} - 3Z_z I_{A1} - (Z_{A1} I_{A0} + Z_{A0} I_{A1} + Z_{A2} I_{A2}) \quad (60)$$

$$E''_{A2} = E_{A2} - 3Z_z I_{A2} - (Z_{A2} I_{A0} + Z_{A1} I_{A1} + Z_{A0} I_{A2}) \quad (61)$$

Also, it is evident that

$$\left. \begin{aligned} E_{A2} &= 0 \\ I_{A0} &= 0 \end{aligned} \right\} \quad (62)$$

The voltages at the terminals of the load are also equal to the drop through the load impedances Z_a , Z_b , Z_c . Remembering that all terms involving I_{A0} drop out since $I_{A0} = 0$:

$$E''_{a1} = Z_{a0} I_{a1} + Z_{a2} I_{a2} \quad (63)$$

$$E''_{a2} = Z_{a1} I_{a1} + Z_{a0} I_{a2} \quad (64)$$

$$E''_{a0} = Z_{a2} I_{a1} + Z_{a1} I_{a2} \quad (65)$$

Converting these expressions into terms of delta quantities by Eqs. (55) and (59)

$$E''_{A1} = (+j\sqrt{3})(Z_{a0} I_{a1}) + (+j\sqrt{3})(Z_{a2} I_{a2}) = 3Z_{a0} I_{A1} - 3Z_{a2} I_{A2} \quad (66)$$

$$E''_{A2} = (-j\sqrt{3})(Z_{a1} I_{a1}) + (-j\sqrt{3})(Z_{a0} I_{a2}) = -3Z_{a1} I_{A1} + 3Z_{a0} I_{A2} \quad (67)$$

$$E''_{A0} = 0 \quad (68)$$

Equating the right-hand members of Eqs. (60) and (66), and of Eqs. (61) and (67), and solving for the voltages on the bus side of the transformer,

$$E_{A1} = 3Z_{a0} I_{A1} - 3Z_{a2} I_{A2} + 3Z_z I_{A1} + (Z_{A1} I_{A0} + Z_{A0} I_{A1} + Z_{A2} I_{A2})$$

$$E_{A2} = -3Z_{a1} I_{A1} + 3Z_{a0} I_{A2} + 3Z_z I_{A2} + (Z_{A2} I_{A0} + Z_{A1} I_{A1} + Z_{A0} I_{A2})$$

Also

$$E_{A0} = 0 = (Z_{A0} I_{A0} + Z_{A2} I_{A1} + Z_{A1} I_{A2}) \quad (69)$$

Therefore,

$$I_{A0} = -\frac{Z_{A2} I_{A1} + Z_{A1} I_{A2}}{Z_{A0}} \quad (70)$$

Eliminating I_{A0} by Eq. (69), and simplifying,

$$E_{A1} = \left[3Z_{a0} + 3Z_s + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}} \right] I_{A1} + \left[Z_{A2} - 3Z_{a2} - \frac{Z_{A1}^2}{Z_{A0}} \right] I_{A2} \quad (71)$$

$$E_{A2} = 0 = \left[Z_{A1} - 3Z_{a1} - \frac{Z_{A2}^2}{Z_{A0}} \right] I_{A1} + \left[3Z_{a0} + 3Z_s + Z_{A0} - \frac{Z_{A2}Z_{A1}}{Z_{A0}} \right] I_{A2} \quad (72)$$

Solving Eq. (72) for I_{A2} and substituting its value in (71) and solving for I_{A1} gives

$$I_{A1} = E_{A1} \left[\frac{3Z_{a0} + 3Z_s + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}}}{\left(3Z_{a0} + 3Z_s + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}} \right)^2 - \left(Z_{A2} - 3Z_{a2} - \frac{Z_{A1}^2}{Z_{A0}} \right) \left(Z_{A1} - 3Z_{a1} - \frac{Z_{A2}^2}{Z_{A0}} \right)} \right] \quad (73)$$

Substituting this value of I_{A1} in the equation for I_{A2} obtained from Eq. (72) gives

$$I_{A2} = E_{A1} \left[\frac{-\left(Z_{A1} - 3Z_{a1} - \frac{Z_{A2}^2}{Z_{A0}} \right)}{K_1} \right] \quad (74)$$

where K_1 is the same denominator as in (73).

The zero-sequence current circulating in the delta-connected transformers can now be found by substituting the values of I_{A1} and I_{A2} from Eqs. (73) and (74) in (70).

The total current I_A in the transformer is obtained by the relation

$$I_A = I_{A0} + I_{A1} + I_{A2} \quad (75)$$

The system of currents I_A , I_B , and I_C in the transformers are completely specified by the equation

$$SI_A = S^0 I_{A0} + S^1 I_{A1} + S^2 I_{A2} \quad (76)$$

It is obvious that the line currents in Fig. 11 are the same on both sides of the transformer bank if the magnetizing current is neglected. The line currents can be determined from the delta currents by applying the delta-star transformation relations, equation (55).

Equations (71), (72), (73), and (74) are general in that no restrictions are placed upon the values of the load and transformer constants. For example, consider the case of a line-to-line fault between phases *b* and *c* at the load.

For this case

$$Z_b = Z_c = 0 \quad \text{and} \quad Z_{a1} = Z_{a2} = Z_{a0} = \frac{Z_a}{3}$$

Substituting in Eq. (73) gives

$$I_{A1} = \frac{Z_a + 3Z_x + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}}}{\left(Z_a + 3Z_x + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}}\right)^2 - \left(Z_{A2} - Z_a - \frac{Z_{A1}^2}{Z_{A0}}\right)\left(Z_{A1} - Z_a - \frac{Z_{A2}^2}{Z_{A0}}\right)} E_{A1} \quad (77)$$

Similarly, substituting in (74) gives

$$I_{A2} = E_{A1} \frac{Z_a - Z_{A1} + \frac{Z_{A2}^2}{Z_{A0}}}{K_2} \quad (78)$$

where K_2 is the same denominator as in Eq. (77)

If, before the line-to-line fault occurred, the circuit was unloaded then $Z_a = \infty$. For this case, Eq. (77) for I_{A1} takes the form ∞/∞ . In order to evaluate this expression it is necessary to differentiate the numerator with respect to Z_a to obtain a new numerator, and the denominator to obtain a new denominator. The derivative of the numerator with respect to Z_a is unity. Differentiating the denominator with respect to Z_a gives

$$\frac{Z_{A0}(6Z_x + 2Z_{A0} + Z_{A2} + Z_{A1}) - (Z_{A1} + Z_{A2})^2}{Z_{A0}}$$

Hence, for $Z_a = \infty$, Eq. (77) becomes

$$I_{A1} = \frac{Z_{A0}E_{A1}}{Z_{A0}(6Z_x + 2Z_{A0} + Z_{A2} + Z_{A1}) - (Z_{A1} + Z_{A2})^2} \quad (79)$$

Equation (78) reduces to the same expression as (79). Therefore,

$$I_{A2} = I_{A1} \quad (80)$$

Since the impedance Z_a is infinite, the line current I_a must be zero. From Eq. (55),

$$I_{A1} = +\frac{j}{\sqrt{3}}I_{a1}$$

$$I_{A2} = -\frac{j}{\sqrt{3}}I_{a2}$$

Substituting these relations in (80),

$$I_{a2} = -I_{a1}$$

The total current $I_a = I_{a1} + I_{a2} = 0$, since the zero-sequence line current I_{a0} must be zero, thus satisfying the conditions for the open phase.

If the transformer impedances are symmetrical,

$$Z_A = Z_B = Z_C$$

and

$$Z_{A0} = Z_A \quad Z_{A1} = 0 \quad Z_{A2} = 0$$

For this case, Eq. (79) reduces to

$$I_{A1} = \frac{Z_A E_{A1}}{Z_A(6Z_x + 2Z_A)} = \frac{E_{A1}}{2(3Z_x + Z_A)} \quad (81)$$

$$I_{A2} = I_{A1}$$

An inspection of Eq. (70) will show that, when the transformer impedances are symmetrical, the zero-sequence delta current I_{A0} becomes zero.

Converting Eq. (81) into terms of star currents,

$$I_{a1} = \frac{E_{a1}}{2Z_x + \frac{2}{3}Z_A} \quad (82)$$

The impedance $\frac{Z_A}{3}$ is simply the line to neutral impedance of the transformer. If $Z'_a = \frac{Z_A}{3}$, Eq. (82) becomes $I_{a1} = \frac{E_{a1}}{2(Z_x + Z'_a)}$.

Another case worthy of investigation is that of a three-phase short circuit at the load where $Z_a = Z_b = Z_c = 0$.

Substituting in equation (73) gives

$$I_{A1} = E_{A1} \left[\frac{3Z_x + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}}}{\left((3Z_x + Z_{A0} - \frac{Z_{A1}Z_{A2}}{Z_{A0}})^2 - \left(Z_{A2} - \frac{Z_{A1}^2}{Z_{A0}} \right) \left(Z_{A1} - \frac{Z_{A2}^2}{Z_{A0}} \right) \right)} \right] \quad (83)$$

If the transformer impedances are symmetrical, Eq. (83) reduces to

$$I_{A1} = E_{A1} \left[\frac{3Z_x + Z_A}{(3Z_x + Z_A)^2} \right] = \frac{E_{A1}}{3Z_x + Z_A} \quad (84)$$

Also,

$$I_{A2} = 0 \quad I_{A0} = 0$$

Applying a delta-star transformation,

$$I_{a1} = \frac{E_{a1}}{Z_x + \frac{Z_A}{3}} \quad (85)$$

$$I_a = I_{a1}$$

$$I_b = a^2 I_{a1}$$

$$I_c = a I_{a1}$$

Open-delta Transformers.—A case which frequently occurs is that of the open delta (Fig. 12a). This figure is, in all respects,

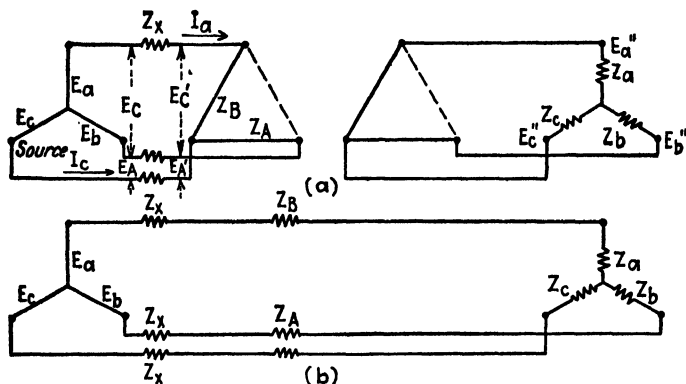


FIG. 12.—(a) Unsymmetrical star-connected impedance load supplied from a bank of open delta-connected transformers; (b) equivalent representation of transformer impedances.

the same as Fig. 11 except that phase *c* transformer is removed. This is an unsymmetrical connection, irrespective of whether the two transformers are duplicate or not. A little thought will show that the representation of Fig. 12b is the same as Fig. 12a. Current flowing in phases *a* and *c* encounter the impedance Z_B , and current flowing in phases *b* and *c* encounter the impedance Z_A . The impedance Z_C corresponding to Z_A and Z_B is zero.

Resolving the impedances of the open-delta bank into zero-, positive- and negative-sequence impedances,

$$Z_{A0} = \frac{Z_A + Z_B}{3} \quad (86)$$

$$Z_{A1} = \frac{Z_A + aZ_B}{3} \quad (87)$$

$$Z_{A2} = \frac{Z_A + a^2Z_B}{3} \quad (88)$$

The voltages at the terminals of the load, in terms of the drops from the bus to the load, are:

$$E''_{a1} = E_{a1} - Z_x I_{a1} - (Z_{A0} I_{a1} + Z_{A2} I_{a2}) \quad (89)$$

$$E''_{a2} = E_{a2} - Z_x I_{a2} - (Z_{A1} I_{a1} + Z_{A0} I_{a2}) \quad (90)$$

and in terms of the drops in the load

$$E''_{a1} = Z_{a0} I_{a1} + Z_{a2} I_{a2} \quad (91)$$

$$E''_{a2} = Z_{a1} I_{a1} + Z_{a0} I_{a2} \quad (92)$$

$$E''_{a0} = Z_{a2} I_{a1} + Z_{a1} I_{a2} \quad (93)$$

Equation (93) gives the displacement of the neutral due to the unsymmetrical load.

Equating the right-hand members of Eqs. (89) and (91) and of (90) and (92), and remembering that $E_{a2} = 0$ and $I_{a0} = 0$,

$$E_{a1} = (Z_x + Z_{A0} + Z_{a0}) I_{a1} + (Z_{A2} + Z_{a2}) I_{a2} \quad (94)$$

$$0 = (Z_x + Z_{A0} + Z_{a0}) I_{a2} + (Z_{A1} + Z_{a1}) I_{a1} \quad (95)$$

From Eq. (95),

$$I_{a2} = \frac{-(Z_{A1} + Z_{a1}) I_{a1}}{(Z_x + Z_{A0} + Z_{a0})} \quad (96)$$

Substituting Eq. (96) in (94) and solving for I_{a1} gives

$$I_{a1} = \frac{E_{a1}(Z_x + Z_{A0} + Z_{a0})}{(Z_x + Z_{A0} + Z_{a0})^2 - (Z_{A2} + Z_{a2})(Z_{A1} + Z_{a1})} \quad (97)$$

Substituting for I_{a1} in (96),

$$I_{a2} = \frac{-E_{a1}(Z_{A1} + Z_{a1})}{(Z_x + Z_{A0} + Z_{a0})^2 - (Z_{A2} + Z_{a2})(Z_{A1} + Z_{a1})} \quad (98)$$

For a line-to-line short circuit between phases b and c , *i.e.*, a closed phase,

$$Z_b = Z_c = 0 \quad \text{and} \quad Z_{a1} = Z_{a2} = Z_{a0} = \frac{Z_a}{3}$$

Substituting in (97) and (98)

$$I_{a1} = \frac{E_{a1} \left(Z_x + Z_{A0} + \frac{Z_a}{3} \right)}{\left(Z_x + Z_{A0} + \frac{Z_a}{3} \right)^2 - \left(Z_{A2} + \frac{Z_a}{3} \right) \left(Z_{A1} + \frac{Z_a}{3} \right)} \quad (99)$$

$$I_{a2} = \frac{-E_{a1} \left(Z_{a1} + \frac{Z_a}{3} \right)}{\left(Z_x + Z_{A0} + \frac{Z_a}{3} \right)^2 - \left(Z_{A2} + \frac{Z_a}{3} \right) \left(Z_{A1} + \frac{Z_a}{3} \right)} \quad (100)$$

If the transformer was unloaded at the time of the fault, $Z_a = \infty$.

In order to evaluate the right-hand members of Eqs. (99) and (100), it is necessary to eliminate Z_a by differentiating the numerators and denominators with respect to Z_a .

After performing this operation these equations become

$$I_{a1} = \frac{E_{a1}}{2(Z_x + Z_{A0}) - (Z_{A2} + Z_{A1})} \quad (101)$$

$$I_{a2} = \frac{-E_{a1}}{2(Z_x + Z_{A0}) - (Z_{A2} + Z_{A1})} = -I_{a1} \quad (102)$$

If the two transformers are similar, $Z_A = Z_B = Z$ and from Eqs. (86) to (88)

$$Z_{A0} = \frac{2Z}{3} \quad (103)$$

$$Z_{A1} = \frac{(1 + a)Z}{3} \quad (104)$$

$$Z_{A2} = \frac{(1 + a^2)Z}{3} \quad (105)$$

Substituting these values in (101),

$$I_{a1} = \frac{E_{a1}}{2Z_x + Z} \quad (106)$$

$$I_{a2} = -I_{a1} \quad (107)$$

$$I_b = I_{b1} + I_{b2} = \frac{-E_A}{2Z_x + Z} \quad (108)$$

If, in Fig. 12, transformer A was removed, instead of C , Eqs. (86) to (88) become

$$Z_{A0} = \frac{2Z}{3} \quad (109)$$

$$Z_{A1} = Z_{A2} = \frac{(a + a^2)Z}{3} \quad (110)$$

Substituting these values in Eq. (101) and simplifying,

$$I_{a1} = \frac{E_{a1}}{2(Z_x + Z)} \quad (111)$$

From Eq. (102),

$$I_{a2} = -I_{a1} \quad (112)$$

Equations (111) and (112) are for a line-to-line fault across the open side of the delta.

Voltage and Current Relations in Star-delta Transformers.—In the example considered, a one-to-one ratio of transformation was assumed. If any ratio other than one to one is present, it is necessary to introduce the transformer ratio in determining the actual or sequence components of currents and voltages on one side of the transformers from those on the other. With star-star or delta-delta connected transformers, the currents in

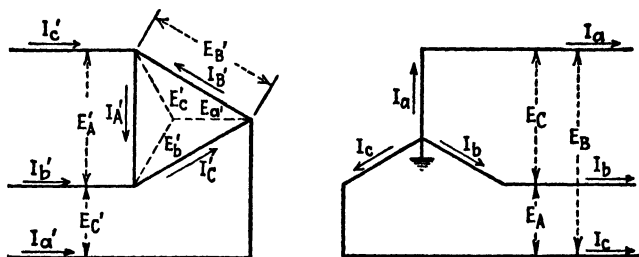


FIG. 13.—Current and voltage relations in a star-delta connected transformer.

the low side are simply equal to the currents in the high side multiplied by the turn ratio of the two windings, and the low-side voltage is equal to the high-side voltage divided by the turn ratio. In the star-delta connection, there is a difference in both magnitude and phase angle between the currents and voltages on one side and those on the other. It is, therefore, necessary to have some convention for representing the relations of quantities involving star-delta transformations. In Fig. 13 is shown the convention adopted, which is the same as that in Fig. 9. The symbols bearing subscripts with single prime refer to the delta side. The magnetizing current is neglected. Thus, the transformer impedance is considered as a series impedance on the voltage base of the winding used in calculating the conditions desired. The turn-ratio constant can then be applied directly

to currents and voltages without the complication of involving the impedance in the expressions.

The assumption will be made that the star winding is the high-voltage side. Let n be the actual turn ratio for the transformers. This is the ratio of the turns on one leg of the star winding to those on one leg of the delta winding.

Determination of Line Currents on the Delta Side from Line Currents on the Star Side.—From Fig. (13),

$$I'_a = I'_b - I'_c = n(I_b - I_c) \quad (113)$$

$$I'_b = I'_c - I'_a = n(I_c - I_a) \quad (114)$$

$$I'_c = I'_a - I'_b = n(I_a - I_b) \quad (115)$$

Equations (113), (114), and (115) hold equally well for the sequence components of currents; therefore,

$$I'_{a1} = n(I_{b1} - I_{c1}) = n(a^2 - a)I_{a1} = -j\sqrt{3}I_{a1} \quad (116)$$

$$I'_{a2} = n(I_{b2} - I_{c2}) = n(a - a^2)I_{a2} = j\sqrt{3}I_{a2} \quad (117)$$

$$I'_{a0} = n(I_{b0} - I_{c0}) = n(1 - 1)I_{a0} = 0 \quad (118)$$

The currents on the star side, in terms of those on the delta side, may now be found:

$$I_{a1} = \frac{j}{n\sqrt{3}}I'_{a1} \quad (119)$$

$$I_{a2} = -\frac{j}{n\sqrt{3}}I'_{a2} \quad (120)$$

$$I_{a0} = \text{indeterminate} \quad (121)$$

The determination of line to neutral voltages on the delta side, in terms of those on star side, is similar:

$$E'_{A1} = \frac{E_{a1}}{n} \quad (122)$$

$$E'_{A1} = E'_{c1} - E'_{b1} = (a - a^2)E'_{a1} \quad (123)$$

Substituting value of E'_{A1} from Eq. (123) in (122),

$$E'_{a1} = -j\frac{E_{a1}}{n\sqrt{3}} \quad (124)$$

Also

$$E'_{a2} = j\frac{E_{a2}}{n\sqrt{3}} \quad (125)$$

$$E'_{a0} = 0 \quad (126)$$

Conversely:

$$E_{a1} = jn\sqrt{3}E'_{a1} \quad (127)$$

$$E_{a2} = -jn\sqrt{3}E'_{a2} \quad (128)$$

$$E_{a0} = \text{indeterminate} \quad (129)$$

The line-voltage relations are

$$E_A = jn\sqrt{3}E'_A$$

$$E_{A1} = jn\sqrt{3}E'_{A1}$$

$$E_{A2} = -jn\sqrt{3}E'_{A2}$$

$$E_{A0} = 0$$

By means of these relations the phase position and magnitude of currents and voltages may be traced, step by step, through any number of transformations that may be encountered. The values of either the actual or the sequence currents and voltages may be used in the transformations. It is frequently desirable to know the magnitude and phase relations of the symmetrical components in all parts of the circuit. For such cases, the conversion expressions will be found convenient.

Unsymmetrical Load on Generator.—The analysis of circuits involving unsymmetrical quantities by the method of sym-

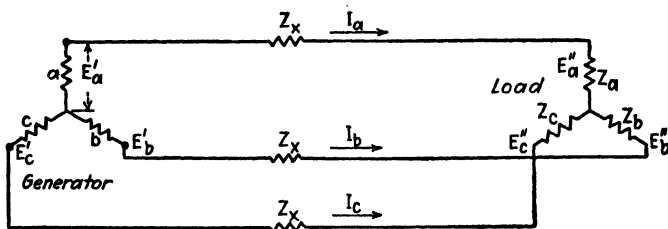


FIG. 14.—Unsymmetrical star-connected load supplied from a symmetrical synchronous generator.

metrical components is not limited to static circuits. A case will now be considered involving a generator supplying an unbalanced star-connected load.¹ The generator is assumed to have a damper winding, such that both stator and rotor may be considered symmetrical. The generator has an impedance Z_1 to positive-sequence currents and an impedance Z_2 to negative-sequence currents. The connections are shown in Fig. 14. The generated internal voltages are E_a , E_b , and E_c . Since the generator is symmetrical, these voltages are symmetrical and

¹Wagner C. F. and R. D. Evans, *op. cit.*

are the positive-sequence-generated voltages. The generated negative- and zero-sequence voltages are, of course, both zero.

The total voltage drop of any sequence must be equal to the generated voltage of that sequence. The following expressions may be written

$$E_{a1} = Z_1 I_{a1} + Z_x I_{a1} + (Z_{a0} I_{a1} + Z_{a2} I_{a2}) \quad (130)$$

$$E_{a2} = 0 = Z_2 I_{a2} + Z_x I_{a2} + (Z_{a1} I_{a1} + Z_{a0} I_{a2}) \quad (131)$$

Solving for I_{a2} in (131),

$$I_{a2} = -\frac{Z_{a1} I_{a1}}{Z_2 + Z_x + Z_{a0}} \quad (132)$$

Substituting (132) in (130),

$$E_{a1} = I_{a1} \left(Z_1 + Z_x + Z_{a0} - \frac{Z_{a2} Z_{a1}}{Z_2 + Z_x + Z_{a0}} \right) \quad (133)$$

$$I_{a1} = E_{a1} \left(\frac{1}{Z_1 + Z_x + Z_{a0} - \frac{Z_{a2} Z_{a1}}{Z_2 + Z_x + Z_{a0}}} \right) \quad (134)$$

Substituting value of I_{a1} from (134) in (132),

$$I_{a2} = -E_{a1} \left[\frac{\frac{Z_{a1}}{Z_2 + Z_x + Z_{a0}}}{Z_1 + Z_x + Z_{a0} - \frac{Z_{a2} Z_{a1}}{Z_2 + Z_x + Z_{a0}}} \right] \quad (135)$$

The symmetrical component voltages at the terminals of the generators are

$$\left. \begin{aligned} E'_{a1} &= E_{a1} - I_{a1} Z_1 \\ E'_{a2} &= 0 - I_{a2} Z_2 \end{aligned} \right\} \quad (136)$$

The phase voltage is the sum of the components:

$$E'_a = E_{a1} - I_{a1} Z_1 - I_{a2} Z_2 \quad (137)$$

The voltages at the terminals of the load are

$$\left. \begin{aligned} E''_{a1} &= Z_{a0} I_{a1} + Z_{a2} I_{a2} \\ E''_{a2} &= Z_{a1} I_{a1} + Z_{a0} I_{a2} \\ E''_{a0} &= Z_{a2} I_{a1} + Z_{a1} I_{a2} \end{aligned} \right\} \quad (138)$$

$$E''_a = (Z_{a0} + Z_{a1} + Z_{a2}) I_{a1} + (Z_{a0} + Z_{a1} + Z_{a2}) I_{a2} \quad (139)$$

The voltage of the other phases may be found by means of the fundamental resolution equations.

The equations developed are perfectly general and hold for any load-impedance constants. Consider the case of a line-to-line

short circuit between phases *b* and *c* at the load. For this condition,

$$Z_b = Z_c = 0$$

Therefore,

$$Z_{a0} = Z_{a1} = Z_{a2} = \frac{Z_a}{3}$$

Substituting these relations in Eqs. (134) and (135) and remembering that $E_{a1} = E_a$,

$$I_{a1} = E_a \left[\frac{1}{Z_1 + Z_2 + \frac{Z_a}{3} - \left(\frac{Z_a^2}{9Z_2 + 9Z_2 + 3Z_a} \right)} \right] \quad (140)$$

$$I_{a2} = -E_a \left[\frac{\frac{Z_a}{3Z_2 + 3Z_2 + Z_a}}{K_2} \right] \quad (141)$$

where K_2 is the same denominator as in (140).

If the generator is unloaded at the instant of fault, $Z_a = \infty$.

Reducing Eq. (140) to a simple fraction and differentiating the numerator and denominator with respect to Z_a , the equations reduce to

$$I_{a1} = E_a \left[\frac{1}{Z_1 + Z_2 + 2Z_x} \right] \quad (142)$$

$$I_{a2} = -I_{a1} \quad (143)$$

$$I_a = I_{a1} + I_{a2} = 0 \quad (144)$$

$$I_b = a^2 I_{a1} + a I_{a2} = \frac{(a^2 - a)E_a}{Z_1 + Z_2 + 2Z_x} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2 + 2Z_x} \quad (145)$$

$$I_c = a I_{a1} + a^2 I_{a2} = \frac{(a - a^2)E_a}{Z_1 + Z_2 + 2Z_x} = \frac{j\sqrt{3}E_a}{Z_1 + Z_2 + 2Z_x} \quad (146)$$

$$I_b = -I_c$$

General Unsymmetrical Network.—Thus far, the only circuit constants considered were simple self-impedances. In practically all polyphase circuits, the phases have mutual coupling. The mutual coupling between conductors in a transmission line is usually large, but it is canceled by the method of calculating the self-inductance. The coupling between phases, in polyphase rotating machines and transformers is large.

It has been shown that when the self-impedances are symmetrical in the phases of a three-phase circuit, the voltage drops

of a particular sequence are caused only by currents of corresponding sequence. It can be shown that for a general static circuit containing both self and mutual impedances, as long as these impedances are symmetrical, the three sequences may be treated independently. This results in a simplification which greatly increases the usefulness of the method of symmetrical components.

The three-phase system may be replaced by three independent single-phase systems, as shown in Fig. 15. The return con-

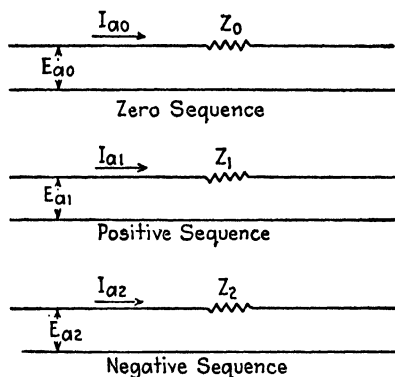


FIG. 15.—Representation of a three-phase system by three independent single-phase systems.

ductors have no impedance, so the impedances Z_0 , Z_1 , and Z_2 may be used in single-line diagrams.

It should be observed that the meaning of the terms Z_0 , Z_1 , and Z_2 given here, when considering a symmetrical system, is different from the same term when considering a system with unsymmetrical impedances. If the distinction between a system containing unsymmetrical impedances and one containing only symmetrical impedances is kept in mind, no ambiguity should result from using the same terminology. This notation will be employed henceforth.

CHAPTER III

CALCULATION OF FAULT CURRENTS AND VOLTAGES

In calculating short-circuit currents and voltages the assumption is generally made that the impedances of the three phases are symmetrical. This includes both self and mutual impedances in each phase, embracing resistance, inductance, and capacitance. It is a practical impossibility to secure perfect phase symmetry, but it is only in special cases that such an assumption would lead to excessive error. In order that phase symmetry may exist, the self and mutual impedances per phase of electrical apparatus must be symmetrical. Phase symmetry in a transmission circuit is secured by transposition of the conductors. This practice is generally resorted to in reducing telephone interference effects. Fortunately, the assumption of phase symmetry is usually permissible, thus greatly simplifying the problem of fault calculations and allowing the application of the method of symmetrical components as a simple practical working tool.

In the calculation of unbalanced faults the system is assumed to be symmetrical up to the point of fault. The fault is the origin of unsymmetrical voltage and current relations. A maximum of three networks is involved in the calculation of unbalanced faults, *viz.*, the positive, negative, and zero sequence. The combinations to be used depend upon the type of fault under consideration. The three networks comprising the fault system have been shown to be independent of each other. The relations of the networks and the connections between them depend upon the particular type of fault and are determined by the terminal conditions at the point of fault. Before proceeding with the derivation of the methods of calculation for the various types of faults, it will be well to describe briefly the characteristics of the three networks.

The positive sequence is the ordinary system impedance network used in the calculation of system performance and three-phase short circuits. The resistances and reactances are the same values to neutral as used in calculating regulation.

The only generated e.m.f. is of positive sequence. The individual synchronous machines are sources of positive-sequence voltages whose magnitude and phase position at the instant of fault depend upon the real and reactive power distribution. The positive-sequence voltage at any point in the network, under fault conditions, is equal to the generated positive-sequence voltage minus the voltage drop due to positive-sequence current flowing through the positive-sequence network. There can be no ground currents in the positive-sequence system, because the system is balanced. Since, in a symmetrical system there is no interaction between the three sequence components and only internal generated voltages of positive sequence exist, the only currents which can react upon the air-gap flux in machines to produce a change in the generated voltage are of positive sequence. This point is of importance in the calculation of transient fault currents.

The negative system of components is a balanced system of opposite-phase sequence to the positive components. Since the negative components are balanced, no ground currents can appear in the negative-sequence network. The negative-sequence network is physically the same as the positive system, differing only in the constants of rotating apparatus. The negative-sequence reactance of transmission circuits and static apparatus, such as transformers, is the same as the positive. The negative-sequence reactance of induction and synchronous machines, however, may be quite different in value. A complete discussion of the nature and determination of machine constants will be given in a succeeding chapter. Generated negative-sequence voltages do not appear in rotating apparatus. Negative-sequence voltage drops appear in the negative-sequence network owing to the flow of currents of negative sequence. If the positive direction of current flow in all networks is taken in a direction from the machines to the fault, the expression for the negative-sequence voltage drop at any point in the system is $0 - I_2 Z_2$ where Z_2 is the equivalent impedance from the input terminals of the negative-sequence network to the particular point where the voltage is measured. It is apparent that the maximum value of the negative-sequence voltage drop will exist at the point of fault. Thus it may be said that a fictitious negative-sequence voltage exists at the point of fault.

The zero-sequence network, in general, differs radically from the positive and negative. Like the negative sequence, there are no generated voltages of zero sequence. The zero-sequence currents in the three phases of a three-phase system are equal and in phase and, therefore, must return through the neutral. The zero-sequence line impedance is the impedance of the three conductors in parallel with earth return in the case of a single circuit line or six conductors for a double circuit line. The zero-sequence line impedance, being a function of the earth-conductivity characteristics, may vary considerably for different localities. The zero-sequence impedance of transformers depends upon the connections, whether star or delta and whether grounded or not. A transformer-connected star, with ungrounded neutral, offers infinite impedance to the flow of zero-sequence currents, while one solidly grounded offers only the ordinary leakage impedance. The zero-sequence impedance of rotating machines depends, principally, upon the design (pitch and breadth factors) and the connections. It will be found that for most calculations for high-voltage systems, the generator zero-sequence impedance does not enter the problem, as the presence of a delta-transformer winding connected to the generator effectively blocks the flow of zero-sequence currents in the generator for faults on the high-voltage side of the transformer.

In the calculation of system fault currents it is general practice to assume the fault to occur at no load. This permits a simplification in that the generated internal voltages of all synchronous machines will be equal in magnitude and in phase with each other. All points in the positive-sequence network, where positive-sequence internal voltages appear, may be connected together. Lumped line constants are used, and the effects of capacitance and transformer magnetizing currents neglected.

Consider the system shown in Fig. 1*a* consisting of two generating stations connected by two transmission circuits. The transformers at both stations are connected delta on the low-voltage side and star, with solidly grounded neutral, on the high-voltage side. In Fig. 1*b*, *c*, and *d* are shown the positive-, negative-, and zero-sequence networks corresponding to the system of Fig. 1*a*. It will be noted that these diagrams are laid out so that for every branch of the physical system there is a corresponding branch of the sequence networks wherever the

sequence currents can flow. The presence of the delta connection on the low-voltage side of the transformers prevents the flow of zero-sequence currents in the generators and forces

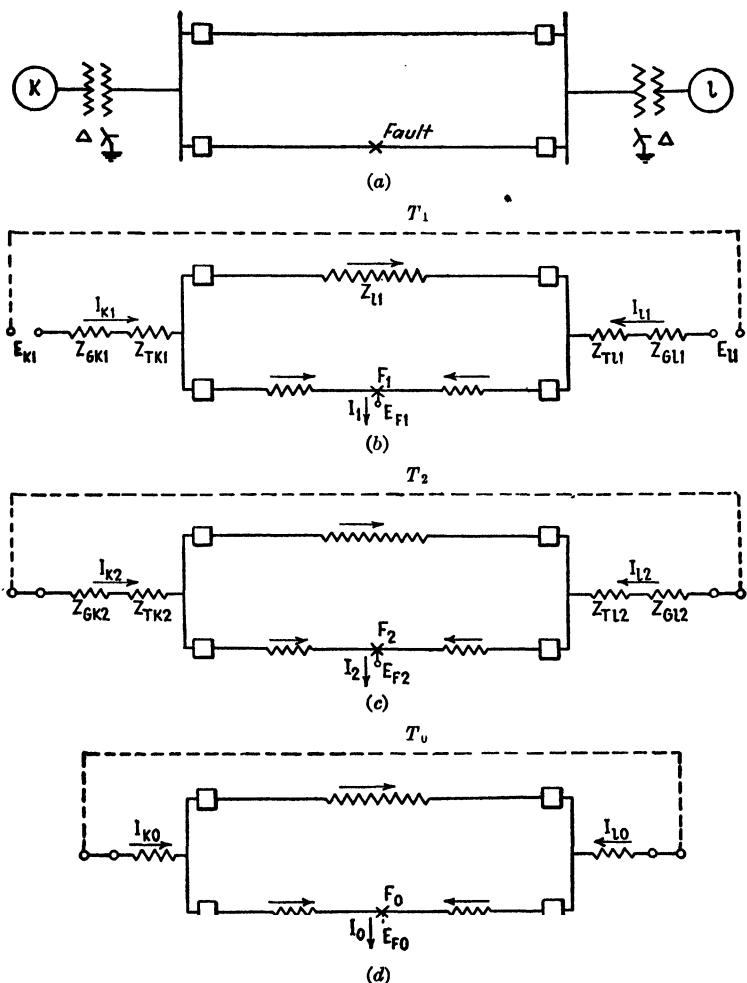


FIG. 1.—(a) Single line diagram; (b) positive-sequence net work; (c) negative-sequence net work; (d) zero-sequence net work.

the zero-sequence currents in the lines to circulate through the neutral of the transformers.

In the diagrams, T_1 , T_2 , and, T_0 are considered the current input terminals of the positive-, negative-, and zero-sequence

networks, respectively. The terminals F_1 , F_2 , and F_0 are the terminals at the point of fault. The positive direction of current flow will always be assumed to be in the same direction in each network, *i.e.*, in a direction from the T terminal to the F terminal. A review of the equations developed will show that the equations stipulate the same direction of current flow in the three networks. It will later be seen that in some cases, such as two line-to-ground faults, I_2 and I_0 flow in an opposite direction to I_1 . This condition is represented by giving negative values to I_2 and I_0 rather than reversing the positive direction of flow.

Since the generated positive-sequence voltages E_{k1} and E_{11} are assumed to be equal and in phase, they may be replaced by an equivalent voltage. This equivalent voltage will simply be the line-to-neutral generated voltage of the reference phase converted to the proper voltage base. Let the equivalent voltage be E_1 . It is also evident that the positive-sequence impedance network between terminals T_1 and F_1 can now be replaced by an equivalent single impedance by properly combining the various branches. Let the equivalent impedance be Z_1 . Referring to the positive sequence, it is evident the relation exists:

$$E_1 = I_1 Z_1 + E_{F1}, \quad \text{or} \quad E_{F1} = E_1 - I_1 Z_1 \quad (1)$$

Similarly, for the negative- and zero-sequence networks, since no generated voltages appear, the expressions are:

For the negative-sequence network,

$$\begin{aligned} 0 &= I_2 Z_2 + E_{F2} \\ E_{F2} &= -I_2 Z_2 \end{aligned} \quad (2)$$

For the zero-sequence network,

$$\begin{aligned} 0 &= I_0 Z_0 + E_{F0} \\ E_{F0} &= -I_0 Z_0 \end{aligned} \quad (3)$$

For the sake of simplicity the sequence networks of Fig. 1 will be represented by their equivalent impedances (Fig. 2). It will be recognized that any network, no matter how complex, can be reduced to an equivalent two-terminal network so long as a fault exists at one point, and the generated voltages can be assumed equal and in phase. Since the networks are independent, the sequence currents for any type of fault can be found by connecting the networks in such a manner that the

terminal conditions at the point of fault are fulfilled and then solving for the individual sequence currents.

In determining the terminal conditions at the point of fault, it is convenient to consider that three imaginary leads of zero

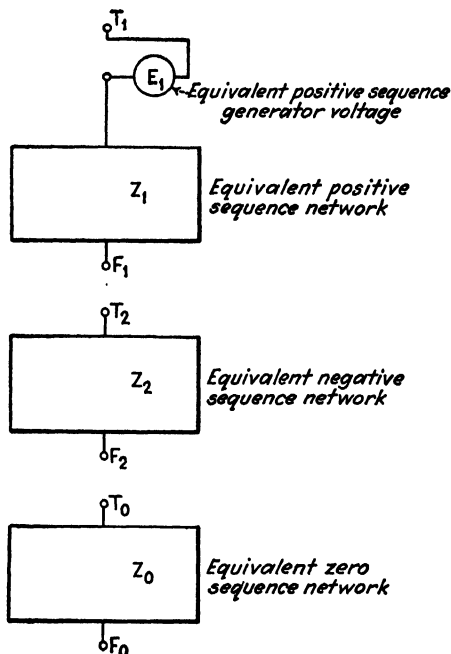


FIG. 2.—Equivalent positive-, negative-, and zero-sequence networks.

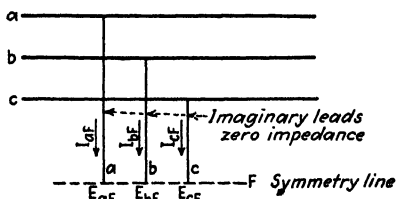


FIG. 3.—Representation of terminal conditions at point of fault.

impedance are connected to the three phases. The faults are applied to the imaginary leads, the system being assumed symmetrical up to the point of fault, as shown in Fig. 3.

Single Line-to-ground Fault with No Fault¹ or Neutral Imped-

¹ WAGNER, C. F., and R. D. EXANS, Symmetrical Components, *Elec. Jour.*, June, 1928, p. 308.

ance.—Throughout these discussions phase *a* will be used as the reference phase. All voltages, unless otherwise specified, are phase to neutral. Assume that the fault occurs on phase *a* and the fault impedance is zero, as shown in Fig. 4.

The terminal conditions at fault point *F* are

$$E_{aF} = 0 \quad (4)$$

$$I_{bF} = I_{cF} = 0 \quad (5)$$

Resolving the phase voltage and current of phase *a* at *F* into symmetrical components:

$$I_{aF} = I_{aF0} + I_{aF1} + I_{aF2} \quad (6)$$

$$I_{aF0} = \frac{1}{3}(I_{aF} + I_{bF} + I_{cF}) = \frac{1}{3}(I_{aF}) \quad (7)$$

$$I_{aF1} = \frac{1}{3}(I_{aF} + aI_{bF} + a^2I_{cF}) = \frac{1}{3}(I_{aF}) \quad (8)$$

$$I_{aF2} = \frac{1}{3}(I_{aF} + a^2I_{bF} + aI_{cF}) = \frac{1}{3}(I_{aF}) \quad (9)$$

$$I_{aF0} = I_{aF1} = I_{aF2} \quad (10)$$

$$E_{aF} = E_{aF0} + E_{aF1} + E_{aF2} = 0 \quad (11)$$

Therefore

$$E_{aF0} + E_{aF2} = -E_{aF1} \quad (12)$$

Equations (10) and (11) must be satisfied in connecting the sequence networks. The voltages E_{aF0} and E_{aF2} in Eq. (12) may be added by connecting the zero and negative networks in series, *i.e.*, connecting F_2 and T_0 of Fig. 2. The voltage T_2 and F_0 by Eq. (12) is equal to $-E_{aF1}$. Now $-E_{aF1}$ is the voltage between F_1 and T_1 , and, therefore, to satisfy the relations, we may connect T_2 and F_1 and T_1 and F_0 as shown in Fig. 5.

Let us now examine the voltage relations in a slightly different manner. Starting with the terminal T_1 in the positive-sequence network and tracing through to the point F_1 , it is seen that $E_{aF1} = E_1 - I_1Z_1$. Similarly, tracing from the *T* terminals of the negative- and zero-sequence networks $E_{aF2} = 0 - I_2Z_2$ and $E_{aF0} = 0 - I_0Z_0$. Substituting these values in Eq. (11),

$$E_1 - I_1Z_1 - I_2Z_2 - I_0Z_0 = 0$$

or

$$E_1 = I_0Z_0 + I_1Z_1 + I_2Z_2 \quad (13)$$

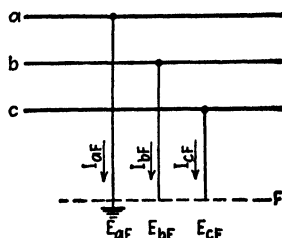


FIG. 4.—Terminal conditions for single line-to-ground fault.

It is obvious that the network connections satisfy Eq. (13) and hence Eq. (12).

The current relations of Eq. (10) are satisfied since the networks are in series; consequently, the currents in the three networks at the fault must be equal.

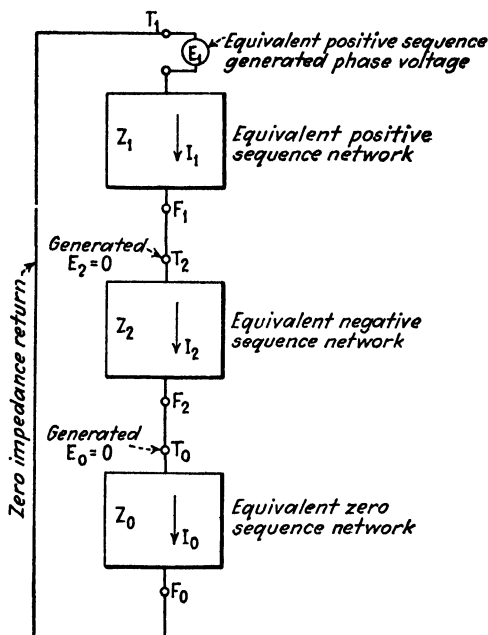


FIG. 5.—Connection of sequence networks for single line-to-ground fault.

From the connections, it is evident that the expression for the positive-, negative-, and zero-sequence currents is:

$$I_{aF1} = I_{aF2} = I_{aF0} = \frac{E_1}{Z_0 + Z_1 + Z_2} \quad (14)$$

The total current in the faulted phase is

$$I_{aF} = I_{aF0} + I_{aF1} + I_{aF2} = \frac{3E_1}{Z_0 + Z_1 + Z_2} \quad (15)$$

$$I_{bF} = I_{cF} = 0 \quad (16)$$

Single Line-to-ground Fault with Fault and Neutral Impedance.—The zero-sequence impedance of a reactance or resistance in a neutral connection is three times the ordinary value. This is because all values of impedance must be on a to neutral base

per phase and any neutral impedance carries the zero-sequence current from all three phases. A neutral impedance appears only in the zero-sequence network, and therefore should be placed in the zero-sequence network at the point in the system where it appears. For example, if there is an impedance in the neutrals of the transformers of Fig 1*a*, these impedances would appear as three times their ordinary values in the zero-sequence network of Fig 1*d*. When the zero-sequence network is reduced to the equivalent Fig. 2, the equivalent neutral impedances are included between the terminals T_0 and F_0 . No difficulty should be encountered in handling neutral impedances. In the following, the assumption will be made that any neutral impedances which may be present in the system have been included in the equivalent zero-sequence network of Fig. 2.

There is frequently considerable fault impedance associated with ground faults, which may change the magnitude and relations of the sequence currents.

Consider such a fault occurring through an impedance Z_F , as shown in Fig. 6. The terminal conditions at the fault F are now

$$E_{aF} = I_{aF}Z_F \quad (17)$$

$$I_{bF} = I_{cF} = 0 \quad (18)$$

The current relations are the same as in Eq. (10):

$$I_{aF0} = I_{aF1} = I_{aF2} = \frac{I_{aF}}{3}$$

$$E_{aF} = E_{aF0} + E_{aF1} + E_{aF2} = I_{aF}Z_F = 3I_{aF1}Z_F \quad (19)$$

Equation (13) now becomes

$$E_1 - I_0Z_0 - I_1Z_1 - I_2Z_2 = 3I_{aF1}Z_F \quad (20)$$

and Eqs. (14) becomes

$$E_1 = I_0Z_0 + I_1Z_1 + I_2Z_2 + 3I_{aF1}Z_F \quad (21)$$

The voltage and current conditions are fulfilled by connecting the networks, as shown in Fig. 7. It should be noted in cal-

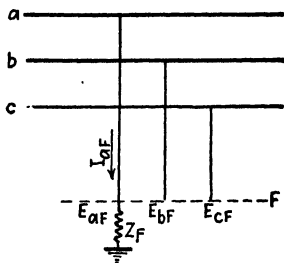


FIG. 6.—Terminal conditions for single line-to-ground fault with fault impedance.

culating the voltage drops that the point F in Fig. 6 is not at ground potential as in Fig. 4. The fault impedance Z_F , therefore, appears as an additional impedance, the equivalent positive-,

negative-, and zero-sequence networks being the same as Fig. 5.

It is evident from Fig. 7 and Eq. (21) that the expressions for the currents are:

$$I_{aF0} = I_{aF1} = I_{aF2} = \frac{E_1}{Z_0 + Z_1 + Z_2 + 3Z_F} \quad (22)$$

$$I_{aF} = \frac{3E_1}{Z_0 + Z_1 + Z_2 + 3Z_F} \quad (23)$$

Double Line-to-ground Fault with no Fault or Neutral Impedance.—For this case the terminal conditions are shown in Fig. 8. Phases b and c are short-circuited to ground. The following relations exist at the point F :

$$E_{bF} = E_{cF} = 0 \quad (24)$$

$$I_{aF} = 0 \quad (25)$$

Resolving Eqs. (24) and (25) into symmetrical components,

$$E_{aF0} = E_{aF1} = E_{aF2} = \frac{E_{aF}}{3} \quad (26)$$

$$I_{aF} = I_{aF0} + I_{aF1} + I_{aF2} = 0 \quad (27)$$

The voltage conditions can be represented by connecting all the T terminals in parallel and the F terminals in parallel, as shown in Fig. 9, i.e., Eq. (26) is satisfied by making

E_{aF1} equal to E_{aF0} and E_{aF2} . This is accomplished by connecting the three points together where these voltages appear. The current relations at the fault, stipulated by Eq. (27) must be satisfied:

$$I_{aF0} + I_{aF1} + I_{aF2} = 0$$

Therefore

$$I_{aF1} = -I_{aF0} - I_{aF2}$$

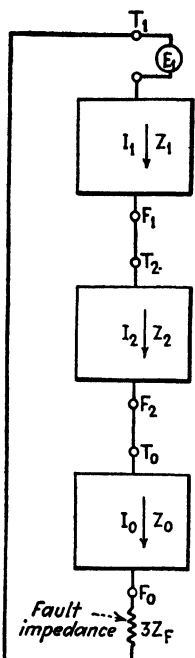


FIG. 7.—Connection of sequence networks for single line-to-ground fault with fault and neutral impedance.

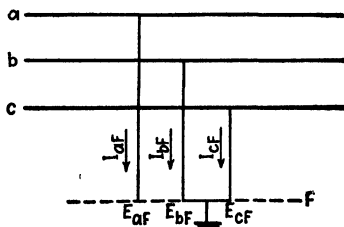


FIG. 8.—Terminal conditions for double line-to-ground fault.

It is evident that these relations exist at the junction of the three F terminals. It should be noted that the assumed positive direction of current flow is retained, but I_0 and I_2 actually are negative in value.

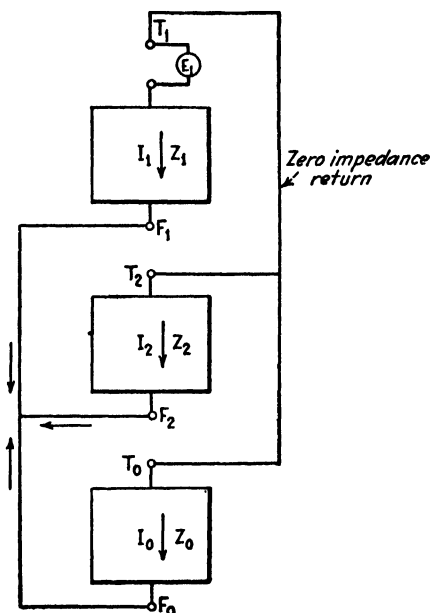


FIG. 9.—Connection of sequence networks for double line-to-ground fault.

From Fig. 9 and Eq. (27) the expression for the symmetrical components of current in the three networks for the unfaulted phase a are

$$I_{aF1} = \frac{E_1}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \quad (28)$$

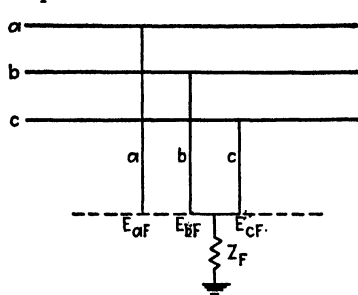
$$I_{aF2} = -I_{aF1} \frac{Z_0}{Z_0 + Z_2} \quad (29)$$

$$I_{aF0} = -I_{aF1} \frac{Z_2}{Z_0 + Z_2} \quad (30)$$

Double Line-to-ground Fault with Neutral and Fault Impedance.—An impedance in any neutral connection to ground, when considering double line-to-ground faults, is handled in exactly the same manner as described for single line-to-ground faults.

Three times the ordinary impedance is included in the equivalent zero-sequence network between the terminals T_0 and F_0 .

Two conditions involving fault impedances will be considered for double line-to-ground faults. In one type the conductors b and c are faulted solidly together and a fault impedance connects their terminals to ground. In the second type a fault impedance exists between the faulted phases as well as between



the phases and ground. The terminal conditions for the first type are shown in Fig. 10.

$$E_{bf} = E_{cf} = Z_F(I_{bf} + I_{cf}) \quad (31)$$

$$I_{af} = 0 \quad (32)$$

$$I_{af} = I_{af0} + I_{af1} + I_{af2} = 0 \quad (33)$$

$$I_{af1} = -I_{af0} - I_{af2}$$

FIG. 10.—Terminal conditions for double line-to-ground fault with fault impedance to ground.

From Eq. (33) it is apparent that the connections of the networks to fulfill the terminal current requirements are the same as the case where no fault impedance was present.

Resolving the voltages into their symmetrical components:

$$E_{af0} = \frac{1}{3}(E_{af} + 2E_{bf}) \quad (34)$$

$$E_{af1} = \frac{1}{3}(E_{af} - E_{bf}) \quad (35)$$

$$E_{af2} = \frac{1}{3}(E_{af} - E_{bf}) \quad (36)$$

$$E_{af1} = E_{af2} \quad (37)$$

An examination of these equations will show that two voltage conditions must be satisfied, $E_{af1} = E_{af2}$ and

$$E_{af0} - E_{af1} = E_{bf} = Z_F(I_{bf} + I_{cf}).$$

$$\text{Since } I_{af0} = \frac{1}{3}(I_{af} + I_{bf} + I_{cf}) \quad \text{and} \quad I_{af} = 0,$$

$$3I_{af0} = I_{bf} + I_{cf}$$

Therefore

$$E_{af0} - E_{af1} = 3I_{af0}Z_F \quad (38)$$

If the networks are connected, as shown in Fig. 11, Eqs. (33), (37), and (38) are satisfied. Equation (37) is fulfilled by connecting F_1 and F_2 together and Eq. (38) by connecting the junction of F_1 and F_2 to F_0 through the fault impedance $3Z_F$.

The following expressions are obtained for the sequence currents in the three networks.

$$I_{aF1} = \frac{E_1}{Z_1 + \frac{Z_2(Z_0 + 3Z_F)}{Z_2 + (Z_0 + 3Z_F)}} \quad (39)$$

$$I_{aF2} = -I_{aF1} \frac{(Z_0 + 3Z_F)}{Z_2 + (Z_0 + 3Z_F)} \quad (40)$$

$$I_{aF0} = -I_{aF1} \frac{Z_2}{Z_2 + (Z_0 + 3Z_F)} \quad (41)$$

The second type of fault impedance, with the terminal conditions, is shown in Fig. 12. A fault impedance Z_s exists between conductors b and c and an impedance Z_F to ground. The impedance Z_F is connected at the mid-point of the phase impedance Z_s . In general, the point of connection of Z_F to Z_s may occur at any part of Z_s . However, by assuming a central location, the problem is simplified by retaining

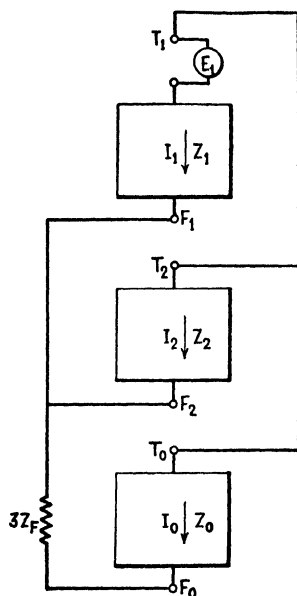


FIG. 11.—Connection of sequence networks for double line-to-ground fault with fault impedance to ground.

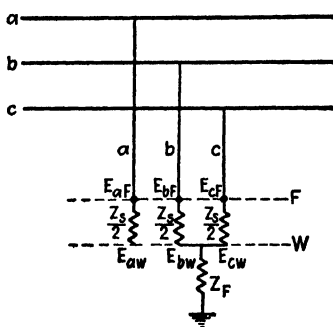


FIG. 12.—Terminal conditions for double line-to-ground fault with fault impedance between phases and to ground.

the specific requirement of phase symmetry. Such a fault impedance, as shown in Fig. 12, might occur with a line-to-line-to-ground arc. It is convenient, in this analysis, to assume a fictitious impedance $Z_s/2$ connected to phase a . This artifice¹

¹ This device is due to W. A. Lewis and L. S. Tippet, *Fundamental Basis for Distance Relaying on Three-phase Systems*, *A.I.E.E., Paper 31-61*, *Abstract Elec. Eng.*, p. 420, June, 1931.

has the effect of making the phases symmetrical up to the point W of Fig. 12. The symmetry line F still retains the meaning as in the preceding discussion. An additional symmetry line W has merely been added.

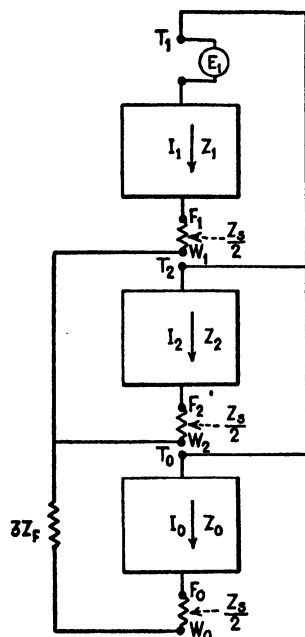


FIG. 13.—Connection of sequence networks for double line-to-ground fault with fault impedance between phases and to ground.

The terminal conditions are:

$$I_{aF} = 0 \quad (42)$$

$$E_{bW} = E_{cW} = Z_F(I_{bF} + I_{cF}) \quad (43)$$

From Eq. (42) the symmetrical components of current are:

$$I_{aF} = I_{aF0} + I_{aF1} + I_{aF2} = 0 \quad (44)$$

$$I_{aF1} = -I_{aF0} - I_{aF2}$$

The symmetrical components of voltage at W are

$$\left. \begin{aligned} E_{aW1} &= \frac{1}{3}(E_{aW} - 3I_{aF0}Z_F) \\ E_{aW2} &= \frac{1}{3}(E_{aW} - 3I_{aF0}Z_F) \\ E_{aW0} &= \frac{1}{3}(E_{aW} + 6I_{aF0}Z_F) \end{aligned} \right\} \quad (45)$$

$$E_{aW1} = E_{aW2} \quad (46)$$

$$E_{aW0} - E_{aW1} = 3I_{aF0}Z_F \quad (47)$$

Equation (44) is satisfied by connecting the network in the same manner as Fig. 11. The impedance $Z_s/2$ per phase can be introduced in each network between the F and W terminals. This can be done since the system is symmetrical up to the point W , as shown in Fig. 13.

The following voltage relations exist at the F terminals:

$$\left. \begin{aligned} E_{aF1} &= E_{aW1} + I_{aF1} \frac{Z_s}{2} \\ E_{aF2} &= E_{aW2} - I_{aF2} \frac{Z_s}{2} \\ E_{aF0} &= E_{aW0} - I_{aF0} \frac{Z_s}{2} \end{aligned} \right\} \quad (48)$$

The fictitious impedance $Z_s/2$ in conductor a does not introduce an error, because actually the voltage of conductor a at F and W is the same since no current is flowing in conductor a .

Line-to-line Fault with no Fault Impedance.—When a line-to-line fault occurs, the zero-sequence network is not involved since there is no path for the flow of zero-sequence currents to earth. Both positive- and negative-sequence currents will exist at the point of fault. The terminal conditions for this type fault are shown in Fig. 14. The fault is assumed to occur between phases *b* and *c*. The current and voltage relations at the terminals are

$$I_{aF} = 0 \quad (49)$$

$$I_{bF} = -I_{cF} \quad (50)$$

$$E_{bF} = E_{cF} \quad (51)$$

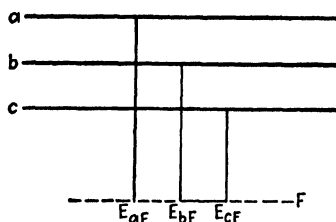


FIG. 14.—Terminal conditions for line-to-line fault.

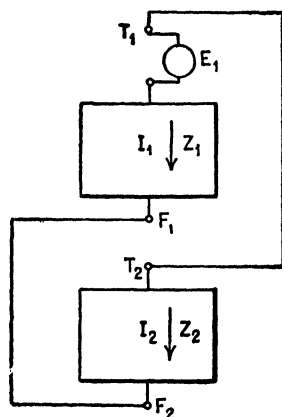


FIG. 15.—Connection of sequence networks for line-to-line fault.

The currents and voltages may be resolved into symmetrical components as follows:

$$I_{aF0} = \frac{1}{3}(I_{aF} + I_{bF} + I_{cF}) = 0 \quad (52)$$

$$I_{aF1} = \frac{1}{3}(I_{aF} + aI_{bF} + a^2I_{cF}) = \frac{1}{3}(a^2 - a)I_{cF} \quad (53)$$

$$I_{aF2} = \frac{1}{3}(I_{aF} + a^2I_{bF} + aI_{cF}) = \frac{1}{3}(a - a^2)I_{cF} \quad (54)$$

Therefore,

$$I_{aF2} = -I_{aF1} \quad (55)$$

Since I_{aF0} is equal to zero, E_{aF0} must also be zero.

$$E_{aF1} = \frac{1}{3}(E_{aF} + aE_{bF} + a^2E_{cF}) = \frac{1}{3}[E_{aF} + E_{cF}(a + a^2)] \quad (56)$$

$$E_{aF2} = \frac{1}{3}(E_{aF} + a^2E_{bF} + aE_{cF}) = \frac{1}{3}[E_{aF} + E_{cF}(a^2 + a)] \quad (57)$$

Therefore,

$$E_{aF1} = E_{aF2} \quad (58)$$

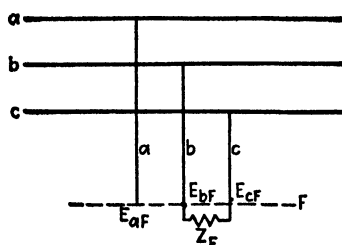
The networks must be connected in such a manner as to satisfy Eqs. (55) and (58). The connections are shown in Fig. 15.

The voltage conditions are satisfied by connecting the terminals F_1 and F_2 .

$$I_{aF1} = \frac{E_1}{Z_1 + Z_2} \quad (59)$$

$$I_{aF2} = -I_{aF1} = -\frac{E_1}{Z_1 + Z_2} \quad (60)$$

Line-to-Line Fault with Fault Impedance.—The terminal conditions for this type fault are shown in Fig. 16. The current relations are the same as for the case with no fault impedance, so the problem reduces to a determination of the disposition of the fault impedance Z_F .



$$I_{aF} = 0 \quad (61)$$

$$I_{bF} = -I_{cF} \quad (62)$$

$$E_{bF} = E_{cF} - I_{cF}Z_F \quad (63)$$

$$I_{aF1} = -I_{aF2} \quad (64)$$

$$E_{aF1} = \frac{1}{3}(E_{aF} + aE_{bF} + a^2E_{cF}) \quad (65)$$

FIG. 16.—Terminal conditions for line-to-line fault with fault impedance.

$$E_{aF2} = \frac{1}{3}(E_{aF} + a^2E_{bF} + aE_{cF}) \quad (65)$$

From Eq. (53) $I_{cF} = 3I_{aF1}/(a^2 - a)$. Substituting the value of I_{cF} from (53) in (63),

$$E_{bF} = E_{cF} - \frac{3I_{aF1}}{(a^2 - a)}Z_F \quad (66)$$

Subtracting (64) and (65),

$$3E_{aF1} - 3E_{aF2} = (a - a^2)E_{bF} + (a^2 - a)E_{cF} \quad (67)$$

Substituting for E_{bF} its value from (66),

$$3E_{aF1} - 3E_{aF2} = (a - a^2)\left(E_{cF} - \frac{3I_{aF1}}{(a^2 - a)}Z_F\right) + (a^2 - a)E_{cF}$$

$$E_{aF1} - E_{aF2} = -\frac{(a - a^2)}{(a^2 - a)}I_{aF1}Z_F = I_{aF1}Z_F$$

$$E_{aF1} = E_{aF2} + I_{aF1}Z_F \quad (68)$$

$$I_{aF1} = -I_{aF2} \quad (69)$$

The necessary conditions to be fulfilled by the network connection are given in Eqs. (68) and (69). The connections are shown in Fig. 17. The voltage conditions are satisfied by connecting the F terminals together through the fault impedance

Z_F . The current conditions are satisfied at the F terminals. It should be observed that the negative-sequence current has a negative value. From Fig. 17 it is evident that the sequence currents can be determined from the expressions.

$$I_{aF1} = \frac{E_1}{Z_1 + Z_2 + Z_F} \quad (70)$$

$$I_{aF2} = -I_{aF1} = -\frac{E_1}{Z_1 + Z_2 + Z_F} \quad (71)$$

THREE-PHASE SHORT CIRCUITS

The three-phase short circuit is the simplest type to calculate, since not only the system but the fault is symmetrical, and, therefore, no negative- or zero-sequence currents and voltages exist. The terminal conditions for a three-phase-to-ground fault are shown in Fig. 18. It will be appreciated that a three-phase fault on the symmetrical system, whether to ground or not, is the same for the condition of zero-fault impedance. It is evident from symmetry that $I_{aF1} = I_a$ and

$$\begin{aligned} E_{aF} &= E_{bF} = E_{cF} = 0 & E_{aF1} &= 0 \\ I_{aF} + I_{bF} + I_{cF} &= 3I_{F0} = 0 & I_{aF2} &= 0 \end{aligned}$$

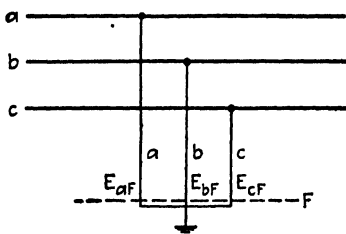


FIG. 18.—Terminal conditions for three-phase-to-ground fault.

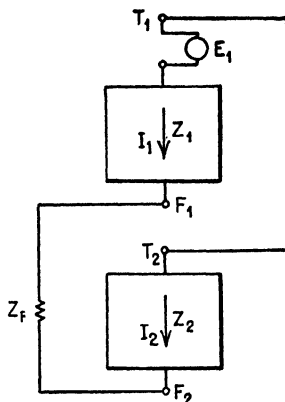


FIG. 17.—Connection of sequence networks for line-to-line fault with fault impedance.

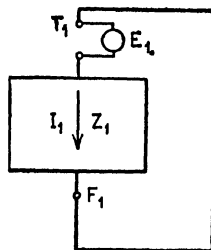


FIG. 19.—Connection of sequence networks for three-phase-to-ground fault.

The network is shown in Fig. 19. The fault current is

$$I_{aF1} = I_a = \frac{E_1}{Z_1} \quad (72)$$

In some cases there may be impedance in a three-phase fault. Such a condition might exist in a three-phase arc to ground. The general case would require a solution for unequal values of impedance in each phase at the fault. For the sake of simplicity it will be assumed that the fault impedance to ground per phase is equal in the three phases, as unsymmetrical impedances make the determination of currents and voltages quite complex. The terminal conditions are shown in Fig. 20:

$$\begin{aligned} I_{aF0} &= I_{aF2} = 0 \\ I_{aF1} &= I_{aF} \\ E_{aF1} &= I_{aF}Z_F = I_{aF1}Z_F \end{aligned}$$

The network connection is shown in Fig. 21:

$$I_{aF1} = I_{aF} = \frac{E_1}{Z_1 + Z_F} \quad (73)$$

The methods just described are equally applicable to analytical calculations or calculating boards. The three sequence

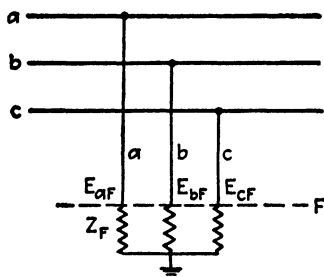


FIG. 20.—Terminal conditions for three-phase-to-ground fault with fault impedance.

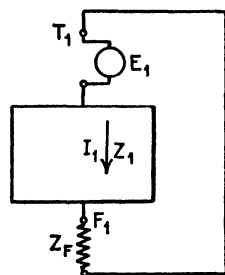


FIG. 21.—Network connection for three-phase-to-ground fault with fault impedance.

networks may be set up simultaneously on a calculating board. The alternating-current board, with several sources of e.m.f. and resistors, reactors, and capacitors, permits the accurate representation of the system characteristics. The magnitude and phase angles of currents and voltages may be found. The networks are connected in accordance with the principles described, depending upon the type of fault under consideration. The phase currents are determined by the addition of the sequence currents at corresponding points in the three sequence networks.

A voltage of a particular sequence appears only in the network of corresponding sequence. The sequence voltages may, therefore, be measured independently of the manner of connection of the networks. Since generated voltages appear only in the positive-sequence network, the voltage at any point in this network is the generated voltage minus the drop to the point considered. In the negative- and zero-sequence networks the voltage at any point is the negative of the drop from the bus of zero potential (*T* terminal) to the point. The bus of zero potential corresponds to the point of application of the generated voltage in the positive-sequence network.

When only the magnitude of the currents and voltages are required, the direct-current calculating board may be used. The elements for representing the system characteristics are all resistance; therefore, the assumption that all generated voltages are of the same phase and magnitude and that all system-impedance constants have the same ratio of reactance to resistance must necessarily be permissible.

Direct-current Calculating Table.—The direct-current, short-circuit calculating table is being extensively used to solve for short-circuit currents where the complexity of the system makes the analytical solution too laborious and involved. The system is set up in miniature form by connecting together a number of adjustable resistor units representing proportioned impedance values of the actual system connections. A known direct-current voltage is impressed between a positive bus, to which all resistor units representing generators are connected, and some point of fault. The direct-current ampere values for total current and branch currents are proportional to the instantaneous symmetrical short-circuit current for a particular fault point.

The multiplier used to convert direct-current readings into alternating-current amperes is easily determined and depends upon the constants of the calculating table and the kilovolt-ampere base used. Usually a base kilovolt-ampere is selected which will give a multiplier of .10 or 100 on the calculating desk, so that the direct-current amperes can be readily converted into alternating-current amperes. This virtually makes it possible to read alternating-current, short-circuit amperes directly.

The direct-current calculating table is also used to determine line-to-ground, short-circuit values. Theoretically, it would

seem necessary to set up three separate networks, the positive, the negative, and the zero sequence, and combine the results analytically. This would necessarily entail a large amount of work. A much simpler method is used which gives quick and accurate results. The standard 40-volt, direct-current, calculating desk has a maximum of 60 volts available. For the determination of line-to-ground short circuits, the calculating desk is operated at 60 volts or 1.5 times normal. For most practical purposes, the positive- and negative-sequence networks are identical. The positive-sequence network, which is usually set up first, in order to obtain three-phase short-circuit values, may be considered to represent both the positive- and negative-sequence networks, each being set up at one-half value. The resistors of the zero-sequence network are then set up at one-half value for each branch. To obtain a reading of line-to-ground currents, it is merely necessary to connect (by a flexible conductor) a particular fault point on the positive-sequence network directly to the point in the zero-sequence network, where all sources of ground current are connected, and then insert the negative plug of the calculating table into the corresponding fault point in the zero-sequence network. The readings in the branches of the zero-sequence network will then give the residual current by applying the same multiplier as used for the three-phase study.

Alternating-current Calculating Table.—There has recently been developed a calculating table¹ which uses alternating current and has provision for setting up a miniature system so as to include resistance, reactance, and capacitance of each branch circuit of a network. Generating stations are represented by separate sources of power, and means are provided so that the actual voltages and phase angles are reproduced accurately. This calculating desk is required only for very special relay applications. It applies particularly where resistor grounds are used or a combination of solid and impedance grounds. The direct-current calculating desk gives sufficiently accurate data for the great majority of relay applications so that the more complicated alternating-current calculating desk need only be used where special problems are being studied.

¹ TRAVERS, H. A., and W. W. PARKER, An Alternating-current Calculating Board, *Elec. Jour.*, May, 1930, p. 266.

SIMULTANEOUS FAULTS AT DIFFERENT LOCATIONS¹

Simultaneous faults are a random effect, occurring most frequently on double-circuit lines, resulting from an arc on one circuit carrying into the second circuit, or due to a disturbance initially involving both circuits at the same time. Faults at different locations may be the result of secondary flashovers from traveling waves or simultaneous lightning strokes at different locations. It is now a recognized fact that such faults do occur, and it is therefore worth while to know the methods applicable to the calculation of the magnitude and distribution of fault currents under such conditions.

The same general method of analysis can be used for simultaneous faults at different locations involving various combinations of conductors, as described for faults at only one point. If the conductors at one fault point C are designated a, b, c , and the corresponding conductors at the other location D are α, β, γ , respectively, then the symmetrical quantities at C are designated $V_{a0}, V_{a1}, V_{a2}, I_{a0}, I_{a1}$ and I_{a2} and those at D will be $V_{\alpha0}, V_{\alpha1}, V_{\alpha2}, I_{\alpha0}, I_{\alpha1}$, and $I_{\alpha2}$. Ten independent equations may be written involving the twelve unknowns. Eight of the unknowns can be eliminated and two equations obtained giving V_{a1} and $V_{\alpha1}$ in terms of the positive-sequence currents I_{a1} and $I_{\alpha1}$ and the known zero- and negative-sequence impedances. All quantities are considered as vectors.

Zero-sequence Network.—When a ground fault occurs at two points on a system, the zero-sequence current in each fault is a function of the zero-sequence voltage at both points of fault. The zero-sequence network for simultaneous faults can generally be represented by an equivalent star (Fig. 22), in which S is the bus of zero-potential and C and D are the fault points. The complete zero-sequence network can be reduced to this simple equivalent circuit by the methods outlined in Chap. IV. In complicated systems it is more convenient to determine the

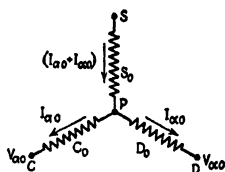


FIG. 22.—Equivalent circuit for zero-sequence network.

¹ Extracted and rearranged from a paper by Edith Clarke, Simultaneous Faults on Three-phase Systems, *Trans. A.I.E.E.*, vol. 50, p. 919, September, 1931.

branches of the equivalent circuit by means of the alternating-current or direct-current calculating table.

When using the calculating table, the complete zero-sequence network is set up and three measurements made to determine the equivalent star network. A voltage V_c to ground is applied at point C with D ungrounded and S grounded and the current I_s at S measured. The same measurement is made with point D grounded and S ungrounded. The third measurement is made for I_s at S with a voltage V_d to ground applied at D with point C ungrounded and S grounded. If the branches of the equivalent star (Fig. 22) are C_0 , D_0 , and S_0 then we may write

$$\frac{V_c}{I_s} = C_0 + S_0 \quad (74)$$

$$\frac{V_c}{I_d} = C_0 + D_0 \quad (75)$$

$$\frac{V_d}{I_s} = D_0 + S_0 \quad (76)$$

Assuming a current flow as shown in Fig. 22, the voltages at the fault points in terms of the two zero-sequence currents in the faults are

$$0 - V_{a0} = I_{a0}(C_0 + S_0) + I_{a0}S_0 \quad (77)$$

$$0 - V_{a0} = I_{a0}(D_0 + S_0) + I_{a0}S_0 \quad (78)$$

From Eqs. (77) and (78),

$$I_{a0} = -V_{a0} \frac{D_0 + S_0}{Z_{00}} + V_{a0} \frac{S_0}{Z_{00}} \quad (79)$$

$$I_{a0} = -V_{a0} \frac{C_0 + S_0}{Z_{00}} + V_{a0} \frac{S_0}{Z_{00}} \quad (80)$$

where

$$Z_{00} = C_0 D_0 + C_0 S_0 + D_0 S_0 \quad (81)$$

Negative-sequence Network.—The negative-sequence network can be reduced to an equivalent star (Fig. 23), by the same procedure as described for the zero-sequence network. All points of zero-potential are bused together at S , but it should be observed that this point does not necessarily represent the same point as in the zero-sequence system.

$$0 - V_{a2} = I_{a2}(C_2 + S_2) + I_{a2}S_2 \quad (82)$$

$$0 - V_{a2} = I_{a2}(D_2 + S_2) + I_{a2}S_2 \quad (83)$$

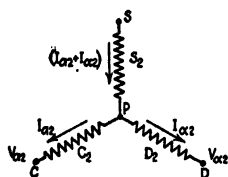


FIG. 23.—Equivalent circuit for negative-sequence network.

Converting into terms of I_{a1} and I_{a1}

$$I_{a2} = -V_{a2} \frac{D_2 + S_2}{Z_{22}} + V_{a2} \frac{S_2}{Z_{22}} \quad (84)$$

$$I_{a2} = -V_{a2} \frac{C_2 + S_2}{Z_{22}} + V_{a2} \frac{S_2}{Z_{22}} \quad (85)$$

where

$$Z_{22} = C_2 D_2 + C_2 S_2 + D_2 S_2 \quad (86)$$

SYMMETRICAL COMPONENTS AT TWO POINTS OF FAULT

The relations between the symmetrical components of voltage and current at the points of fault are independent of the remainder of the system. For a single point of fault, three equations may be written connecting the components of voltage or current at the fault for different types of faults. If there are two or more points of fault, there will be three equations relating components of current or voltage at each point. Such relations for a single point of fault are given in Table I. Substituting α , β , and γ for a , b , and c , respectively, gives equations for a fault at point D .

TABLE I.—SYMMETRICAL COMPONENTS AT POINT OF FAULT FOR SINGLE FAULTS

Reference	Type fault	Phases involved	Current relations	Voltage relations
(a)	Single line to ground	a	$I_{a0} = I_{a1} = I_{a2}$	$V_{a1} = -(V_{a0} + V_{a2})$
(b)	Single line to ground	b	$I_{a0} = a^2 I_{a1}; I_{a1} = I_{a0}$ $I_{a2} = a I_{a1}; I_{a1} = a^2 I_{a2}$	$V_{a0} = -(a V_{a2} + a^2 V_{a1})$
(c)	Single line to ground	c	$I_{a0} = a I_{a1}; I_{a1} = a^2 I_{a0}$ $I_{a2} = a^2 I_{a1}; I_{a1} = a I_{a2}$	$V_{a1} = -(a^2 V_{a0} + a V_{a2})$
(d)	Line to line	b, c	$I_{a0} = 0; I_{a2} = -I_{a1}$	$V_{a2} = V_{a1}$
(e)	Line to line	a, c	$I_{a0} = 0; I_{a2} = -a I_{a1}$ $I_{a1} = -a^2 I_{a2}$	$V_{a2} = a V_{a1}; V_{a1} = a^2 V_{a2}$
(f)	Line to line	a, b	$I_{a0} = 0; I_{a2} = -a^2 I_{a1}$ $I_{a1} = -a I_{a2}$	$V_{a2} = a^2 V_{a1}; V_{a1} = a V_{a2}$
(g)	Double line to ground	b, c	$I_{a1} = -(I_{a0} + I_{a2})$	$V_{a0} = V_{a1}; V_{a2} = V_{a1}$
(h)	Double line to ground	a, c	$I_{a1} = -(a I_{a0} + a^2 I_{a2})$	$V_{a0} = a^2 V_{a1}; V_{a1} = a V_{a0}$ $V_{a2} = a V_{a1}; V_{a1} = a^2 V_{a2}$
(i)	Double line to ground	a, b	$I_{a1} = -(a^2 I_{a0} + a I_{a2})$	$V_{a0} = a V_{a1}; V_{a1} = a^2 V_{a0}$ $V_{a2} = a^2 V_{a1}; V_{a1} = a V_{a2}$
(j)	Three phase	a, b, c	$I_{a0} = 0$	$V_{a1} = V_{a2} = 0$
(k)	Three phase to ground	a, b, c		$V_{a1} = V_{a2} = V_{a0} = 0$

General Simultaneous Equations.—Three equations giving the relations between components of current or voltage at each point of fault, two relating negative-sequence currents and voltages,

and two relating zero-sequence currents and voltages provide the ten equations necessary for the elimination of the eight unknowns V_{a0} , I_{a0} , $V_{\alpha 0}$, $I_{\alpha 0}$, V_{a2} , I_{a2} , $V_{\alpha 2}$, and $I_{\alpha 2}$. The positive-sequence components of voltage at the two faults can be expressed in terms of the positive-sequence components of current and the zero- and negative-sequence impedances

$$V_{a1} = kI_{a1} + mI_{\alpha 1} \quad (87)$$

$$V_{\alpha 1} = nI_{a1} + lI_{\alpha 1} \quad (88)$$

The coefficients k , l , m , and n depend upon the branch impedances of the equivalent star negative- and zero-sequence networks and the combination of simultaneous faults. The positive-sequence impedance is not involved in Eqs. (85) and (86).

Derivation of Coefficients k , l , m , and n . *Single-line-to-ground faults at two points, conductors a and α .*

Adding Eqs. (77) and (82), also (78) and (83) and substituting the relations from Table I for a single-line-to-ground fault on phase a gives

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)I_{a1} + (S_0 + S_2)I_{\alpha 1} \quad (89)$$

$$V_{\alpha 1} = (D_0 + S_0 + D_2 + S_2)I_{a1} + (S_0 + S_2)I_{\alpha 1} \quad (90)$$

Conductors b and α .

Multiplying Eq. (77) by a and (82) by a^2 and adding, also adding (78) and (83) and substituting the relations from Table I for single-line-to-ground fault on phase a and b gives

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)I_{a1} + (aS_0 + a^2S_2)I_{\alpha 1} \quad (91)$$

$$V_{\alpha 1} = (D_0 + S_0 + D_2 + S_2)I_{a1} + (a^2S_0 + aS_2)I_{\alpha 1} \quad (92)$$

Conductors c and α .

Multiplying (77) by a^2 , and (82) by a , and following the same procedure as in the preceding case,

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)I_{a1} + (a^2S_0 + aS_2)I_{\alpha 1} \quad (93)$$

$$V_{\alpha 1} = (D_0 + S_0 + D_2 + S_2)I_{a1} + (aS_0 + a^2S_2)I_{\alpha 1} \quad (94)$$

Line-to-line faults at two points, conductors a , c , and β , γ .

Substituting from Table I the relations for line-to-line faults on a , c , and b , c , in Eqs. (82) and (83), and multiplying the result, after substituting in (82), by a^2 gives

$$V_{a1} = (C_2 + S_2)I_{a1} + a^2S_2I_{\alpha 1} \quad (95)$$

$$V_{\alpha 1} = (D_2 + S_2)I_{a1} + aS_2I_{\alpha 1} \quad (96)$$

Double-line-to-ground faults at two points, conductors a, c and β, γ .

Multiplying (79) by a and (84) by a^2 and adding; also adding (80) and (85), and substituting from Table I, the relations for fault combinations a, c , and b, c :

$$I_{a1} = V_{a1} \left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}} \right) - V_{a1} \left(\frac{aS_0}{Z_{00}} + \frac{aS_2}{Z_{22}} \right) \quad (97)$$

$$I_{a1} = V_{a1} \left(\frac{C_0 + S_0}{Z_{00}} + \frac{C_2 + S_2}{Z_{22}} \right) - V_{a1} \left(\frac{a^2S_0}{Z_{00}} + \frac{aS_2}{Z_{22}} \right) \quad (98)$$

Solving for V_{a1} and V_{a1} in the above equation,

$$V_{a1} = \frac{[Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)]I_{a1} + (aS_0Z_{22} + a^2S_2Z_{00})I_{a1}}{(C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2) + 3S_0S_2} \quad (99)$$

$$V_{a1} = \frac{[Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)]I_{a1} + (a^2S_0Z_{22} + aS_2Z_{00})I_{a1}}{(C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2) + 3S_0S_2} \quad (100)$$

Three-phase faults at two points:

$$V_{a1} = 0 \quad (101)$$

$$V_{a1} = 0 \quad (102)$$

Line-to-line fault at C and single-line-to-ground fault at D, conductors b, c, and α .

Substituting from Table I in Eq. (82), and also substituting in the sum of Eqs. (78) and (83),

$$V_{a1} = I_{a1}(C_2 + S_2) - I_{a1}S_2 \quad (103)$$

$$V_{a1} = I_{a1}(D_0 + S_0 + D_2 + S_2) - I_{a1}S_2 \quad (104)$$

Double-line-to-ground fault at C, single-line-to-ground fault at D, conductors b, c, and α .

Adding Eqs. (79) and (84) and substituting from Table I, the relations for a double-line-to-ground fault on conductors b, c are:

$$I_{a1} = V_{a1} \left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}} \right) - V_{a1} \frac{S_0}{Z_{00}} - V_{a1} \frac{S_2}{Z_{22}} \quad (105)$$

Rearranging Eqs. (80) and (85) to express V_{a0} in terms of V_{a0} and I_{a0} and V_{a2} in terms of V_{a2} and I_{a2} and then replacing V_{a0} and V_{a2} by V_{a1} from Table I (g) and I_{a0} and I_{a2} by I_{a1} from (a).

$$V_{a0} = V_{a1} \frac{S_0}{C_0 + S_0} - I_{a1} \frac{Z_{00}}{C_0 + S_0} \quad (106)$$

$$V_{a2} = V_{a1} \frac{S_2}{C_2 + S_2} - I_{a1} \frac{Z_{22}}{C_2 + S_2} \quad (107)$$

Substituting these two equations in (105) and replacing Z_{00} and Z_{22} by their values from (81) and (86):

$$V_{a1} = \frac{(C_0 + S_0)(C_2 + S_2)}{C_0 + S_0 + C_2 + S_2} I_{a1} - \frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{a1} \quad (108)$$

Adding (106) and (107) and replacing $(V_{a0} + V_{a2})$ by V_{a1} from Table I (a), and V_{a1} by its value from (108),

$$V_{a1} = \frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{a1} + \left[(D_0 + D_2 + S_0 + S_2) - \frac{(S_0 - S_2)^2}{C_0 + S_0 + C_2 + S_2} \right] I_{a1} \quad (109)$$

Three-phase fault at C, single-line-to-ground fault at D on phase α .

Substituting, from Table I (j), $I_{a0} = 0$ in Eq. (77) and (78), and $V_{a2} = 0$ in Eq. (82),

$$-V_{a0} = I_{a0} S_0 \quad (110)$$

$$-V_{a0} = I_{a0}(D_0 + S_0) \quad (111)$$

$$I_{a2} = -I_{a2} \frac{S_2}{C_2 + S_2} \quad (112)$$

Substituting (112) in (83) and rearranging,

$$V_{a2} = I_{a2} \left(D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (113)$$

Adding (111) and (113) and substituting from Table I (a),

$$V_{a1} = I_{a1} \left(D_0 + S_0 + D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (114)$$

$$V_{a1} = 0 \quad (115)$$

The values of k , l , m , and n for most fault combinations are given in Table II.

Positive-sequence Network.—In the positive-sequence network there is a voltage V_{a1} to ground at C and a current I_{a1} flowing into the fault and at point D a positive-sequence voltage V_{a1} and current I_{a1} . The relations between voltages and currents are given in Eqs. (87) and (88). These equations may be written in a different form:

$$V_{a1} = (k - n)I_{a1} + \frac{m + n}{2}(I_{a1} + I_{a1}) + \frac{m - n}{2}(I_{a1} - I_{a1}) \quad (116)$$

$$V_{a1} = (l - m)I_{a1} + \frac{m + n}{2}(I_{a1} + I_{a1}) + \frac{m - n}{2}(I_{a1} - I_{a1}) \quad (117)$$

The last term of (116) and (117) may be written

$$\frac{m - n}{2}(I_{a1} - I_{a1})$$

A general equivalent circuit for representing two faults in the positive-sequence network is given in Fig. 24. The equivalent circuit is of Y form having branches $(k - n)$, $(l - m)$, and $\left(\frac{m + n}{2}\right)$ between points C, D, and F and an impedance branch

$\left(\frac{m - n}{2}\right)$ or $\left(\frac{n - m}{2}\right)$ paralleled by an adjustable voltage V_s between F and ground. Equations (116) and (117) will be satisfied if $(I_{a1} - I_{a1})$ is forced to flow through the branch $\left(\frac{m - n}{2}\right)$, or if $(I_{a1} - I_{a1})$ flows through $\left(\frac{n - m}{2}\right)$. The current flowing in the impedance $\left(\frac{m - n}{2}\right)$ will be $(I_{a1} - I_{a1})$ if the voltage V_s is adjusted in phase and magnitude until the current flowing through it is double I_{a1} . If $\left(\frac{n - m}{2}\right)$ is used, the current through V_s must be double I_{a1} .

GENERAL PROCEDURE FOR CALCULATING SIMULTANEOUS FAULTS

The distribution of positive-sequence currents and voltages may be found by using a calculating table or by analytical methods. In the analytical solution, equations may be written expressing the internal machine voltages and positive-sequence fault voltages V_{a1} and V_{a1} in terms of the positive-sequence impedance and positive-sequence currents of the system. If there are n sources of internal generated voltage, there will be a minimum of n unknown currents in the machines and two unknown currents at the fault points or a total of $(n + 2)$. There will also be a minimum of $(n + 2)$ voltage equations. Each

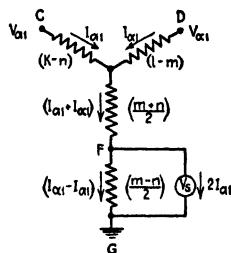


FIG. 24.—General equivalent circuit for positive-sequence network.

closed loop in the system introduces an additional unknown current but also an additional equation, since the voltage drop around the loop is zero. In the equations that may be set up, the machine internal voltages are known or can be determined from the known operating conditions at the instant of fault, and the $(n + 2)$ currents and the two fault voltages are unknown. With Eqs. (87) and (88) there will be $(n + 4)$ equations and $(n + 4)$ unknowns, therefore the positive-sequence currents can be solved for in terms of the internal voltages and positive-, negative-, and zero-sequence impedances of the system. The positive-sequence voltages can then be found from Eqs. (87) and (88) or from the known internal voltages, positive-sequence current, and positive-sequence impedance.

Miss Clarke, in her paper, gives a general method for avoiding the additional equations and unknowns due to loops in the positive-sequence system. Let $E_1, E_2, \dots, E_n$ be the internal voltages of machines 1, 2, $\dots, n$ and $I_1, I_2, \dots, I_n$ the corresponding currents. Positive direction of current flow is considered as from the generators toward the fault:

$$\begin{aligned} E_1 &= I_1 Z_{11} + I_2 Z_{12} + \dots + I_n Z_{1n} + I_{a1} Z_{1c} + I_{a1} Z_{1d} \\ E_2 &= I_1 Z_{21} + I_2 Z_{22} + \dots + I_n Z_{2n} + I_{a1} Z_{2c} + I_{a1} Z_{2d} \\ E_n &= I_1 Z_{n1} + I_2 Z_{n2} + \dots + I_n Z_{nn} + I_{a1} Z_{nc} + I_{a1} Z_{nd} \\ V_{a1} &= I_1 Z_{c1} + I_2 Z_{c2} + \dots + I_n Z_{cn} + I_{a1} Z_{cc} + I_{a1} Z_{cd} \\ V_{a1} &= I_n Z_{d1} + I_2 Z_{d2} + \dots + I_n Z_{dn} + I_{a1} Z_{dc} + I_{a1} Z_{dd} \end{aligned}$$

In these equations the impedance constants have the values:

$$\begin{aligned} Z_{11} &= \frac{E_1}{I_1} \text{ when no current is flowing except } I_1 \\ Z_{1n} &= \frac{E_1}{I_n} \text{ when no current is flowing except } I_n \\ Z_{1c} &= \frac{E_1}{I_{a1}} \text{ when no current is flowing except } I_{a1} \\ Z_{cc} &= \frac{V_{a1}}{I_{a1}} \text{ when no current is flowing except } I_a \\ Z_{cd} &= \frac{V_{a1}}{I_{a1}} \text{ when no current is flowing except } I_{a1} \\ Z_{nn} &= \frac{E_n}{I_n} \text{ when no current is flowing except } I_n \end{aligned}$$

If there is a point of zero-potential in the positive-sequence network, such as the neutral of a shunt impedance load, there will

be a return path for the current when only one current is flowing and the impedance can be easily obtained on a calculating board. When there is no such neutral point it is necessary to ground one machine, say No. 2, to provide a return path when only one current is flowing. The voltage $E_2 = I_1 Z_{a1} + \dots I_{a1} Z_{2a}$ then becomes indeterminate but can be replaced by the equation

$$I_1 + I_2, \dots, I_n = I_{a1} + I_{a1}.$$

The current distribution is first determined with machine 2 grounded, $E_2 = 0$, and then with all machines except No. 2 grounded. The distribution of positive-sequence currents can then be obtained by superposing¹ the two sets of currents.

After determining the positive-sequence currents and voltages at the fault points, four of the eight unknowns V_{a0} , V_{a0} , V_{a2} , V_{a2} , I_{a0} , I_{a0} , I_{a2} , and I_{a2} may be found using the relations in Table I. Two of the remaining unknowns are in the negative- and two in the positive-sequence system and may be found from Eqs. (77), (78), (82), and (83), or (79), (80), (84) and (85).

In general the voltages at the two points of fault in the negative- and zero-sequence systems will not be in phase with each other or with the positive-sequence reference voltage. The currents in the negative- or zero-sequence networks may be found by adding vectorially the currents due to the fault voltage at *C* with point *D* grounded, and the currents due to the fault voltage at *D* with *C* grounded.

The current and voltage distribution in the zero-sequence system can be determined with an alternating-current calculating table by applying V_{a0} and V_{a0} at points *C* and *D*, respectively. The distribution of negative-sequence currents and voltages is determined in a similar manner.

The general method described for the calculation of simultaneous faults at two points can be extended to cover the case of three or more simultaneous faults. There is little practical necessity for discussing this case. The effect of fault impedance can readily be incorporated in the equations by making use of the relations developed for single faults with fault impedance and following the same procedure as outlined for the case of zero fault impedance with simultaneous faults.

¹ EVANS, R. D., Analytical Solution of Networks, *Elec. Jour.*, April, 1924, p. 149.

TABLE II.—VALUES OF k , l , m , AND n TO BE SUBSTITUTED IN EQS. (87) AND (88) FOR SIMULTANEOUS FAULTS AT TWO POINTS

Type fault	Phases involved	k	l	m	n
Single-line-to-ground faults at two points	a and α b - α c - α	$\frac{Z_{aa}}{Z_{aa} + Z_{bb}}$	$\frac{Z_{aa}}{Z_{aa}}$	$\frac{S_1 + S_2}{a^2 S_1 + a^2 S_2}$	$\frac{S_1 + S_2}{a^2 S_1 + a^2 S_2}$
Line-to-line faults at two points	b - $\beta\gamma$ a - $\beta\gamma$ ab - $\beta\gamma$	$\frac{C_1 + S_1}{C_1 + S_1}$	$\frac{D_1 + S_1}{D_1 + S_1}$	$\frac{S_1}{a^2 S_1}$	$\frac{S_1}{a^2 S_1}$
Double-line-to-ground faults at two points	b - $\beta\gamma$ a - $\beta\gamma$ ab - $\beta\gamma$	$\frac{Z_{aa}(C_1 + S_1) + Z_{bb}(C_2 + S_2)}{Z_{aa}(C_1 + S_1) + Z_{bb}(C_2 + S_2)}$	$\frac{Z_{aa}(D_1 + S_1) + Z_{bb}(D_2 + S_2)}{Z_{aa}(D_1 + S_1) + Z_{bb}(D_2 + S_2)}$	$\frac{Z_{aa} S_1 + Z_{bb} S_2}{a^2 S_1 Z_{aa} + a^2 S_2 Z_{bb}}$	$\frac{Z_{aa} S_1 + Z_{bb} S_2}{a^2 S_1 Z_{aa} + a^2 S_2 Z_{bb}}$
Three-phase faults at two points	abc - $\alpha\beta\gamma$	0	0	0	0
Line-to-line fault at C and single-line-to-ground fault at D	b - α a - α ab - α	$\frac{C_1 + S_1}{C_1 + S_1}$	$\frac{Z_{aa}}{Z_{aa}}$	$\frac{-S_1}{-a^2 S_1}$	$\frac{-S_1}{-a^2 S_1}$
Double-line-to-ground fault at C , Single-line-to-ground fault at D	b - α a - α ab - α	$\frac{(C_1 + S_1)(C_2 + S_2)}{Z_{aa}}$	$\frac{Z_{aa} - \frac{(S_1 - S_2)^2}{Z_{aa}}}{Z_{aa} - \frac{S_1^2 + S_2^2}{Z_{aa}}}$	$\frac{-S_1(C_1 + S_1) + S_2(C_2 + S_2)}{a^2 S_1(C_1 + S_1) + a^2 S_2(C_2 + S_2)}$	$\frac{-S_1(C_1 + S_1) + S_2(C_2 + S_2)}{a^2 S_1(C_1 + S_1) + a^2 S_2(C_2 + S_2)}$
Three-phase fault at C , Single-line-to-ground at D on phase α	abc to ground abc to ground	0	$\frac{D_1 + S_1 + D_2 + S_2}{D_1 + S_1 + D_2 + S_2}$	0	0

$$Z_{aa} = C_0 + S_0 + D_1 + S_1 \quad Z_{bb} = (C_1 + S_1)(D_2 + S_2) + (C_2 + S_2)(S_0 + S_1) + (D_1 + S_1)(S_0 + S_2)$$

CHAPTER IV

NETWORK REDUCTION AND THE DETERMINATION OF CURRENT AND VOLTAGE DISTRIBUTION

The method of connecting the sequence networks for the different types of faults and the analytical expressions for the current at the point of fault in each network has been determined. From the fundamental relations of sequence quantities, the phase currents and voltages at the fault may be determined. Usually it is necessary to know the distribution of the fault current through the networks. The distribution of the fault current in each phase in all branches of the original network can be found only from a knowledge of the distribution of the sequence currents in the sequence networks.

The following characteristics of the three sequences should be kept in mind:

1. When the impedances are symmetrical in the three phases, zero-sequence voltage drops are produced by zero-sequence currents only; negative-sequence voltage drops by negative-sequence currents; and positive-sequence voltage drops by positive-sequence currents.
2. Only positive-sequence voltages are generated by rotating machines.
3. Negative- and zero-sequence voltages appear due to the flow of negative- and zero-sequence currents through their respective impedances.
4. No positive- or negative-sequence currents can flow to ground as both systems are balanced.
5. The zero-sequence currents in the three phases are all in phase and the sum of these currents in the three phases act as though the conductors were connected to ground. One third the ground current flows in each phase. It is found in taking the summation of the three components in the ungrounded phase that the positive- and negative-sequence currents cancel the zero, thus satisfying the conditions.
6. A particular sequence current distributes through its network independent of the distribution of the other sequence currents through their networks.

The general procedure to be followed in making short-circuit calculations will now be outlined. The preliminary work necessary is the same for the calculation of any type of fault. This may be outlined as follows:

a. The preparation of a single-line impedance diagram showing every branch of the system which would influence the magnitude or distribution of the fault current, including generators, transformers, and transmission lines.

b. The preparation of a single-line impedance diagram for the zero-, positive-, and negative-sequence networks.

c. Reduction of all impedance values to some common base, either percentage impedance on a common kilovolt-ampere base or ohms on a common voltage base.

Impedance Values.—In connection with item *a* above, note that the system impedance diagram is also the positive-sequence network. It is necessary to exercise care in the selection of the value of impedance to represent the generators. If the sustained value of current is desired, the synchronous reactance should be used; if the transient value, the transient reactance should be used; and if the maximum instantaneous value, the subtransient should be used. This matter will be discussed in subsequent chapters.

The question arises whether to neglect resistance and base all calculations on reactance alone, or use impedance values. This is often a matter of judgment and depends upon the characteristics of the transmission circuits. On long, small-capacity circuits for 66,000 volts and lower, the resistance may comprise an appreciable part of the total impedance. In such cases, it is advisable to use impedance values. It should be noted that in combining series circuits which have different ratios of reactance to resistance, the use of scalar impedance values introduces an error. In general the ratio of reactance to resistance is so nearly equal for the different circuits that it is not necessary to use complex expressions for the calculations in determining short-circuit currents.

The ratio of reactance to resistance is so high for transformers and synchronous apparatus that it is customary to neglect the resistance and use only reactance values.

In connection with item *c*, short-circuit calculations may be made by using percentage impedance on an arbitrary kilovolt-ampere base, or by using ohms on a voltage base. If percentage impedance is used on some kilovolt-amperes base, the result of the calculations will give the magnitude of the short circuit in kilovolt-amperes. It is then necessary to find the current knowing the kilovolt-amperes. If the calculations are made

using ohms on a certain voltage base, the current will be obtained directly at the chosen voltage. If percentage impedance is used, the impedance of each part of the network, including synchronous machines and transformers, must be converted to the same kilovolt-ampere base. Likewise, if using ohms impedance, it is necessary to convert all impedance values to one voltage base.

Reactances of machines and transformers are usually given as a percentage, based on the kilovolt-ampere rating of the particular machine. Since there will be various ratings of machines on any system, it is necessary to reduce all percentage reactances to the same kilovolt-ampere base. To convert a reactance of x_A per cent on A kva. base to B kva. base, it is only necessary to multiply the percentage reactance by the ratio of the desired base to the original, thus:

$$\frac{B}{A}x_A = x_B\% \quad (1)$$

This follows directly from the fact that the short-circuit kilovolt-amperes for a given machine with a given reactance are the same for any kilovolt-ampere base. The percentage reactance is simply the percentage voltage drop due to the reactance of the circuit for a given kilovolt-ampere load. The percentage reactance on a given kilovolt-ampere base can be converted to ohms reactance on any voltage base by the formula

$$\begin{aligned} x \text{ (ohms)} &= \frac{\frac{E_L}{\sqrt{3}}x \frac{\%}{100}}{\frac{Kva1000}{E_L\sqrt{3}}} = \frac{x(\%)E_L^2}{Kva1000} \\ &= \frac{x(\%)(KV_L1000)^2}{Kva1000} \end{aligned}$$

or

$$x \text{ (ohms)} = \frac{(KV_L)^2 10x\%}{Kva} \quad (2)$$

where x is the reactance on the line-to-line voltage base KV_L or E_L .

To convert ohms reactance or resistance on a voltage base E_a to the equivalent value on a base E_b it is necessary to multiply by the square of the voltage ratio:

$$x_a \left(\frac{E_b}{E_a} \right)^2 = x_b \quad R_a \left(\frac{E_b}{E_a} \right)^2 = R_b \quad (3)$$

Example.—As an example, consider the calculation for three-phase faults on the system shown in Fig. 1 using percentage reactance and ohms reactance, neglecting resistance.

1. One 20,000-kva. generator, 12,000 volts, 12 per cent transient reactance.

2. One 25,000-kva. transformer, 12,000/66,000 volts, 7.5 per cent reactance.

3. A transmission line 50 miles long, 300,000 c.m. aluminum, 6 ft. equivalent spacing.

Percentage Reactance Method Using 25,000-kva. Base.

Generator reactance on 25,000-kva. base = $25,000/20,000 \times 12 = 15$ per cent.

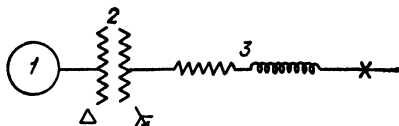


FIG. 1.—A fault on a simple transmission system.

From tables, the reactance of the transmission circuit = 32.7 ohms. Converting to percentage reactance on 25,000-kva. base by formula (2),

$$x_3\% = \frac{32.7 \times 25,000}{(66)^2 \times 10} = 18.8\%$$

The total reactance to the point of short circuit is the sum of the individual reactances, since all are in series.

$$x_1 + x_2 + x_3 = 15 + 7.5 + 18.8 = 41.3\%$$

Short-circuit kilovolt-amperes = $25,000/41.3 \times 100 = 60,500$.

$$I = \frac{60,500}{66 \times 1.73} = 530 \text{ amp.}$$

Calculation Using Ohms Reactance on a 66-kv. Base.

$$x_1 = \frac{12 \times (66)^2 \times 10}{20,000} = 26.1 \text{ ohms}$$

$$x_2 = \frac{7.5 \times (66)^2 \times 10}{25,000} = 13.08 \text{ ohms}$$

$$x_3 = 32.7$$

$$x_1 + x_2 + x_3 = 26.1 + 13.08 + 32.7 = 71.88 \text{ ohms}$$

$$I = \frac{E_n}{71.88} = \frac{66,000/\sqrt{3}}{71.88} = 530 \text{ amp.}$$

After the preliminary work is completed, the steps are as follows:

1. Determination of equivalent single impedance networks to replace the positive-, negative-, and zero-sequence networks.
2. Determination of the total sequence currents at the fault.
3. Determination of the distribution of the sequence currents in their respective networks.
4. Determination of sequence voltages at any desired point by calculating the voltage drop in each individual network to corresponding points using relations developed in Chap. III.
5. Conversion of sequence currents and voltages in each part of the network from common base to actual base, taking into account any phase shift introduced by transformer connections.
6. Determination of actual currents and voltages in all parts of the system by combining the sequence quantities at corresponding points in the networks.

In performing the operations connected with items 1, 2, and 3, the networks are reduced by successive simplification until equivalent networks are obtained. The sequence current flowing out of each network to the fault is determined by combining the networks in a manner to represent the type of fault considered. The distribution of each sequence current in its network is then determined by working back, step by step, through the original network reduction in reverse manner. If calculations are to be made for more than one type of fault at a particular location, it is advantageous to introduce *distribution factors*. This consists of introducing unit current in each network at the fault and working back through the network to determine the division of the unit current through the branches. The sequence current in any branch for any type fault may be found immediately by multiplying the sequence current at the fault for the particular network by the distribution factor for the branch considered. It is evident that distribution factors must be determined for each fault location. If only one type of fault is to be considered at a particular location, there is no advantage in the use of distribution factors.

In making short-circuit calculations there are certain shortcuts which materially decrease the amount of labor in reducing a network to the simplest form. It is usually sufficiently accurate in three-phase calculations to use either reactance or scalar impedance values. In calculations for ground currents, if

the system neutrals are grounded through resistance or reactance to limit the current, it is necessary to use the more accurate solution involving complex quantities, or the alternating-current calculating board, otherwise the error may be excessive. However, the methods applicable to network simplification hold for either method.

The first step in any network analysis is to make the simplest possible single-line diagram showing the circuit constants involved in the problem.

NETWORK REDUCTION

When two or more impedances are connected in series, the total impedance is equal to the sum of the individual impedances. Thus, in Fig. 2,

$$Z_T = Z_1 + Z_2 + Z_3 \quad (4)$$

For two or more circuits in parallel, the equivalent impedance is equal to the reciprocal of the sum of the reciprocals of the individual impedances. For two impedances in parallel, this becomes

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



FIG. 2.—Combination of impedances in series.

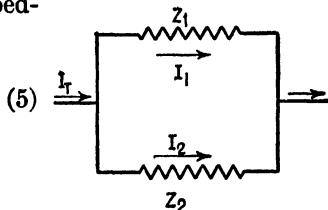


FIG. 3.—Combination of impedances in parallel.

The current divides between the branches inversely as the impedances. The total current is first found and then the division through the various branches. For this case

$$I_1 = \frac{Z_2}{Z_1 + Z_2} I_T \quad I_2 = \frac{Z_1}{Z_1 + Z_2} I_T$$

Where there are more than two branches in parallel, it is convenient to determine the total equivalent impedance by a step-by-step process. That is, branches 1 and 2 are first combined and then this equivalent combined in like manner with branch 3, etc.

Series-parallel circuits are combined and the current division determined by the method described. For example, in a circuit such as Fig. 4,

Combine Z_3 , Z_4 , and Z_5 , and let Z_a = the equivalent

$$Z_a = \frac{Z_3 Z_4}{Z_3 + Z_4} + Z_5$$

Combine Z_a with Z_2 , and let Z_b = the equivalent

$$Z_b = \frac{Z_a Z_2}{Z_a + Z_2}$$

The total equivalent impedance of the entire network is then

$$Z_T = Z_1 + Z_b + Z_6$$

The current division is then obtained by taking the steps in reverse order and tracing back the division through the various branches.

The network should be carefully examined for points of symmetry. Any points of equal potential may be connected together in the network, and all branches connecting these

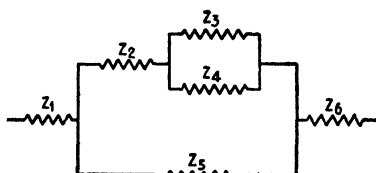


FIG. 4.—Combination of series-parallel impedances.

points entirely disregarded. This frequently results in considerable simplification.

In most complex networks the simple combinations of series and parallel impedances will not result in simplification to such an extent that the problem can be completely solved. In such cases it is necessary to resort to the use of a solution involving simultaneous equations, or by substituting equivalent networks for certain parts of the original network. The latter method in general is much simpler, as solutions involving simultaneous equations are very laborious in problems of this type.

Star-delta Transformation.—The substitution of an equivalent star of impedances for a delta, or vice versa, is very frequently useful as it may permit the combination in series or parallel with other parts of the network which otherwise could not be combined.

In order for two networks to be equivalent it is necessary that the terminal conditions of current and voltage be the same at corresponding terminals in the networks. For any similar combinations of loading on the two networks, the voltage drop and the total equivalent impedance between corresponding pairs of terminals in the two networks must be the same. Therefore, to change a star group of impedances to an equivalent delta, it is necessary to set up sufficient equations expressing the relations of corresponding quantities in the two networks, to maintain the specified measure of equivalence.

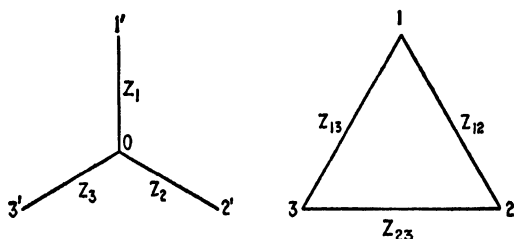


FIG. 5.—Substitution of an equivalent delta for a star group of impedances.

The star with branches Z_1 , Z_2 , and Z_3 and the desired equivalent delta with branches Z_{12} , Z_{23} , Z_{13} are shown in Fig. 5. The conditions at the terminals 1, 2, 3 must be the same as those at 1', 2', 3', respectively. Loads will now be applied to the two networks and sufficient equations written to satisfy the network relations. Assume a unit current enters terminal 1' and leaves 2', and also a unit current enters 1 and leaves 2. The voltage drop $V_{1'2'}$ and V_{12} between terminals 1'-2' and 1-2, respectively, can be equated thus,

$$\frac{1}{V_{12}} = \frac{1}{Z_{12}} + \frac{1}{Z_{13} + Z_{23}} \quad (6)$$

$$\frac{1}{V_{1'2'}} = \frac{1}{Z_1 + Z_2} \quad (7)$$

$$\frac{1}{V_{12}} = \frac{1}{V_{1'2'}} \quad (8)$$

The voltage drop and equivalent impedance between terminals are the same since unit current flows.

Since no current flows in the branch Z_3 , the potential of terminal 3' is the same as the junction O. By the network relations, the potential of terminal 3 must be the same as 3' and

hence of the junction O . If i is the portion of the unit current flowing through branches Z_{13} and Z_{23} , we may write

$$1(Z_1 + Z_2) = i(Z_{13} + Z_{23}) \quad (9)$$

$$Z_1 = iZ_{13} \quad \text{and} \quad Z_2 = iZ_{23} \quad (10)$$

$$\frac{Z_1}{Z_2} = \frac{Z_{13}}{Z_{23}} \quad (11)$$

Similarly with unit current flowing between terminals 1-3 and 2-3 in both networks

$$\frac{1}{V_{13}} = \frac{1}{V_{1'3'}} = \frac{1}{Z_1 + Z_3} = \frac{1}{Z_{13}} + \frac{1}{Z_{12} + Z_{23}} \quad (12)$$

also

$$\frac{Z_1}{Z_3} = \frac{Z_{12}}{Z_{23}}$$

for unit current between terminals 1'-3' and 1-3.

$$\frac{1}{V_{23}} = \frac{1}{V_{2'3'}} = \frac{1}{Z_2 + Z_3} = \frac{1}{Z_{23}} + \frac{1}{Z_{12} + Z_{13}} \quad (13)$$

also

$$\frac{Z_2}{Z_3} = \frac{Z_{12}}{Z_{13}}$$

for unit current between terminals 2'-3' and 2-3.

If the network relations are satisfied by the equations for this combination of loads, then the relations will be satisfied for all combinations of loads.

Solving the equations for the impedance branches of the delta in terms of the impedance of the star we find:

Conversion from star to equivalent delta,

$$\left. \begin{aligned} Z_{12} &= \frac{Y}{Z_3} \\ Z_{23} &= \frac{Y}{Z_1} \\ Z_{13} &= \frac{Y}{Z_2} \end{aligned} \right\} \quad (14)$$

where

$$Y = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$$

Conversely, to change the delta impedance network to an equivalent star,

$$\left. \begin{aligned} Z_1 &= \frac{Z_{12}Z_{13}}{\Delta} \\ Z_2 &= \frac{Z_{12}Z_{23}}{\Delta} \\ Z_3 &= \frac{Z_{23}Z_{13}}{\Delta} \end{aligned} \right\} \quad (15)$$

where

$$\Delta = Z_{12} + Z_{23} + Z_{31}$$

The substitution of a star group for a delta group does not result in a change of terminal conditions; consequently, as

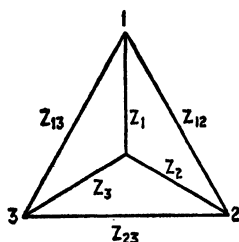


FIG. 6.—Equivalent star and delta groups of impedances.

far as the remainder of the network is concerned, there is no change. When one group is substituted for the other, the substituted group must be inserted in the network with the terminals 1, 2, and 3 occupying the same relative positions as in Fig. 6. It will be noted in the substitution process that, although the terminal conditions remain the same, the identity of junction points and branches within the network is lost.

To determine the division of current through the original network it is necessary to work back step by step through the derived networks to the original. Where a delta group of impedances is replaced by an equivalent star, the current division in the star is first found. When the current in two of the star branches is known, the current in the original corresponding delta branch can be found by dividing the sum of the voltage drops of the two star branches, with due regard to sign, by the delta impedance. In like manner the currents in other sides of the original delta are determined from the voltage drops in the equivalent star. Where a star group is replaced by a delta group, the current division, in working back through the network, is first found in the delta. The current in the corresponding legs of the star group can then be found by taking the summation of the delta currents at the terminal points.

Multipoint Star-mesh Transformation.—Some networks are too complex to be reduced by the simplification methods described. The principles of the star-delta transformation are readily extended to cover the case of converting a star with

any number of branches to an equivalent mesh. There is no general transformation, however, from a mesh to a multipoint star network.

Since the multipoint star impedances frequently occur in complex network systems, the derivation of an equivalent mesh will be given. The substitution of a mesh for a multipoint star eliminates the junction point of the star. This method can be applied only when the identity of the junction point is not required. In most cases the identity of the junction point is not important. In changing from a star group to a mesh group, the equivalent mesh will have a branch from each

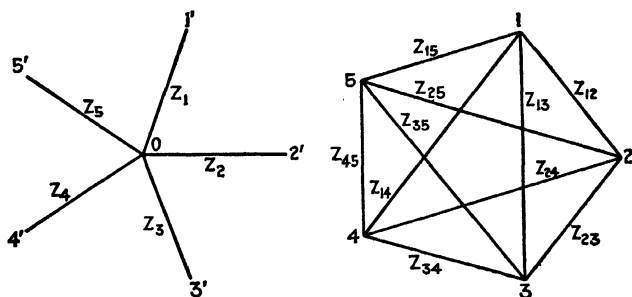


FIG. 7.—Conversion of a multipoint star group of impedances to an equivalent mesh.

terminal to every other terminal. The number of branches in an equivalent mesh substituted for an n point star is given by the expression $\frac{n}{2}(n - 1)$. It might appear that such a transformation would result in an equivalent mesh network that is more complicated than the original star because of additional branches. The simplification comes about by the elimination of the junction point in the original star. The result may be such that branches of the network which contained the star can then be paralleled with the sides of the equivalent mesh network which has been substituted for the star.

Assume that the star network containing $n = 5$ branches as shown in Fig. 7 is to be changed to the equivalent mesh having n terminal points and

$$(n - 1) + (n - 2) + (n - 3) + (n - 4) + (n - 5) = 10 \text{ branches.}$$

The points in the two networks having corresponding numbers are the same. The branches of the star are designated by single subscript and those in the mesh by double numbers. That is, Z_{12} in the mesh is the impedance of the branch between points 1 and 2, etc.

The same method of attack may be used in determining Z_{13} , Z_{14} , etc., knowing Z_1 , Z_2 , Z_3 , etc., as was used in deriving the delta equivalent of the three-point star. Sufficient combinations of loads must be applied to corresponding points in the two networks and enough equations obtained for corresponding factors in the two networks to fulfill the relations between networks necessary for equivalence.

The loads will be set up between terminal 1 and each of the other terminals in turn. Assume that a unit current enters at terminal 1' in the multipoint star (Fig. 7) and leaves at point 2'. Since no current flows in branches 3', 4', and 5', the potentials of these three points are the same and are equal to the potential at the junction point O . The total voltage drop between 1' and 2' is

$$1(Z_1 + Z_2).$$

This is also the equivalent impedance between the points since the current is of unit value.

Circulating unit current between terminals 1 and 2 of the mesh, the potentials of points 3, 4, and 5 must be equal to the potentials of 3', 4', 5', respectively, and hence equal to each other. These relations must be true to fulfill the conditions for equivalence. Since points 3, 4, and 5 must be at equal potential there cannot be a flow of current in any branches connecting the points of equal potential. The branches connecting points of equal potential may be eliminated and the mesh of Fig. 7 then reduces to Fig. 8. Let V_{12} be the voltage drop or impedance between 1 and 2 and $V_{1'2'}$ the voltage drop between terminals 1' and 2'. We may write

$$\frac{1}{V_{12}} = \frac{1}{Z_{12}} + \frac{1}{Z_{13} + Z_{23}} + \frac{1}{Z_{14} + Z_{24}} + \frac{1}{Z_{15} + Z_{25}} \quad (16)$$

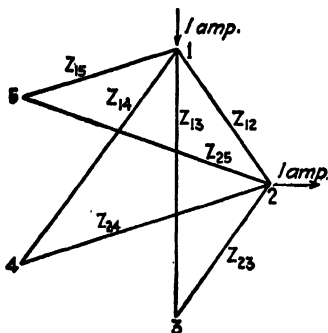


FIG. 8.

$$\frac{1}{V_{1'2'}} = \frac{1}{V_{12}} = \frac{1}{Z_1 + Z_2} \quad (17)$$

Each term in Eq. (16) represents a branch between 1 and 2 and, if multiplied by its respective current, must produce a voltage drop equal to V_{12} . For example, assume the second term $\frac{1}{Z_{13} + Z_{23}}$ carries a part i of the unit current, then

$$i(Z_{13} + Z_{23}) = 1(Z_1 + Z_2) \quad (18)$$

Since points O , $3'$, $4'$, and $5'$ of Fig. 7, and 3, 4, and 5 of Fig. 8, have equal potentials,

$$iZ_{13} = Z_1 \quad (19)$$

$$iZ_{23} = Z_2 \quad (20)$$

Dividing (19) by (20)

$$\frac{Z_{13}}{Z_{23}} = \frac{Z_1}{Z_2} \quad (21)$$

Also

$$\frac{Z_1}{Z_2} = \frac{Z_{13}}{Z_{23}} = \frac{Z_{14}}{Z_{24}} = \frac{Z_{15}}{Z_{25}} \quad (22)$$

Equations may be set up in a similar manner for unit current entering 1 and leaving 3, 4, and 5 in turn. This gives the following:

Current out at 3,

$$\frac{Z_1}{Z_3} = \frac{Z_{12}}{Z_{23}} = \frac{Z_{14}}{Z_{34}} = \frac{Z_{15}}{Z_{35}} \quad (23)$$

Current out at 4,

$$\frac{Z_1}{Z_4} = \frac{Z_{12}}{Z_{24}} = \frac{Z_{13}}{Z_{34}} = \frac{Z_{15}}{Z_{45}} \quad (24)$$

Current out at 5,

$$\frac{Z_1}{Z_5} = \frac{Z_{12}}{Z_{25}} = \frac{Z_{13}}{Z_{35}} = \frac{Z_{14}}{Z_{45}} \quad (25)$$

It is not necessary to set up equations for current flowing in at terminal 2 and out at all other terminals, as an examination of the network will show that all terminals and branches have been considered. Therefore, if the networks satisfy the equations set up, they will satisfy any other combinations. It is now only necessary to solve for the unknowns in terms of the knowns using the relations developed.

$$Z_{23} = \frac{Z_3}{Z_1} Z_{13} \quad (26)$$

$$Z_{34} = \frac{Z_3}{Z_1} Z_{14} = \frac{Z_4 Z_3}{Z_2 Z_1} Z_{12} \quad (27)$$

$$Z_{45} = \frac{Z_5}{Z_1} Z_{14} = \frac{Z_5 Z_4}{Z_1 Z_2} Z_{12} \quad (28)$$

$$Z_{13} = Z_{23} = \frac{Z_1}{Z_2} Z_{23} = \frac{Z_3}{Z_2} Z_{12} \quad (29)$$

$$Z_{24} = \frac{Z_4}{Z_1} Z_{12} \quad (30)$$

$$Z_{35} = \frac{Z_5}{Z_1} Z_{13} = \frac{Z_3 Z_5}{Z_1 Z_2} Z_{12} \quad (31)$$

$$Z_{14} = \frac{Z_1}{Z_2} Z_{24} = \frac{Z_4}{Z_2} Z_{12} \quad (32)$$

$$Z_{25} = \frac{Z_5}{Z_1} Z_{12} \quad (33)$$

$$Z_{15} = \frac{Z_1}{Z_2} Z_{25} = \frac{Z_5}{Z_2} Z_{12} \quad (34)$$

Substituting in Eq. (16) the expressions obtained in (26), (29), (30), (32), (33), and (34),

$$\frac{1}{Z_1 + Z_2} = \frac{1}{Z_{12}} \left[1 + \frac{Z_1 Z_2}{Z_3(Z_1 + Z_2)} + \frac{Z_1 Z_2}{Z_4(Z_1 + Z_2)} + \frac{Z_1 Z_2}{Z_5(Z_1 + Z_2)} \right] \quad (35)$$

Solving for Z_{12} ,

$$Z_{12} = Z_1 Z_2 \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5} \right] \quad (36)$$

Let

$$Y = \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5} \right]$$

substituting the value of Z_{12} in Eqs. (26) to (34), the expression is obtained for all the branch impedances in the mesh network in terms of the star network impedances.

$$\begin{aligned} Z_{12} &= Z_1 Z_2 Y & Z_{23} &= Z_2 Z_3 Y & Z_{34} &= Z_3 Z_4 Y & Z_{45} &= Z_4 Z_5 Y \\ Z_{13} &= Z_1 Z_3 Y & Z_{24} &= Z_2 Z_4 Y & Z_{35} &= Z_3 Z_5 Y \\ Z_{14} &= Z_1 Z_4 Y & Z_{25} &= Z_2 Z_5 Y \\ Z_{15} &= Z_1 Z_5 Y \end{aligned}$$

In general to change a star of any number of branches n into an equivalent general network of mesh form, take the sum

of the reciprocals of all the star branch impedances and determine this constant. The impedance of a branch of the equivalent mesh network between any two points 1 and 2 of the original network is obtained by multiplying the constant by the product of the two star impedances which connect 1 and 2.

In making a multistar mesh transformation, it may be advantageous to use admittances rather than impedances because the resultant admittances, after the transformation is made, parallel directly with other branches in the original network containing the star. The expression for admittance is as follows: y_1, y_2 , etc., is the admittance of the branches of the star and y_{12}, y_{13} , etc., the branches of the mesh. Let

$$\Delta = \frac{1}{y_1} + \frac{1}{y_2} + \frac{1}{y_3} + \cdots \frac{1}{y_n}$$

then

$$y_{12} = y_1 y_2 \Delta$$

$$y_{13} = y_1 y_3 \Delta$$

$$y_{1n} = y_1 y_n \Delta$$

Circuits with Mutual Coupling.—Mutual coupling between two or more circuits sometimes appears in networks. The

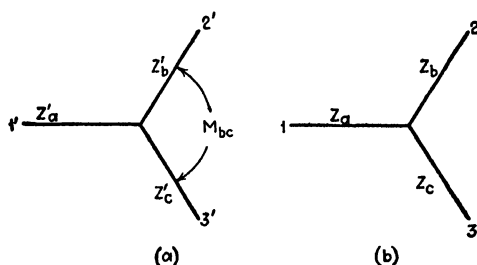


FIG. 9.—(a) Original star with mutual between two branches; (b) equivalent star without mutual.

positive- and negative-sequence currents being balanced and the phase impedances assumed symmetrical, there is no interaction between the currents in two or more branches of their networks. In the zero-sequence network, however, there may be a mutual coupling of considerable magnitude between closely spaced parallel circuits. In simplifying a network containing a mutual impedance coupling, it is necessary to eliminate the mutual by substituting an equivalent network containing only self-impedances.

A star group of impedances (Fig. 9a) with a mutual impedance between two branches may be replaced by a star without mutual (Fig. 9b).

Equating the voltage drops for unit current flowing between terminals 1-2, 1-3, and 2-3 of each network:

$$V_{1'2'} = Z'_a + Z'_b = V_{12} = Z_a + Z_b \quad (37)$$

$$V_{1'3'} = Z'_a + Z'_c = V_{13} = Z_a + Z_c \quad (38)$$

$$Z_{2'3'} = Z'_b + Z'_c - 2M_{bc} = V_{23} = Z_b + Z_c \quad (39)$$

Solving for Z_a , Z_b , and Z_c in terms of Z'_a , Z'_b and Z'_c

$$\left. \begin{aligned} Z_a &= Z'_a + M_{bc} \\ Z_b &= Z'_b - M_{bc} \\ Z_c &= Z'_c - M_{bc} \end{aligned} \right\} \quad (40)$$

If the original network consists of only two branches Z'_b and Z'_c with mutual impedance between them, then $Z'_a = 0$ and in the equivalent star $Z_a = M_{bc}$. The other terms are unchanged.

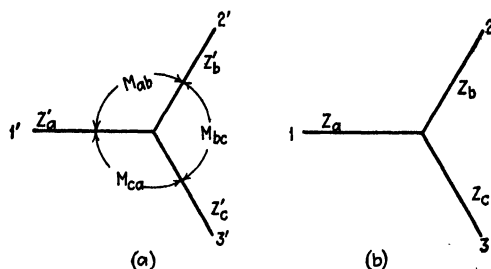


FIG. 10.—(a) Original star with mutual between each pair of branches; (b) equivalent star without mutuals.

A three-point star such as Fig. 10a with mutual impedance between adjacent branches can be replaced by a three-point star without mutual impedance (Fig. 10b). The derivation of the conversion will not be given, as the same methods are used as in the previous example.

$$\left. \begin{aligned} Z_a &= Z'_a - M_{ab} - M_{ca} + M_{bc} \\ Z_b &= Z'_b - M_{bc} - M_{ab} + M_{ca} \\ Z_c &= Z'_c - M_{ca} - M_{bc} + M_{ab} \end{aligned} \right\} \quad (41)$$

A delta group of impedances containing a mutual impedance between two branches (Fig. 11a) can be replaced by a delta without mutual impedance (Fig. 11b).

$$\left. \begin{aligned} Z_a &= \frac{Z'_a(Z'_b + Z'_c) + M_{ac}(M_{ac} - 2Z'_a)}{Z'_b + Z'_c - M_{ac}} \\ Z_b &= Z'_b \\ Z_c &= \frac{Z'_c(Z'_b + Z'_a) + M_{ac}(M_{ac} - 2Z'_c)}{Z'_b + Z'_a - M_{ac}} \end{aligned} \right\} \quad (42)$$

Example of System Short-circuit Calculations.—In order to demonstrate the application of the method of symmetrical components to the determination of currents and voltages during short circuits, an example will be worked out for the four types of faults. The system chosen for calculation is shown in Fig. 12a consisting of a hydroelectric generating station having generators A and B and a steam station with generator F. The two stations are connected by two 110,000-volt transmission circuits.

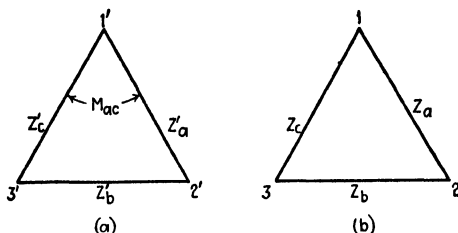


FIG. 11.—(a) Delta group of impedances with single mutual; (b) equivalent delta without mutual.

The apparatus is assumed to have the following characteristics:

Station 1

Generator A. 50,000 kva., 12,000 volts; transient reactance 40 per cent, negative-sequence reactance 25 per cent.

Generator B. 20,000 kva., 12,000 volts; transient reactance 35 per cent, negative-sequence reactance 25 per cent.

Transformer C. 50,000 kva., 12,000 volts delta to 110,000 volts star, solidly grounded neutral, 8 per cent reactance.

Transformer D. 20,000 kva., 12,000 volts delta to 110,000 volts star, ungrounded neutral, 8 per cent reactance.

Station 2

Generator G. 25,000 kva., 12,000 volts; transient reactance 20 per cent, negative-sequence reactance 15 per cent.

Transformer E. 60,000 kva., 12,000 volts delta to 110,000 volts star, solidly grounded neutral, 8 per cent reactance.

Line 1. 50 miles long, reactance per mile 0.8 ohm, 110,000 volts.

Line 2. Same as line 1.

The zero-sequence reactance is assumed to be three times the positive-sequence reactance for the transmission lines.

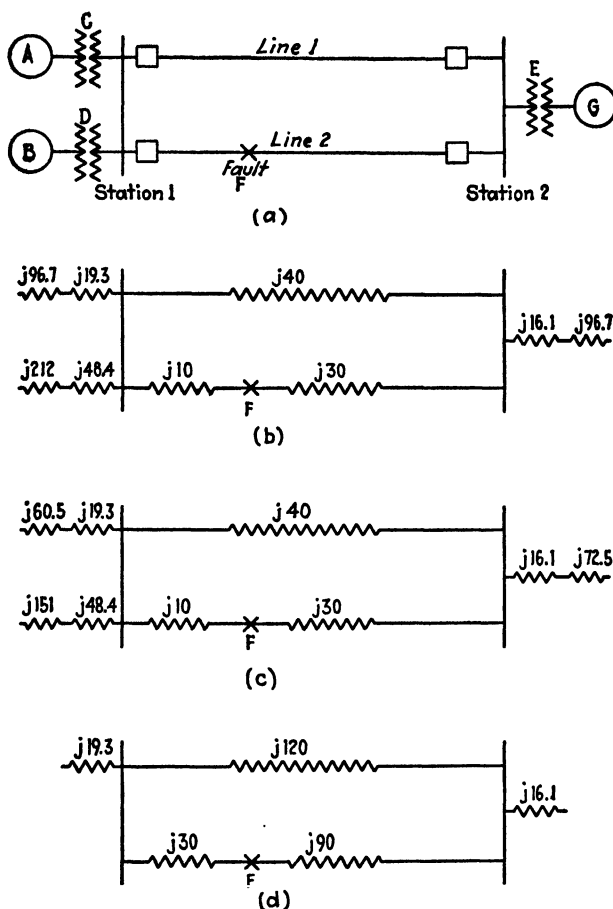


FIG. 12.—(a) Single-line system diagram; (b) positive-sequence network; (c) negative-sequence network; (d) zero-sequence network.

The accuracy desired in the calculations is such that the following assumption can be made:

1. All generated voltages are equal and in phase.
2. The resistance components are neglected, and lumped reactance constants used.
3. The faults occur with no load on the system, and transformer magnetizing shunt admittance and transmission-line capacitance is neglected.

4. The transmission circuits are separated by a sufficient distance that the mutual impedance between lines for zero-sequence currents can be neglected.

These assumptions are the same as are usually made when using the direct-current calculating table. The scalar impedance could be used just as readily as reactance.

All faults will be considered as occurring on line 2 at one-fourth of the distance from station 1 to station 2. The first step is to reduce all reactances to a common base. For this case the reactances will be converted to ohms on 110-kv. base.

$$\begin{aligned}x_A &= \frac{0.40 (110,000)^2}{50,000,000} = 96.7 \text{ ohms on 110-kv. base} \\x_B &= 212 \\x_C &= 19.3 \\x_D &= 48.4 \\x_E &= 16.1 \\x_G &= 96.7\end{aligned}$$

The impedance networks for the positive, negative, and zero sequences are then set up in single-line form in Fig. 12*b*, *c* and *d*, respectively. The negative-sequence network is the same as the positive except for the value of reactance used for the generators. Transformer *D* does not appear in the zero-sequence network because the neutral is ungrounded. The zero-sequence reactance of the generator does not appear in the zero-sequence network because the delta low-voltage windings of the transformers effectively block the flow of zero-sequence currents.

The next step is to reduce the positive-, negative-, and zero-sequence networks each to a single equivalent impedance. Since more than one type of fault is to be calculated for the same fault location, it is also desirable to work back through the networks to determine the current distribution factors for the branch currents in the three networks. The current distribution factor for any branch of any network is the ratio of the current in that branch to the current at the fault in the same network. The most convenient method to determine the distribution is to assume unit current flowing out of each network at the fault and work back through each network to determine the distribution of the unit current in the various branches. After the total currents at the fault in the sequence networks

are determined by the proper combination of the network equivalent impedances, all that is necessary to find the current in any branch is to multiply the total current at the fault in the particular network by the distribution factor for the branch considered. After the sequence currents at the fault and the distribution in the networks are found, the phase currents, sequence voltages, phase voltages, and line voltages may be readily calculated.

Network Reduction.—Consider the reduction of the positive-sequence network Fig. 12b. This network is redrawn and shown

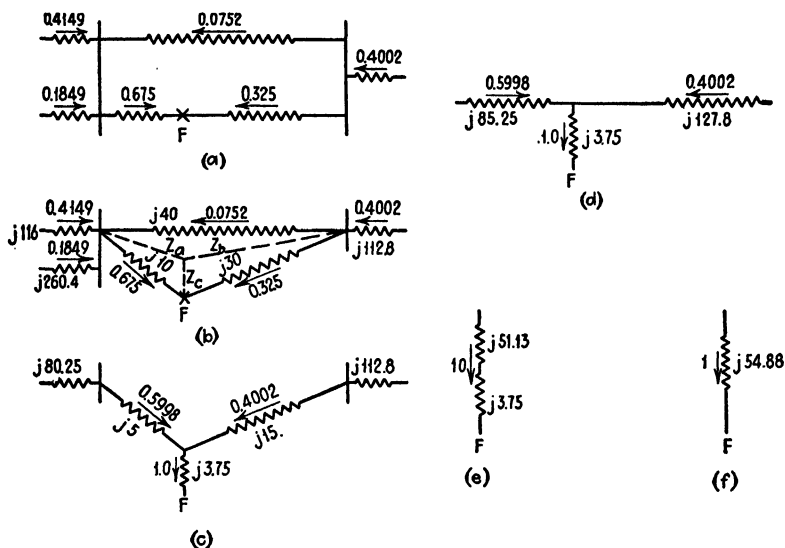


FIG. 13.—Positive-sequence network reduction and unit-current distribution.

in simplified form in Fig. 13b, the generator and transformer reactances of each branch being combined. Figure 13b is reduced to 13c by paralleling the two reactances on the left and substituting an equivalent group of star reactances for the delta. The equivalent star is shown dotted in the delta of Fig. 13b. The equivalent star branches are calculated by means of Eqs. (14). Thus,

$$Z_a = \frac{j10 \times j40}{j80} = j5$$

$$Z_b = \frac{j40 \times j30}{j80} = j15$$

$$Z_e = \frac{j30 \times j10}{j80} = j3.75$$

The star of Fig 13c can be changed to 13d by adding the series reactances. Paralleling the reactances of 13d and adding the series reactances of 13e, the final equivalent impedance is shown in 13f.

Unit-current Distribution.—The distribution factors for the positive-sequence network can now be found by assuming unit current flowing out of the network into the fault and working back through the network. The determination of the distribution is self-evident from an inspection of Fig. 13b, c, d, e, and

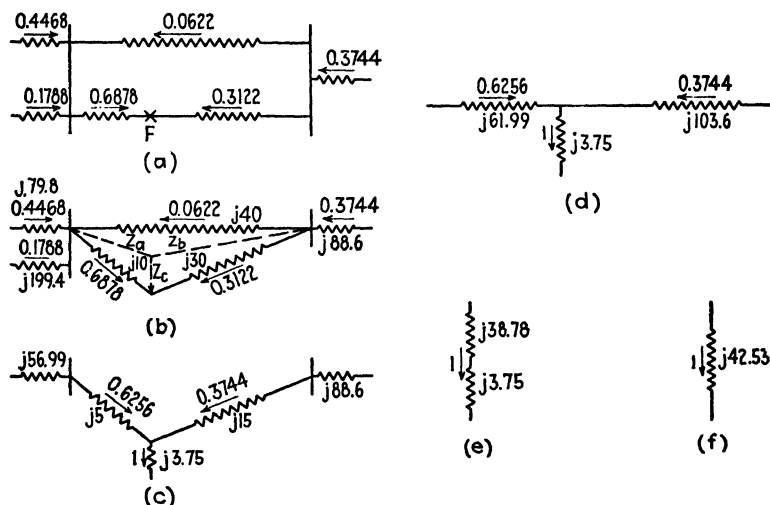


FIG. 14.—Negative-sequence network reduction and unit-current distribution.

f. The current in the delta of 13b is found from the current in the equivalent star by making use of the fact that for equivalence the potential drop between terminals must be the same for the star or delta. Thus, equating the drop between two terminals of the star and dividing by the corresponding delta impedance gives the delta current. For example, the current in the right-hand side of the delta (Fig. 13b) is equal to

$$\frac{1.0 \times j3.75 + 0.4002 \times j15}{j30} = 0.325 \text{ amp.}$$

The currents in the other sides of the delta may be found similarly or by equating the currents at the junction points.

The reduction of the zero- and negative-sequence networks and the determination of their distribution factors are accom-

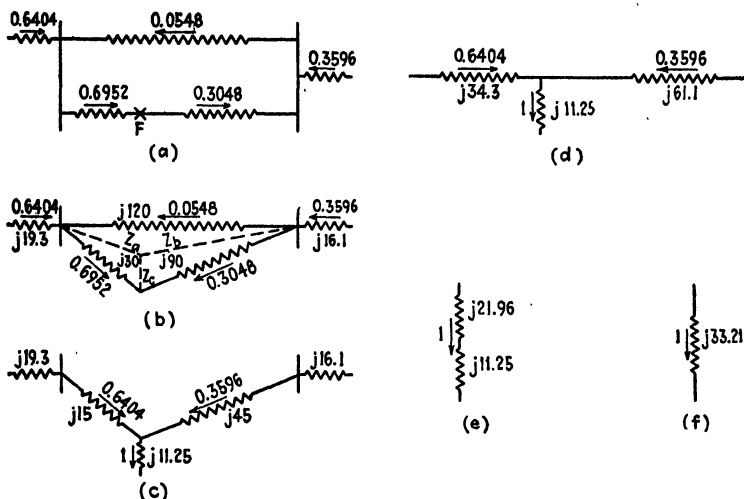


FIG. 15.—Zero-sequence network reduction and unit-current distribution.

plished in the same way. The steps for the negative- and zero-sequence networks are shown in Figs. 14 and 15, respectively.

Single-line-to-ground Fault on Phase a Sequence Currents.—The positive-, negative-, and zero-sequence equivalent impedances are connected in series for determining the fault current for a single-line-to-ground fault as shown in Fig. 16. The sequence currents at the fault are given by the expression

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_a}{Z_0 + Z_1 + Z_2}$$

All impedances are based on the line voltage of 110 kv.; therefore the voltage to neutral E_a is 63,584. If the position of the voltage vector E_a is taken along the imaginary axis, i.e., $E_a = jE_a$, since the impedances are all reactance, the currents will all be real. The

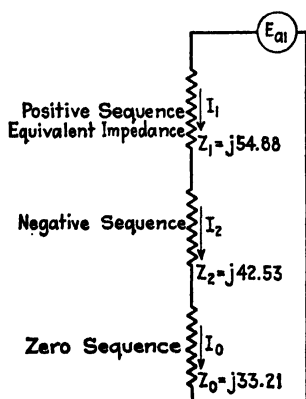
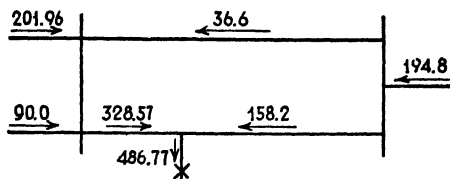


FIG. 16.—Connection of equivalent impedances for single-phase-to-ground fault.

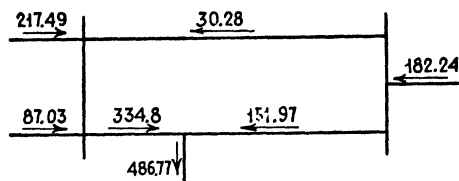
facility of this device is that the algebraic operations are simpler.

$$I_{a0} = I_{a1} = I_{a2} = \frac{j63,584}{j(54.88 + 42.53 + 33.21)} = 486.77 \text{ amp.}$$

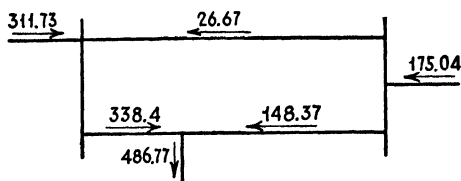
The assumed direction of current flow in the three networks is toward the fault. The distribution of the fault current



(a) Positive Sequence Current Distribution



(b) Negative Sequence Current Distribution



(c) Zero Sequence Current Distribution

FIG. 17.—Current distribution in the three sequence networks for single-line-to-ground fault.

in any network is found by multiplying the current in that network at the fault by the distribution factor for the particular branch. Thus the current in the left-hand faulted branch of the positive-sequence network (Fig. 13b) is

$$486.77 \times 0.675 = 328.57 \text{ amp.}$$

The distribution of all sequence currents can be found in like manner. The complete system is shown in Fig. 17a, b, and c.

Phase Currents.—The current in the two unfaulted phases at the fault is zero.

$$I_a = 3 \times 486.77 = 1460.31$$

$$I_b = (1 + a^2 + a)I_{a1} = 0$$

$$I_c = (1 + a + a^2)I_{a1} = 0$$

The phase currents at any point in the system may be found from the relations

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} = I_{a0} - 0.5(I_{a1} + I_{a2}) - j0.866(I_{a1} - I_{a2})$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} = I_{a0} - 0.5(I_{a1} + I_{a2}) + j0.866(I_{a1} - I_{a2})$$

For example, referring to Fig. 17, the current flowing in the faulted line from the left-hand bus is

$$I_a = 338.4 + 328.57 + 334.8 = 1001.77$$

$$I_b = 338.4 - 0.5(328.57 + 334.8) - j0.866(328.57 - 334.8) = +6.72 + j5.39$$

$$I_c = 6.72 - j5.39.$$

The residual current at any point is equal to the vector sum of the three-phase currents and is also equal to three times the zero-sequence current:

$$I_R = I_a + I_b + I_c = 1015.21$$

$$I_R = 3I_{a0} = 3 \times 338.4 = 1015.20$$

The complete phase-current distribution is shown in Fig. 18.

Actual Currents.—All currents found thus far have been in terms of the 110-kv. voltage base which is the voltage of the lines upon which the faults occur. The currents in passing through the transformers are changed in magnitude due to the turn ratio and in phase angle by the delta-star winding connections. The actual currents on the 12-kv. side of the transformers can be found from the relations

$$I'_a = n(I_b - I_c)$$

$$I'_b = n(I_c - I_a)$$

$$I'_c = n(I_a - I_b)$$

Consider generator *A* and transformer *C*. The transformer turn ratio $n = 63,584/12,000 = 5.30$. The current in the lines on the delta side of the transformer in terms of 12,000 volts is:

$$I'_a = [(102.01 + j13.45) - (102.01 - j13.45)]5.3 = +j142.57$$

$$I'_b = [(102.01 - j13.45) - (731.18)]5.3 = -3334.6 - j71.28$$

$$I'_c = [(731.18) - (102.01 + j13.45)]5.3 = +3334.6 - j71.28$$

The currents on the low-voltage side of the other transformers are found in a similar manner. The phase currents in all parts of the circuit are shown in Fig. 18. The symmetrical com-

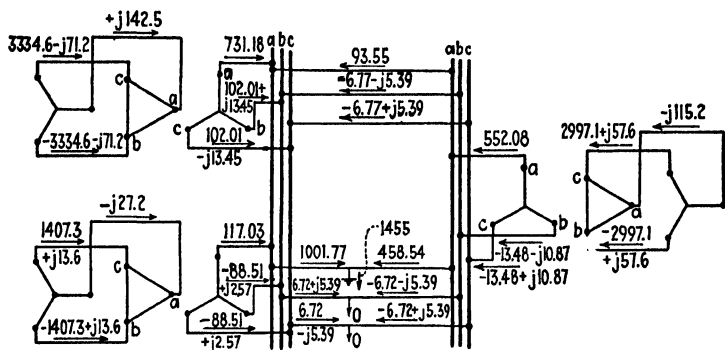


FIG. 18.—Phase-current distribution for single line-to-ground fault.

ponents of the currents on the 12-kv. side of the transformers could have been found by the relations of Chap. II and these components combined to find the phase currents and the same results obtained.

The currents in the delta winding of transformer *C* are:

$$I'_A = nI_a = 5.3(731.18) = 3875.2$$

$$I'_B = nI_b = 5.3(102.01 + j13.45) = 540.6 + j71.28$$

$$I'_C = nI_c = 5.3(102.01 - j13.45) = 540.6 - j71.28$$

Phase-sequence Voltages.—The phase-sequence voltages at any point in the network may be found after the distribution of the current in the network is determined. For example, the positive-sequence voltage to neutral at the left-hand high-voltage bus is equal to the positive-sequence generated voltage minus the drop due to the positive-sequence current flowing through the generator and transformer,

$$E'_{a1} = j63,584 - 201.96 \times j116 = +j40,157$$

Since no generated negative- or zero-sequence voltages exist, the negative- and zero-sequence voltages at the bus are merely the negative drop to the point considered:

$$E'_{a2} = 0 - 217.49 \times j79.8 = -j17,355$$

$$E'_{a0} = 0 - 311.73 \times j19.3 = -j6016$$

Phase Voltages.—The phase voltages are found by combining the phase-sequence voltages:

$$E'_a = E'_{a0} + E'_{a1} + E'_{a2} = +j16,786$$

$$\begin{aligned} E'_b &= E'_{a0} + a^2 E'_{a1} + a E'_{a2} = E'_{a0} - 0.5(E'_{a1} + E'_{a2}) - \\ &\quad j0.866(E'_{a1} - E'_{a2}) \\ &= -6016 - 0.5(j40,157 - j17,355) - \\ &\quad j0.866(j40,157 + j17,355) \\ &= 49,805 - j17,417 \end{aligned}$$

$$\begin{aligned} E'_c &= E'_{a0} + a E'_{a1} + a^2 E'_{a2} = E'_{a0} - 0.5(E'_{a1} + E'_{a2}) + \\ &\quad j0.866(E'_{a1} - E'_{a2}) \\ &= -49,805 - j17,417 \end{aligned}$$

Voltages at the fault:

$$E''_{a1} = j40,157 - 328.57 \times j10 = +j36,872$$

$$E''_{a2} = -j17,355 - 334.80 \times j10 = -j20,703$$

$$E''_{a0} = -j6016 - 338.4 \times j30 = -j16,168$$

$$E''_a = 0$$

$$E''_b = 49,860 - j24,252$$

$$E''_c = -49,860 - j24,252$$

Line voltages at the fault:

$$E''_A = E''_c - E''_b = -99,720$$

$$E''_B = E''_a - E''_c = 49,860 + j24,252$$

$$E''_C = E''_b - E''_a = 49,860 - j24,252$$

Line-to-line Fault, Phase b to c.—The zero-sequence network does not enter the line-to-line-fault calculations. The same equivalent positive- and negative-sequence impedances are used as for the single-line-to-ground case. The impedances are connected as shown in Fig. 19. It is important that the positive direction of current flow be as shown. It will be observed that the negative-sequence current is equal to the positive but is of negative value.

The sequence currents at the fault are

$$I_{a1} = \frac{j63,584}{j(54.88 + 42.53)} = \frac{j63,584}{j97.41} = 652.72$$

$$I_{a2} = -I_{a1} = -652.72$$

$$I_{a0} = 0$$

Phase currents at fault:

$$I_a = 0$$

$$I_b = a^2(652.72) - a(652.72) = (a^2 - a)652.72 = -j1129.2$$

$$I_c = (a - a^2)652.72 = +j1129.2$$

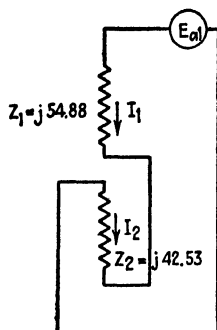


FIG. 19.—Connection of equivalent impedances for line-to-line faults.

The sequence-current distribution is found in the same manner as previously described. The sequence currents are shown in Fig. 20.

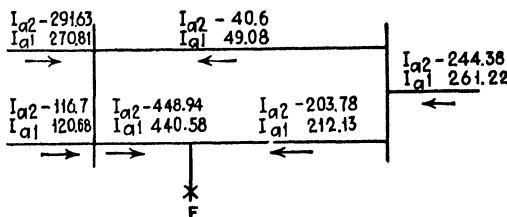


FIG. 20.—Sequence-current distribution for line-to-line fault.

Voltages at left-hand bus:

$$E'_{a1} = j63,584 - 270.81 \times j116 = +j32,170$$

$$E'_{a2} = 0 - (-291.63)j79.8 = +j23,272$$

$$E'_{a0} = 0$$

$$E'_a = j32,170 + j23,272 = +j55,442$$

$$E'_b = -0.5(j32,170 + j23,272) - j0.866(j32,170 - j23,272) = +7705 - j27,721$$

$$E'_c = -7705 - j27,721$$

Voltages at the fault:

$$E''_{a1} = +j32,170 - (440.58)j10 = +j27,765$$

$$E''_{a2} = +j23,272 - (-448.94)j10 = +j27,761$$

$$E''_a = +j55,526$$

$$E''_b = -j27,763$$

$$E''_c = -j27,790$$

Line voltages:

$$E'_A = 0$$

$$E'_B = +j83,316$$

$$E'_C = -j83,316$$

Double-line-to-ground Fault, Phase *b* to *c*.—The equivalent impedances for this type of fault are connected as shown in Fig. 21. The negative- and zero-sequence currents are both negative for this case. The sequence currents at the fault are:

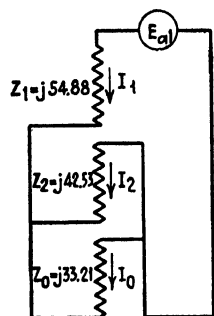


FIG. 21.—Connection of equivalent impedances for double-line-to-ground fault.

$$I_{a1} = \frac{j63,584}{j\left(54.88 + \frac{33.21 \times 42.53}{75.74}\right)}$$

$$= \frac{j63,584}{j(54.88 + 18.649)} = 864.75$$

$$I_{a2} = -I_{a1} \frac{Z_0}{Z_0 + Z_2}$$

$$= (-864.75) \frac{j33.21}{j75.74} = -379.15$$

$$I_{a0} = -I_{a1} \frac{Z_2}{Z_0 + Z_2} = -485.60$$

The sequence-current distribution is shown in Fig. 22.

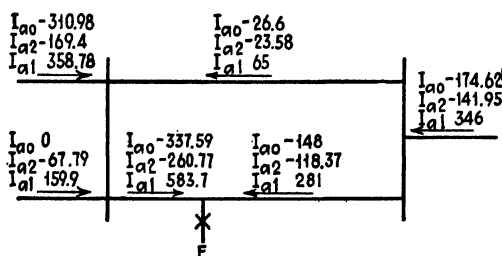


FIG. 22.—Sequence-current distribution for double-line-to-ground fault.

The phase currents at the fault are:

$$I_a = 0$$

$$I_b = -485.6 + a^2(864.7) + a(-379.1) = -628.4 - j1077.1$$

$$I_c = -628.4 + j1077.1$$

Voltages at the left-hand bus:

$$E'_{a1} = j63,584 - 358.78 \times j116 = +j21,966$$

$$E'_{a2} = 0 - (-169.4)j79.8 = +j13,518$$

$$E'_{a0} = 0 - (-310.98)j19.3 = +j6002$$

$$E'_a = +j41,486$$

$$E'_b = +7316 - j11,740$$

$$E'_c = -7316 - j11,740$$

Voltages at fault:

$$E''_{a1} = j21,966 - 583.7 \times j10 = +j16,129$$

$$E''_{a2} = +j16,125$$

$$E''_{a0} = +j16,129$$

$$E''_a = +j48,383$$

$$E''_b = 0$$

$$E''_c = 0$$

Line voltages:

$$E'_A = 0$$

$$E'_B = +j48,383$$

$$E'_C = -j48,383$$

Three-phase Short Circuit.—Only the positive-sequence equivalent impedance is involved for this case.

$$I_{a1} = \frac{j63,584}{j54.88} = 1158.6$$

$$I_{a2} = I_{a0} = 0$$

$$I_a = 1158.6$$

$$I_b = a^2 I_a = -579 - j1003$$

$$I_c = a I_a = -579 + j1003$$

The current distribution is shown in Fig. 23.

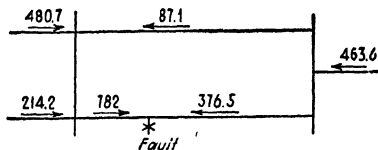


FIG. 23.—Current distribution for three-phase fault.

The voltage at the left-hand bus is:

$$E'_{a1} = j63,584 - 480.7 \times j116 = +j7823$$

$$E'_{a2} = E'_{a0} = 0$$

The sequence voltages, phase voltages, and line voltages are all zero at the fault.

CHAPTER V

DETERMINATION OF PHASE-SEQUENCE CONSTANTS OF TRANSFORMERS AND AUTOTRANSFORMERS AND THEIR CONNECTIONS IN THE SEQUENCE NETWORKS

TWO-WINDING TRANSFORMERS

In the determination of the sequence-impedance networks for calculating fault currents it is necessary to substitute equivalent impedances to represent transformers. The positive- and negative-sequence impedances of transformers are the same, since phase symmetry can usually be assumed. The transformer is a static device; consequently its impedance is not influenced by the phase sequence of voltages and currents. The positive- or negative-sequence equivalent impedance of a transformer is exactly the same as the ordinary equivalent impedance with all values reduced to a common base. In general, for short-circuit calculations, the mutual or magnetizing impedance of the transformer may be neglected, as the magnetizing current is negligible in comparison with the fault current. The equivalent impedance of the two-winding transformer can be represented by two series impedances, one for the high-voltage and one for the low-voltage winding. Each of these impedances represents the separate leakage reactance and resistance of the individual windings. Actually it is not necessary to determine the separate impedances, as the equivalent impedance is equal to the sum of these impedances. The equivalent impedance is simply the impedance measured by short-circuiting one winding and applying a voltage to the other, of such value as will produce full-load currents in both windings.

The zero-sequence impedance of a circuit involving a transformer depends upon the external connections from the transformer to the remainder of the circuit. If no neutral connection is present there is no path for the flow of zero-sequence currents

to earth, and consequently the impedance is infinite. The equivalent zero-sequence impedance of a transformer can be represented in the same manner as the positive and negative impedance, but the connections to the external circuit in the equivalent network may differ.

In order to obtain a thorough understanding of the representation of transformers in the sequence-impedance networks it is convenient to resort to a consideration of the fundamentals of coupled circuits. Consider the two-winding transformer of Fig. 1.¹ H and L are the high- and low-voltage windings,

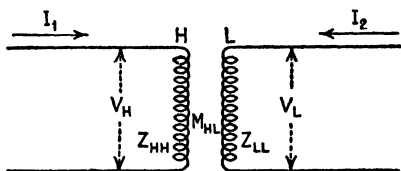


FIG. 1.—Representation of two-winding transformer.

respectively. In this discussion a one-to-one ratio of transformation will be assumed for simplicity. The following terminology will be used:

L_H = self-inductance of high-voltage winding.

L_L = self-inductance of low-voltage winding.

L_{HL} = mutual inductance between H and L winding.

$Z_{HH} = R_H + j\omega L_H$ = self-impedance of H winding.

$Z_{LL} = R_L + j\omega L_L$ = self-impedance of L winding.

$M_{HL} = j\omega L_{HL}$ = mutual impedance between H and L windings.

I_0 = exciting or magnetizing current.

Assuming a current flow as in Fig. 1, the following equations may be written for the terminal voltages:

$$V_H = Z_{HH}I_1 + M_{HL}I_2 \quad (1)$$

$$V_L = Z_{LL}I_2 + M_{HL}I_1 \quad (2)$$

The difference between the terminal voltages is the effective impedance drop.

$$V_H - V_L = (Z_{HH} - M_{HL})I_1 - (Z_{LL} - M_{HL})I_2 \quad (3)$$

¹ DAHL, O. G. C., "Electric Circuits," vol. I. Short-circuit Calculations and Steady-state Theory, p. 38, McGraw-Hill Book Company, Inc., New York.

The sum of the currents in the two windings must be equal to the exciting current.

$$I_1 + I_2 = I_e \quad (4)$$

An examination of Eqs. (3) and (4) will show that the two-winding transformer can be represented by an equivalent T circuit. The leakage impedance of the H winding is $(Z_{HH} - M_{HL})$ and that of the L winding is $(Z_{LL} - M_{HL})$. These are the separate leakage impedances of the two windings and make up the two arms of the T equivalent circuit (Fig. 2). The pillar impedance, which carries the exciting current, is obtained by the relations

FIG. 2.—Representation of two-winding transformer with equivalent T circuit.

$$Z_M = \frac{V_H - (Z_{HH} - M_{HL})I_1}{I_e} \quad (5)$$

Substituting the value of V_H from Eq. (1) in (5) gives

$$Z_M = \frac{M_{HL}(I_1 + I_2)}{I_e} = M_{HL} \quad (6)$$

Equation (6) shows that the pillar impedance is the mutual impedance between windings. This term is purely reactance since the classical equations for coupled circuits do not include the effect of core losses. This effect may be taken into account by the addition of a resistance component to the pillar impedance.

In calculating short-circuit currents the exciting current can be neglected since it usually does not exceed 5 per cent of the full-load current. In this case, Eq. (4) becomes

$$I_1 + I_2 = 0$$

or

$$I_2 = -I_1 \quad (7)$$

Substituting Eq. (7) in (3) gives

$$V_L = I_1(Z_{HH} - M_{HL}) + (Z_{LL} - M_{HL})I_1 = (Z_{HH} + Z_{LL} - 2M_{HL})I_1 \quad (8)$$

The equivalent circuit then reduces to that shown in Fig. 3. Now $(V_H - V_L)$ represents the impedance drop through the

transformer. Assume the L winding is short-circuited, then $V_L = 0$. Therefore,

$$V_H = (Z_{HH} + Z_{LL} - 2M_{HL})I_1 = Z_{HL}I_1 \quad (9)$$

It is evident that $Z_{HL} = (Z_{HH} + Z_{LL} - 2M_{HL})$ is the equivalent impedance of the transformer.

The representation of the two-winding transformer impedance in the positive-, negative-, and zero-sequence networks can now be derived. In the following discussion the approximate equivalent circuit of Fig. 3 will be used, the exciting current being neglected. However, the voltage E_ϕ , which would exist if the exciting current were present owing to the flux in the core, will be assumed to exist. This approximation will simplify the derivation without detracting from the value of the results because of the negligible effect of the exciting current.

Consider the star-delta connected transformers shown in Fig. 4. Assume for simplicity that all constants are expressed in terms of the high-voltage star-connected winding in the usual manner by multiplying self-impedances by the square of the

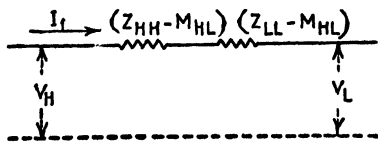


FIG. 3.—Equivalent circuit for two-winding transformer neglecting magnetizing current.

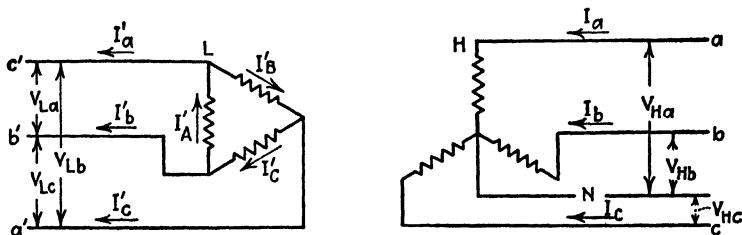


FIG. 4.

turn ratio and mutual impedances by the direct ratio, *i.e.*, if each transformer has the constants of Fig. 2 and all impedances are in terms of the H winding, the actual impedances in ohms are Z_{LL}/n^2 , M_{HL}/n , and Z_{HH} . In Fig. 4 it is assumed that the transformers are single phase and have identical electrical characteristics. The notation used in Fig. 4 is self-explanatory and is consistent with that previously used. The following

general equations¹ may be written expressing the terminal voltages.

$$\left. \begin{aligned} V_{Ha} &= (Z_{HH} - M_{HL})I_a + E_{\phi a} \\ V_{Hb} &= (Z_{HH} - M_{HL})I_b + E_{\phi b} \\ V_{Hc} &= (Z_{HH} - M_{HL})I_c + E_{\phi c} \end{aligned} \right\} \quad (10)$$

$$\left. \begin{aligned} V_{La} &= (Z_{LL} - M_{HL})I'_a + E_{\phi a} \\ V_{Lb} &= (Z_{LL} - M_{HL})I'_b + E_{\phi b} \\ V_{Lc} &= (Z_{LL} - M_{HL})I'_c + E_{\phi c} \end{aligned} \right\} \quad (11)$$

$$\left. \begin{aligned} V_{Ha} - V_{La} &= Z_H I_a - Z_L I'_a \\ V_{Hb} - V_{Lb} &= Z_H I_b - Z_L I'_b \\ V_{Hc} - V_{Lc} &= Z_H I_c - Z_L I'_c \end{aligned} \right\} \quad (12)$$

where

$$\begin{aligned} Z_H &= (Z_{HH} - M_{HL}) \\ Z_L &= (Z_{LL} - M_{HL}) \\ Z_H + Z_L &= Z_{HL} \quad \text{leakage impedance} \end{aligned}$$

Equations (10), (11), and (12) are perfectly general and hold for both the actual quantities and the phase-sequence components of voltage or current. This is evident, since the transformer is assumed to be symmetrical; consequently, currents of positive, negative and zero sequence produce drops of corresponding sequence only.

The equivalent circuit for any transformer connection may be obtained for any particular sequence by circulating current of that sequence through the transformer. Consider the positive-sequence impedance of the transformer in Fig. 4. Assume only currents of positive sequence are circulated in the *H* winding by applying a positive-sequence voltage to its terminals. Then

$$\begin{aligned} I_a &= I_{a1} & V_{Ha} &= V_{Ha1} \\ I'_a &= I'_{a1} & V_{La} &= V_{La1} \\ I'_a &= I'_{a1} \end{aligned}$$

Substituting in the first equation of (12),

$$V_{Ha1} = V_{La1} + Z_H I_{a1} - Z_L I'_{a1} \quad (13)$$

But

$$I'_{a1} = -I_{a1},$$

¹ This method of introducing the voltage E_{ϕ} due to the flux in the core was presented by W. V. Lyon, *Trans. A.I.E.E.*, 1925, p. 318, in discussing a paper by A. Boyajian, Theory of Three Circuit Transformers, *Trans. A.I.E.E.*, 1924, p. 508.

so Eq. (13) becomes

$$V_{Ha1} = V_{La1} + Z_{HL}I_{a1} \quad (14)$$

and

$$I_{a1} = \frac{V_{Ha1} - V_{La1}}{Z_{HL}}$$

Equation (14) indicates that the positive-sequence impedance of the transformer on the H -winding base is the ordinary leakage impedance referred to the H winding. Therefore, Z_{HL} may be inserted in any positive-sequence network to represent a two-winding transformer or autotransformer. The negative-sequence impedance will be the same, since the transformer is a static device and is unaffected by phase sequence.

The zero-sequence equivalent circuit may now be determined by applying voltages of zero sequence to the terminals a , b , and c of Fig. 4 and circulating zero-sequence currents through the windings. For this case the following expressions may be written:

$$V_{Ha} = V_{Hb} = V_{Hc} = V_{H0} \quad (15)$$

$$I_a = I_b = I_c = I_0 \quad (16)$$

$$I'_a = I'_b = I'_c = -I_a = -I_0 \quad (17)$$

From Eq. (17) it follows that $I'_a = I'_b = I'_c = 0$.

This shows that no zero-sequence currents flow in the lines a' , b' , c' , due to zero-sequence currents in the lines a , b , c . This holds irrespective of the terminal conditions of the lines on the delta side. It therefore follows that

$$V_{La} = V_{Lb} = V_{Lc} = 0$$

$$V_{La} + V_{Lb} + V_{Lc} = 0 \quad (18)$$

Making use of the first equations in (10) and (11), expressions for the zero-sequence terminal voltages are obtained:

$$V_{H0} = (Z_{HH} - M_{HL})I_{a0} + E_{\phi 0} \quad (19)$$

$$V_{L0} = (Z_{LL} - M_{HL})I'_{a0} + E_{\phi 0} \quad (20)$$

From Eq. (18), $V_{L0} = 0$.

$$\text{therefore} \quad E_{\phi 0} = -(Z_{LL} - M_{HL})I'_{a0} \quad (21)$$

Substituting (21) in (19) gives

$$V_{H0} = (Z_{HH} - M_{HL})I_{a0} - (Z_{LL} - M_{HL})I'_{a0} \quad (22)$$

$$I'_{a0} = -I_{a0}$$

Therefore (22) becomes

$$V_{H0} = (Z_{HH} + Z_{LL} - 2M_{HL})I_{a0} = Z_{HL}I_{a0} \quad (23)$$

$$I_{a0} = \frac{V_{H0}}{Z_{HL}} = \frac{V_{Ha}}{Z_{HL}} \quad (24)$$

Equations (23) and (24) are independent of delta voltages or terminal conditions. Since no zero-sequence currents can flow from the external circuit to the delta or vice versa, the equivalent circuit on the delta side is open as shown in Fig. 5a. The currents $I_a + I_b + I_c = 3I_0$ flow from the neutral, so the equivalent circuit on the star side must be connected to ground. The complete zero-sequence equivalent circuit for the delta-grounded star connection is shown in Fig. 5a.

The currents circulating in the delta are compensating currents for the currents in the star, which are all in phase. Since the zero-sequence currents meet the impedance of both windings, in the zero-sequence network the ground or T_0 terminal should be shown on the delta side of the transformer.

It is evident from Eq. (23) that, when the transformer is considered without reference to the external circuit, the zero-sequence impedance of the transformer is the same as the positive and negative.

Figure 5b is similar to 5a except that a neutral impedance is connected between the neutral of the star and ground. The zero-sequence impedance of an impedance in the neutral connection is three times the ordinary value. Therefore, in the zero-sequence network, this impedance will be represented by an impedance of three times its ordinary value inserted in the ground connection.

The representation of a delta-delta connection follows from the delta-star. In the case of a delta winding, the delta allows the flow of ground current by providing compensating currents; therefore, the zero-sequence connection of the equivalent circuit is a ground on the delta side of the equivalent impedance. With two deltas, therefore, there is a ground connection for each delta on respective sides of the equivalent delta. However, there can be no flow of zero-sequence currents from the delta to the external circuit or vice versa, and therefore there must be an open circuit in the zero-sequence network between the deltas and the external circuit. The connections are shown in Fig. 5c. It is

evident that the delta-delta connection effectively blocks the flow of zero-sequence currents.

	Three-phase connection	Zero sequence circuit	Positive or negative sequence
a			
b			
c			
d			
e			
f			
g			
h			
L			

FIG. 5.—Equivalent circuits for the positive-, negative-, and zero-sequence impedance of two-winding transformers.

The connections for a star-star transformer with both windings grounded are shown in Fig. 5d. There is a path for the flow of

zero-sequence current through both the high- and low-voltage windings since there is a neutral connection for each winding. If there is a path for the flow of zero-sequence currents through the external circuit on both sides of the transformer, zero-sequence currents can flow through the external circuits and the transformer windings. However, there can be no circulation of compensating currents within the transformer as was the case for the star-delta connection. The representation of the equivalent circuit requires a through feed of zero-sequence current from the external circuit on one side of the transformer to the external circuit on the other. Obviously, unless there is a path for the flow of zero-sequence current in both external circuits, there can be no flow of zero-sequence current in the transformer. This follows from the fact that no compensating currents can circulate in the transformer winding itself. Therefore, there must not be a ground connection in the equivalent circuit representing the transformer in the zero-sequence network. The equivalent circuit is shown in Fig. 5*d*. The requirements are fulfilled by this circuit, since, if there is no path for zero-sequence currents in one of the external circuits, then the connection of the transformer terminal on that side will be open thus effectively blocking the flow of all zero-sequence currents through the transformer.

In case there is an impedance in either or both neutrals of the star-star connected transformer, the equivalent circuit will be as shown in Fig. 5*e*. The zero-sequence current in passing through the low-voltage winding will encounter the winding impedance and the neutral impedance, and the same holds for the high-voltage winding. This is represented by placing an impedance equal to three times the L -winding neutral impedance in series with the L winding, and an impedance equal to three times the H -winding neutral impedance in series with the H winding. The neutral impedance does not appear in the positive- or negative-sequence equivalent networks since these currents are symmetrical.

The circuits for a star-star ungrounded transformer and a star-star transformer with only one winding grounded are shown in Fig. 5*f* and *g*. There is no complete path for the flow of zero-sequence currents in either case.

The equivalent circuits for the delta-star transformer with ungrounded neutral is shown in Fig. 5*h*. Since there is no

ground connection to allow the flow of zero-sequence current, the terminal of the equivalent impedance on the star side is shown open.

In Fig. 5i is shown the circuit for the zigzag or interconnected-star grounding transformer. Each phase of this type of transformer has two separate identical windings on the same core. The individual windings are interconnected between phases. The transformer is used to establish a grounded neutral for a system and supply ground current during faults. When a ground fault occurs, equal zero-sequence currents flow in all phases and all are in the same direction. The connections are such that the currents in the two windings on the same core are in opposite directions and produce opposing m.m.fs. The zero-sequence impedance is therefore the leakage impedance from one winding to the other on the same core. The interconnected star transformer offers an exciting admittance to voltages of positive and negative sequence applied to the terminals. This can be neglected in the equivalent circuit, as the value of the magnetizing current is usually negligible. The same connections apply for a grounded star-delta transformer when used as a grounding transformer.

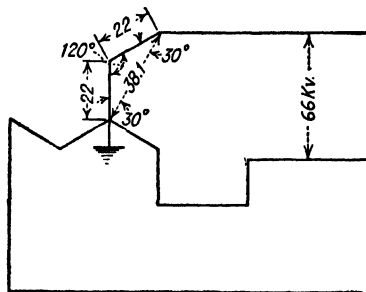


FIG. 6.

With the interconnected-star transformer, care should be used in reducing the percentage reactance to ohms on a selected voltage base for inclusion in an equivalent circuit. The vector diagram (Fig. 6) should serve to clarify this point. The line-to-line voltage is 66 kv. corresponding to a line-to-neutral voltage of 38.1 kv. The voltage across the individual windings of each phase is 22 kv., because the line-to-neutral voltage is made up of the voltage of two windings supplied from different phases. Since the leakage impedance is that between two windings on the same core, the ohms value should be based upon the voltage of the individual windings, or 22 kv. This will then be the proper value of impedance to use with the 38.1 kv. to neutral voltage corresponding to 66 kv. line to line for calculations. If the

impedance is expressed in per cent, it should be in terms of the line-to-neutral voltage and not the voltage of the individual windings. This will be more evident from a study of the inter-connected-star-delta case at the end of the chapter.

Example.—Consider the case of a 3000-kva., three-phase, interconnected-star grounding transformer. The kilovolt-amperes per phase is 1000, and the impedance between windings on each phase is 10 per cent. The line voltage is 66 kv. The voltage across the individual windings of the inter-connected star is 22 kv. corresponding to the line voltage of 66 kv. Converting the percentage impedance per phase to ohms at 22 kv. gives

$$x = \frac{j 0.10(22,000)^2}{1,000,000} = j 48.5 \quad \text{ohms to neutral on 22-kv. base}$$

This is the value of impedance to be used in calculating short-circuit currents when using the line-to-neutral voltage of 38,100 as base.

THREE-WINDING TRANSFORMERS

The three-winding transformer consists of three windings per phase involving the self-impedance of each winding and the

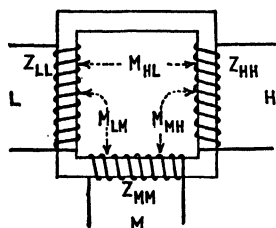


FIG. 7.—Three-winding transformer.

mutual impedance between pairs of windings. The three-winding transformer is shown in Fig. 7 in which H , M , and L represent, respectively, the high-, medium-, and low-voltage windings. In general the load ratings and voltages of the three windings are different. It is very important to observe this and reduce all windings to a common base when making calculations.

The impedances between windings of a three-winding transformer, when reduced to the same base, can be replaced by an equivalent group of star-connected impedances. In determining the effective impedances between pairs of windings, the assumption will be made that all impedances have been converted to the H -winding base. The exciting current will be neglected, and the same notation used as for the two-winding transformer.

The general equations for the terminal voltages of the three-winding transformer may be written in the same manner as for the two-winding transformer:

$$V_H = Z_{HH}I_1 + M_{HL}I_2 + M_{HM}I_3 + E_\phi \quad (25)$$

$$V_L = Z_{LL}I_2 + M_{HL}I_1 + M_{ML}I_3 + E_\phi \quad (26)$$

$$V_M = Z_{MM}I_3 + M_{HM}I_1 + M_{ML}I_2 + E_\phi \quad (27)$$

where I_1 , I_2 , and I_3 are the currents in the H , L , and M windings, respectively.

The effective impedance drops between windings are found by taking the differences of the terminal voltages.

$$V_H - V_L = (Z_{HH} - M_{HL})I_1 - (Z_{LL} - M_{HL})I_2 + (M_{HM} - M_{ML})I_3 \quad (28)$$

$$V_L - V_M = (Z_{LL} - M_{ML})I_2 - (Z_{MM} - M_{ML})I_3 + (M_{HL} - M_{HM})I_1 \quad (29)$$

$$V_M - V_H = (Z_{MM} - M_{HM})I_3 - (Z_{HH} - M_{HM})I_1 + (M_{ML} - M_{HL})I_2 \quad (30)$$

Since the exciting current is neglected

$$I_1 + I_2 + I_3 = 0 \quad (31)$$

Substituting Eq. (31) in (28), (29), and (30):

$$V_H - V_L = (Z_{HH} - M_{HL} - M_{HM} + M_{ML})I_1 - (Z_{LL} - M_{HL} - M_{ML} + M_{HM})I_2 \quad (32)$$

$$V_L - V_M = (Z_{LL} - M_{ML} - M_{HL} + M_{HM})I_2 - (Z_{MM} - M_{ML} - M_{HM} + M_{HL})I_3 \quad (33)$$

$$V_M - V_H = (Z_{MM} - M_{HM} - M_{ML} + M_{HL})I_3 - (Z_{HH} - M_{HM} - M_{HL} + M_{ML})I_1 \quad (34)$$

In Eqs. (32), (33), and (34) are only three distinct impedance terms. Let these terms be represented as follows:

$$Z_H = Z_{HH} - M_{HL} - M_{HM} + M_{ML} \quad (35)$$

$$Z_L = Z_{LL} - M_{ML} - M_{HL} + M_{HM} \quad (36)$$

$$Z_M = Z_{MM} - M_{HM} - M_{ML} + M_{HL} \quad (37)$$

It will be noted that Z_H consists of the self-impedance of the H winding ($Z_{HH} - M_{HL}$) and the mutual effect of the M winding on the H and L windings. Z_L and Z_M are similarly composed.

Equations (32), (33), and (34) may then be rewritten using the relations in (35), (36), and (37):

$$V_H - V_L = Z_H I_1 - Z_L I_2 \quad (38)$$

$$V_L - V_M = Z_L I_2 - Z_M I_3 \quad (39)$$

$$V_M - V_H = Z_M I_3 - Z_H I_1 \quad (40)$$

The form of these equations is exactly the same as for the two-winding transformer.

The effective impedances of the three pairs of windings, Eqs. (38), (39), and (40), may be determined by short-circuiting one

winding of each pair and applying voltage to the other winding. If the L , M , and H windings are short-circuited in turn and voltage applied to the H , L , and M windings in the same order:

$$V_H = (Z_H + Z_L)I_1 = Z_{HL}I_1 \quad (41)$$

$$V_L = (Z_L + Z_M)I_2 = Z_{LM}I_2 \quad (42)$$

$$V_M = (Z_M + Z_H)I_3 = Z_{MH}I_3 \quad (43)$$

Therefore the equivalent leakage impedances Z_{HL} , Z_{LM} , and Z_{MH} are

$$Z_H + Z_L = Z_{HL} \quad (44)$$

$$Z_L + Z_M = Z_{LM} \quad (45)$$

$$Z_M + Z_H = Z_{MH} \quad (46)$$

The characteristics of the three-winding transformer are usually given by the manufacturer in terms of the equivalent leakage impedance between pairs of windings Z_{HL} , Z_{LM} , and Z_{MH} .

The equivalent star-connected group of impedances to represent the three-winding transformer may be determined by the solution of Eqs. (44), (45) and (46):

$$Z_H = \frac{Z_{HL} + Z_{MH} - Z_{LM}}{2} \quad (47)$$

$$Z_L = \frac{Z_{LM} + Z_{HL} - Z_{MH}}{2} \quad (48)$$

$$Z_M = \frac{Z_{MH} + Z_{LM} - Z_{HL}}{2} \quad (49)$$

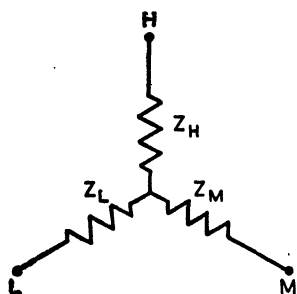


FIG. 8.—Equivalent star group of impedances for representing the impedances between windings of a three-winding transformer.

It is evident from the nature of the equations that an equivalent impedance may be negative in value. In such a case the impedance acts as a capacitance.

The equivalent star of impedances is shown in Fig. 8.

The junction of the equivalent star should not be confused with the system neutral. The star is purely an equivalent group of impedances to represent the impedances of the three transformer windings, and, for the same combination of loads at the terminals of the two devices, the terminal conditions must be the same. Any or all windings of the transformer may be connected to external circuits.

The star impedance of Fig. 8 is the equivalent circuit of the three-winding transformer for the positive and negative sequence. This equivalent circuit is simply inserted in the positive- and negative-sequence network in the same manner that the transformer is connected.

The zero-sequence circuit for the transformer is the same as the positive and negative, but the same rules governing its connection in the external circuit in the zero-sequence network must be followed as in the two-winding transformer.

The zero-sequence equivalent impedance circuit for a three-winding transformer may be determined from the general equations. Consider the star-delta-star transformer of Fig. 9.

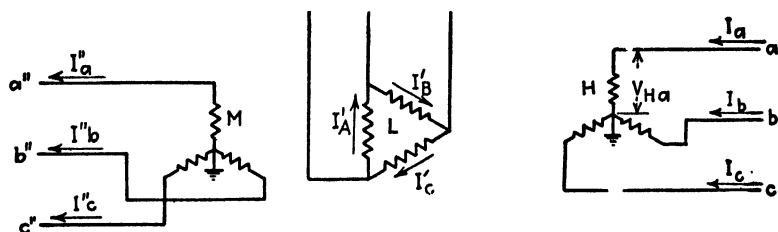


Fig. 9.—Current and voltage relations in three-winding transformer.

Assume that a zero-sequence voltage is applied to conductors a, b, c on the H winding. If there is a path for the flow of zero-sequence currents through conductors $a'', b'',$ and c'' , the following expressions hold:

$$V_{Ha} = V_{Hb} = V_{Hc} = V_{H0} \quad (50)$$

$$I_a = I_b = I_c = I_0 \quad (51)$$

$$I_{a0} + I'_{A0} + I''_{a0} = 0 \quad (52)$$

$$V_{H0} = Z_{HH}I_{a0} + M_{HL}I'_{A0} + M_{HM}I''_{a0} + E_{\phi_0} \quad (53)$$

$$V_{L0} = Z_{LL}I'_{A0} + M_{HL}I_{a0} + M_{ML}I''_{a0} + E_{\phi_0} \quad (54)$$

$$V_{M0} = Z_{MM}I''_{a0} + M_{HM}I_{a0} + M_{ML}I'_{A0} + E_{\phi_0} \quad (55)$$

$$V_{L0} = 0 \quad (56)$$

$$E_{\phi_0} = -Z_{LL}I'_{A0} - M_{HL}I_{a0} - M_{ML}I''_{a0} \quad (56)$$

$$I'_{A0} = -I_{a0} - I''_{a0} \quad (57)$$

Substituting the values of E_{ϕ_0} and I'_{A0} from Eqs. (56) and (57) in Eqs. (53) and (54),

$$V_{H0} = (Z_H + Z_L)I_{a0} + Z_L I''_{a0} \quad (58)$$

$$V_{M0} = (Z_M + Z_L)I''_{a0} + Z_L I_{a0} \quad (59)$$

$$V_{H0} - V_{M0} = Z_H I_{a0} - Z_M I''_{a0} \quad (60)$$

For the conditions assumed, $I''_{a0} = -I_{a0}$, therefore,

$$V_{H0} - V_{M0} = (Z_H + Z_M)I_{a0} \quad (61)$$

Assume that the neutral connection is removed from the M winding. Then $I''_{a0} = 0$ and Eq. (58) becomes

$$V_{H0} = (Z_H + Z_L)I_{a0} \quad (62)$$

Assume that the neutral of the M winding is connected to earth and the H winding isolated, and circulate zero-sequence currents through the M winding. For this case,

$$V_{M0} = (Z_M + Z_L)I''_{a0} \quad (63)$$

Equations (61), (62), and (63) are satisfied by connecting the zero-sequence network as shown in Fig. 10c.

Circuits for typical connections of three-winding transformers are shown in Fig. 10. A delta-delta-star three-winding transformer is shown in Fig. 10a. The zero-sequence circuit can best be seen by first considering the H and M windings as a two-winding transformer and then the H and L windings in the same manner. Using this method, it is evident that both delta branches of the equivalent circuit must be connected to ground in the zero-sequence network. The two delta-winding impedances, Z_M and Z_L , are therefore in parallel. This is apparent since both delta windings should furnish a circuit for the circulation of current to compensate for the currents flowing in the star winding to ground.

If an impedance is present in the neutral connection of the star winding, the impedance appears in the zero-sequence circuit as shown in Fig. 10b. Since in the actual circuit all the zero-sequence current flows through the impedance to ground, the neutral impedance in the equivalent circuit must be connected to ground in such a manner that all the zero-sequence current flows through it. The connections of Fig. 10b fulfill these requirements.

The star-delta-star connections are shown in Fig. 10c and d. In Fig. 10c the neutral of both star windings is grounded; therefore there may be a path from the external circuits through the two star windings and also between each star winding and the delta. The equivalent circuit for the zero-sequence system can readily be determined by considering the circuit for the windings taken in pairs, as though the three-winding transformer

consisted of three two-winding transformers. Thus the connections for H - M must be the same as H - L in Fig. 5d and of H - L and L - M the same as H - L in Fig. 5a.

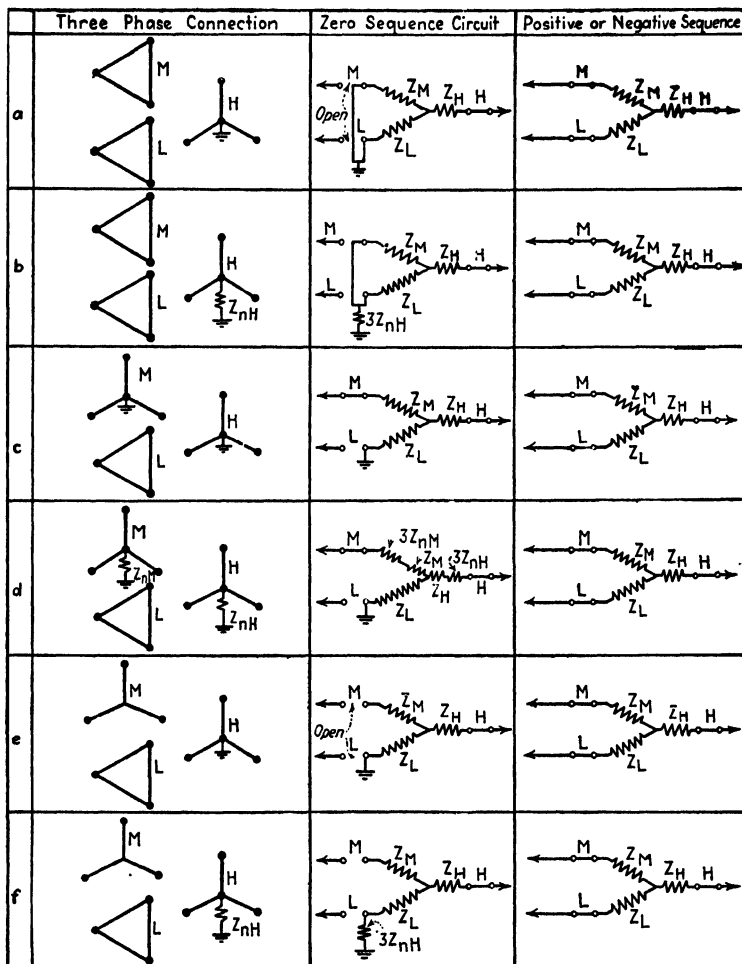


FIG. 10.—Three-winding transformer connections with equivalent circuits for positive-, negative-, and zero-sequence impedances.

If an impedance is present in the neutral connections of the two star windings, the equivalent circuits will be as shown in Fig. 10d.

For the case of the three-winding transformer having one delta winding and two star windings with grounded neutrals, the current in the delta winding will be equal to the difference of the zero-sequence currents in the two star windings when all windings are reduced to the same base. This is obvious since the vector sum of the zero-sequence currents in the three windings must be zero, otherwise unbalanced ampere turns would exist. The delta winding supplies sufficient current to compensate for the difference of the ampere turns produced by the currents in the two star windings.

The equivalent circuit for a star-delta-star three-winding transformer in which only one star winding is solidly grounded is shown in Fig. 10e. In Fig. 10f the connection is the same as in Fig. 10e except for an impedance placed in the neutral of the star winding that is grounded. It will be noted that, as far as the transformer-circuit representation is concerned, it does not matter whether the neutral impedance is shown connected adjacent to the ground or the *H* terminal network, as in Fig. 10d. This same statement applies to the other cases considered.

Example.—In order to illustrate the application of the equations developed to the determination of the equivalent impedances for a three-winding



FIG. 11.—Three-winding transformer.

transformer, consider the transformer of Fig. 11. The transformer windings are assumed to have the following characteristics:

Winding *M*. 25,000 kva., 3 phase, star connected, 13,200 volts, line to line.

Winding *H*. 20,000 kva., three phase, star connected, 110,000 volts, line to line.

Winding *L*. 5000 kva., three phase, delta connected, 2300 volts, line to line.

The percentage resistance drops in the three windings are:

Winding *M*. 0.85 per cent resistance drop on 25,000-kva. base.

Winding *H*. 0.68 per cent resistance drop on 20,000-kva. base.

Winding *L*. 1.0 per cent resistance drop on 5000-kva. base.

The percentage reactance drops between windings are:

Winding *M-H*. 9 per cent reactance drop on 25,000-kva. base.

Winding *H-L*. 9.5 per cent reactance drop on 5000-kva. base.

Winding *M-L*. 4.8 per cent reactance drop on 20,000-kva. base.

In order to determine the impedances of the equivalent circuits, it is necessary to convert all impedances to a common kilovolt-ampere base if using percentage impedance, or a common voltage base if using ohms impedance.

DETERMINATION OF PHASE-SEQUENCE CONSTANTS 123

In this example, all impedances will be converted to percentage on a 25,000-kva. base and then to ohms on the 110,000-volt base. In terms of 25,000 kva. and 110,000 volts,

$$\frac{110,000}{1.73} = 63,500 \text{ line-to-neutral voltage}$$

and

$$\frac{25,000,000}{1.73 \times 110,000} = 131 \text{ amp. phase current.}$$

$$R_H = 0.68 \text{ per cent on 20,000-kva.} = 0.85 \text{ per cent on 25 000-kva. base.}$$

$$R_H = \frac{0.0085 \times 63,500}{131} = 4.12 \text{ ohms on 110-kv. base.}$$

$$R_M = \frac{0.0085 \times 63,500}{131} = 4.12 \text{ ohms}$$

$$R_L = \frac{0.05 \times 63,500}{131} = 24.2 \text{ ohms}$$

The winding reactances may be reduced in a similar manner.

$$X_{HM} = \frac{0.09 \times 63,500}{131} = 43.7 \text{ ohms}$$

$$X_{HL} = 9.5 \text{ per cent on 5000 kva. or 47.5 per cent on 25,000 kva.} = \frac{0.475 \times 63,500}{131} = 230 \text{ ohms.}$$

$$X_{ML} = 4.8 \text{ per cent on 20,000 kva. or 6 per cent on 25,000 kva.} = \frac{0.06 \times 63,500}{131} = 29.1 \text{ ohms on 110-kv. base.}$$

The total impedance between the three windings includes both resistance and reactance.

$$Z_{MH} = R_M + R_H + X_{MH} = 8.24 + j43.7$$

$$Z_{HL} = R_H + R_L + X_{HL} = 28.32 + j230$$

$$Z_{ML} = R_M + R_L + X_{ML} = 28.32 + j29.1$$

The impedances of the equivalent star to replace the three-winding transformer are found by substituting the above expressions in Eqs. (47), (48), and (49):

$$Z_H = 4.12 + j122.3$$

$$Z_M = 4.12 - j78.6$$

$$Z_L = 24.2 + j107.7.$$

These impedances are the proper ones to represent the three-winding transformer. The resistance terms in these expressions are the same as R_H , R_M , and R_L .

AUTOTRANSFORMERS IN SEQUENCE NETWORKS

The usual types of autotransformers used in power systems consist of a winding tapped to obtain two voltages and a tertiary winding. The tapped windings for the three phases are star connected and usually grounded and the tertiary windings are

connected delta. In low-voltage circuits, autotransformers are sometimes used which do not have tertiary windings.

In short-circuit calculations, autotransformers may be treated in a manner similar to two- and three-winding transformers. The determination of an equivalent impedance for an autotransformer is considerably more complex than for an ordinary transformer because part of the current is transferred from the low-voltage to the high-voltage circuit, or vice versa, by simple conduction, and part by transformation. The common and series windings offer some difficulty, since they are part of the same winding but carry currents of different magnitude. Furthermore, the inclusion of line and neutral impedances must be given careful consideration for the same reasons.

In this analysis, the simple autotransformer without delta tertiary will be treated in a manner similar to that for the ordinary two-winding transformer. The equivalent circuits for practically

all combinations will be given. The positive- and negative-sequence reactance of an autotransformer is simply the ordinary leakage reactance between windings. In Fig. 12 is shown one phase of a simple autotransformer. The leakage reactance most frequently used is that between ac and bc . This

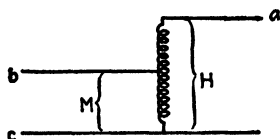


FIG. 12.—Single-winding autotransformer; ab series, bc common.

can be measured by short-circuiting bc and applying voltage to ac . Obviously a leakage reactance can also be measured between ab and bc .

The zero-sequence impedance of the autotransformer of Fig. 12 is the same as the positive and negative, but the impedance of the complete transformer circuit depends upon the connections of the external circuit. Since no delta winding is present, there is no path for the flow of compensating currents of zero sequence. All connections for autotransformer equivalent circuits will be shown in conjunction with the external supply circuit.

In Fig. 13a is shown an autotransformer with grounded neutral and the neutral of the supply source ungrounded. There is no path for the circulation of zero-sequence currents, and, therefore, the terminals of the zero-sequence impedances are shown open.

In Fig. 13b there is a path for the flow of zero-sequence current in the external circuit, and, therefore, the current flows through

the autotransformer, encountering the reactance Z_{HM} in the transformer. The impedance of the generator and transformer should be reduced to the same kilovolt-ampere base if dealing

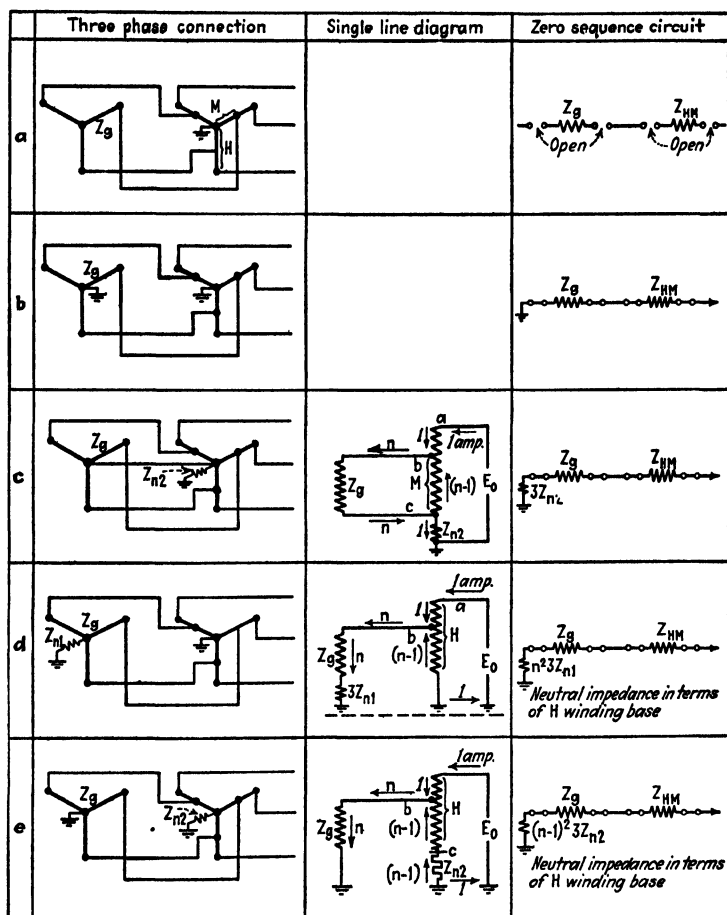


FIG. 13.—Autotransformer connections with equivalent circuits for zero-sequence network.

with percentage, or to a common voltage base if dealing with ohms.

The neutrals of the generator and transformer are interconnected and the neutral of the transformer grounded through an impedance in Fig. 13c. This circuit appears more complex than 13b, but, reducing the diagram to a single-phase basis, as

shown, it is seen that the two are the same with the exception of the neutral impedance. A zero-sequence voltage E_0 of such value as to cause 1 amp. to flow in the high-voltage circuit is shown applied to the high-voltage winding. The actual currents on their respective voltage bases are shown in the other circuits. The ratio of transformation is the ratio of the turns ac to bc , i.e., $N_{ac}/N_{bc} = E_H/E_M = n$. With 1 amp. in the a conductor, the current in b is n . It should be noted that the current in the neutral is per phase the same as that in the high-voltage circuit; therefore the impedance Z_{n2} should be used in the equivalent circuit with its actual *ohmic* value since it is already on the voltage base of the H winding. This is assuming that the voltage base chosen for calculation is the H winding. In the zero-sequence network the impedance Z_{n2} must be multiplied by 3 since it is in the neutral.

In Fig. 13*d* the neutral of the transformer is solidly grounded and the neutral of the generator grounded through an impedance. The zero-sequence-current flow for a single phase is shown for 1 amp. in the H circuit. The current through the neutral impedance Z_{n1} is the same as that in the b conductor; therefore, this impedance is on the voltage base of the M winding as indicated by the current n . If the H -winding voltage is chosen for calculations, the actual neutral impedance Z_{n1} must be multiplied by n^2 .

The connections shown in Fig. 13*e* might at first appear as simple as the preceding cases, but an examination of the diagram will show this is not true. The current in the neutral impedance is that of neither the high- nor low-voltage circuit, and, therefore, the impedance cannot be said to be already on either base. The impedance must be evaluated for the equivalent circuit. It will be observed that the current through the neutral impedance is $(n - 1)$. The effective neutral impedance $3Z_{n2}$ in ohms can be converted to ohms in terms of the high-voltage circuit by multiplying by $(n - 1)^2$.

If the generator neutral is grounded through an impedance, the impedance should be evaluated as in Fig. 13*d*.

AUTOTRANSFORMERS WITH DELTA TERTIARY WINDINGS

This type of autotransformer can be treated in the same manner as an ordinary three-winding transformer, but the determination

of the zero-sequence equivalent circuit is generally more complex. The type of equivalent circuit, neglecting the exciting current, is the same for the auto as the three-winding transformer.

A study of Figs. 14, 15, and 16 will clarify the problem of determining the zero-sequence equivalent circuit for an autotransformer. In Fig. 14, a generator with ungrounded neutral is connected to the medium voltage or common winding of a grounded autotransformer. The fault is on the high-voltage line. The impedance encountered by the current in flowing

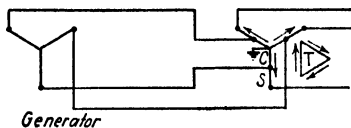


FIG. 14.

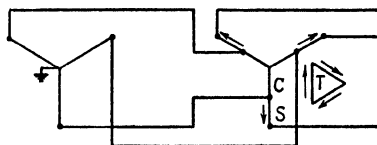


FIG. 15.

from the transformer neutral to the fault is that of the full-tapped winding with the delta as secondary. The impedance should be based on the total turns in the series and common windings. In Fig. 15 the generator is grounded and the transformer ungrounded. The fault location is the same as Fig. 14. The impedance encountered for this case is that of the series winding with the delta as secondary. The impedance should be based on the turns in the series winding. The connections for Fig. 16 are the same as Fig. 14, except that the fault occurs on the low-voltage line. The impedance encountered is that of the common winding with the delta as secondary, and the impedance is based on the turns in the common winding.

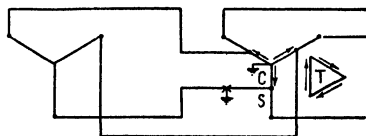


FIG. 16.

It is evident that the impedance of an autotransformer to zero-sequence currents does not involve actual transformation from one circuit to another. The zero-sequence impedance may be thought of as a series-impedance coil with short-circuited secondary, the secondary representing the delta winding of the autotransformer.

In representing an autotransformer by a group of three star-connected impedances,¹ there are two combinations of pairs of

¹ The general method of analysis used for autotransformers with tertiary windings is taken from *Symmetrical Components* by C. F. Wagner and R. D. Evans, *Elec. Jour.*, December, 1929, p. 576.

terminals that may be used for the tapped winding. The two combinations are shown in Figs. 17a and 18a. The *S-C-T* combination, shown in Fig. 17a, involves the impedances between the series and common, series and tertiary, and common and tertiary windings. In Fig. 18a is shown the *H-M-L* combination, involving the impedances between the series-common and

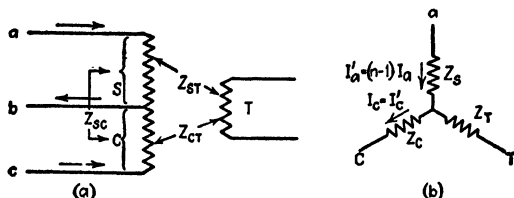


FIG. 17.—(a) Designation of series, common, and tertiary windings of three-winding autotransformers; (b) equivalent circuit for the *S-C-T* combination.

common, series-common and tertiary, and common and tertiary windings. It should be noted that in the *S-C-T* combination the terminal *b* is common; therefore its identity will be lost in the equivalent circuit. Similarly, in the *H-M-L* combination of Fig. 18a the terminal *c* is common.

If the impedances are given as in Figs. 17a and 18a, the equivalent star impedances for representing the two transformers are found by the same method as for the three-winding transformer.

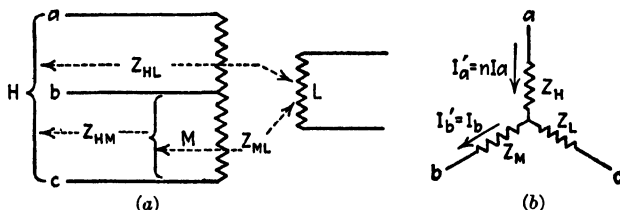


FIG. 18.—(a) Designation of high-, medium- and low-voltage windings of three-winding autotransformers; (b) equivalent circuit for the *H-M-L* combination.

The equivalent star impedances are shown in Figs. 17b and 18b. In some cases it may be more convenient to use one combination and in other cases the other. The impedances may be given in terms of one combination, and it may be desirable to use the other combination; it is therefore convenient to know the relations between the two combinations so that if given one set of impedances the other may be found.

The equivalent star impedances of Figs. 17b and 18b are on a common voltage base, and the currents shown flowing in the equivalent circuits are on the same common voltage base. The actual currents can be determined from the equivalent currents by converting from the common base to the actual voltage of the particular winding in which the currents flow. The currents in branches Z_s and Z_c of Fig. 17b, after being converted to the S and C voltage base, respectively, are the actual currents in conductors a and c Fig. 17a and b. The identity of conductor b has been lost but the actual current in conductor b can be found from the actual currents in a and c . The current in b is the sum of the actual currents in a and c . The currents in branches Z_H and Z_M of Fig. 18b, after converting to their actual values, represent the current in conductors a and b . The identity of conductor c is lost, but the actual current in this conductor can be determined from the difference of the actual currents in conductors a and b .

In the following analysis the C or M winding will be used as the reference, and all impedances will be expressed in terms of the voltage of this winding as a base. All voltages are line to neutral. The ratio of transformation of the main winding will be assumed according to the convention

$$n = \frac{E_H}{E_M} = \frac{E_s + E_c}{E_c} \quad (64)$$

$$(n - 1) = \frac{E_s}{E_c} \quad (65)$$

The expressions for the equivalent impedances in the S - C - T combinations, using the three-winding transformer-conversion formulas, are

$$Z_s = \frac{1}{2}(Z_{ST} + Z_{SC} - Z_{CT}) \quad (66)$$

$$Z_c = \frac{1}{2}(Z_{CT} + Z_{CS} + Z_{ST}) \quad (67)$$

$$Z_T = \frac{1}{2}(Z_{TC} + Z_{TS} - Z_{CS}) \quad (68)$$

The equivalent impedances in the H - M - L combination are

$$Z_H = \frac{1}{2}(Z_{HL} + Z_{HM} - Z_{LM}) \quad (69)$$

$$Z_M = \frac{1}{2}(Z_{ML} + Z_{MH} - Z_{LH}) \quad (70)$$

$$Z_L = \frac{1}{2}(Z_{LM} + Z_{LH} - Z_{MH}) \quad (71)$$

All impedances in Eqs. (66) to (71), inclusive, are on the same kilovolt-ampere base and the voltage base of the C or M winding.

Z_{HL} in Terms of S - C - T Constants.—The impedance Z_{HL} in terms of the S - C - T constants can now be derived.

In Fig. 19a assume winding L short-circuited and a voltage applied to the H winding of such a value as will cause unit current to flow in that winding. The equivalent circuit is shown in Fig. 19b. The notation will be used that a symbol without a prime represents actual currents or voltages in the actual circuit in terms of the voltage of that circuit, and symbols with single prime represent currents or voltages in the equivalent circuit on the voltage base of the C or M winding. For the condition of unit current flowing in the H winding, the actual currents in the

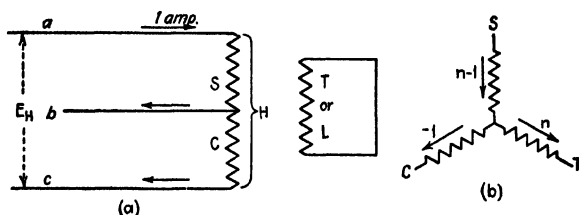


FIG. 19.

S and C circuits are the actual currents in the terminals a and c , or unity

$$I_s = I_a \quad \text{and} \quad I_c = I_c \quad (72)$$

The currents in the equivalent circuit Fig. 19b can be found from the known currents in the actual circuit.

$$I'_s = \frac{E_s}{E_c}(1) = n - 1 \quad (73)$$

$$I'_c = -1 \quad (74)$$

The negative sign in Eq. (74) follows from the convention that current is positive when flowing into the star and negative when flowing away from the junction. In the equivalent circuit, the current in the T branch must equal the difference of the currents in the other two branches:

$$I'_T = I'_s - I'_c = (n - 1) - (-1) = n \quad (75)$$

The voltage relations can now be determined. The actual voltage at the terminals of the T winding is zero since this winding is short-circuited. The equivalent voltage in the equivalent circuit must also be zero at the T terminal for this condition.

The equivalent voltages across the S and C windings are equal to the voltage drops to the S and C terminals of the equivalent circuit (Fig. 19b):

$$E'_s = nZ_T + (n - 1)Z_s \quad (76)$$

$$E'_c = nZ_T + Z_c \quad (77)$$

The junction of the equivalent impedance star should not be confused with the system neutral. The equivalent voltage given in Eq. (76) is the voltage between terminals a and b in Fig. 19a and the equivalent voltage Eq. (77) is the voltage between terminals b and c , both referred to the C - or M -winding base. The voltage E'_s is that portion of the total voltage drop between terminals a and b when unit current flows in a and out of c with the T winding short-circuited.

The actual voltages across the S and C windings can be found by multiplying the equivalent voltages by the proper transformation ratios. Thus,

$$\begin{aligned} E_s &= \frac{E'_s}{E'_c} [nZ_T + (n - 1)Z_s] = (n - 1)[nZ_T + (n - 1)Z_s] \\ &= n(n - 1)Z_T + (n - 1)^2Z_s \end{aligned} \quad (78)$$

$$E_c = 1(nZ_T + Z_c) = nZ_T + Z_c \quad (79)$$

The actual voltage E_H is obviously equal to the sum of Eqs. (78) and (79):

$$E_H = E_s + E_c = n^2Z_T + (n - 1)^2Z_s + Z_c \quad (80)$$

Since unit current is flowing in the main winding, the value of E_H is equal to the impedance Z_{HL} in terms of the H -winding base. This can be converted to impedance on the C -winding base by dividing by n^2 .

$$Z_{HL} = Z_T + \left(\frac{n - 1}{n} \right)^2 Z_s + \frac{Z_c}{n^2} \text{ in terms of } C\text{-winding base} \quad (81)$$

Substituting the values of Z_s , Z_c , and Z_T from Eqs. (66), (67), and (68) in Eq. (81) gives

$$Z_{HL} = \frac{Z_{TC}}{n} + \frac{n - 1}{n} Z_{TS} - \frac{n - 1}{n^2} Z_{CS} \quad (82)$$

From the equivalence of the two combinations it is evident that Z_{ML} and Z_{CT} are the same.

$$Z_{ML} = Z_{CT} \quad (83)$$

In order to determine Z_{ML} in terms of the S - C - T constants, assume the L or T winding open and the M or C winding short-circuited (Fig. 20a). Assume a voltage E_H applied across terminals a and c of such value that unit current flows into a . The current in the T winding is zero since the circuit is open. The actual currents in the S and C windings are

$$I_s = 1 \quad (84)$$

$$I_c = \frac{E_s}{E_c}(1) = n - 1 \quad (85)$$

The equivalent currents in the equivalent circuit (Fig. 20b) on the common voltage base C or M are

$$I'_s = \frac{E_s}{E_c}(1) = n - 1 \quad (86)$$

$$I'_c = n - 1 \quad (87)$$

The equivalent voltage across the S winding is equal to the voltage drop to the S terminal of the equivalent circuit. The

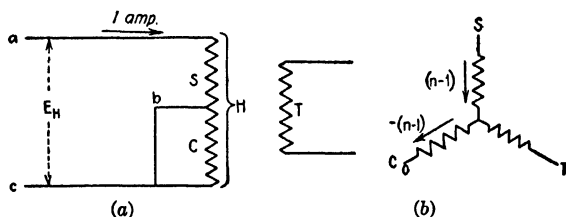


FIG. 20.

equivalent voltage across the terminals of the C winding is zero since this winding is short-circuited. The equivalent voltage at the C terminal of the equivalent circuit is therefore zero.

$$E'_c = 0 \quad (88)$$

$$E'_s = (n - 1)(Z_s + Z_c) \quad (89)$$

The actual voltages are

$$E_c = 0 \quad (90)$$

$$E_s = (n - 1)^2(Z_s + Z_c) = (n - 1)^2 Z_{sc} \quad (91)$$

$$E_H = E_c + E_s \quad (92)$$

Since unit current was assumed to flow, E_H is equivalent to Z_{HM} on the H -winding base. This can be converted to the C -winding base by dividing by n^2 .

$$Z_{HM} = \frac{E_H}{n^2} = \left(\frac{n-1}{n}\right)^2 Z_{SC} \quad \text{on } C\text{-winding base} \quad (93)$$

The relations between the equivalent star impedances representing the two combinations may be found by substituting Eqs. (82), (83), and (93) in Eqs. (66) to (71), inclusive. For convenience all the relations are tabulated. It should be remembered that all impedances appearing in the equations are on the *C*- and *M*-winding base.

$$\left. \begin{aligned} Z_S &= \frac{1}{2}(Z_{ST} + Z_{SC} + Z_{CT}) \\ Z_C &= \frac{1}{2}(Z_{CT} + Z_{CS} - Z_{ST}) \\ Z_T &= \frac{1}{2}(Z_{TC} + Z_{TS} - Z_{CS}) \end{aligned} \right\} \quad (94)$$

$$\left. \begin{aligned} Z_H &= \frac{1}{2}(Z_{HL} + Z_{HM} - Z_{LM}) \\ Z_M &= \frac{1}{2}(Z_{ML} + Z_{MH} - Z_{LH}) \\ Z_L &= \frac{1}{2}(Z_{LM} + Z_{LH} - Z_{MH}) \end{aligned} \right\} \quad (95)$$

$$\left. \begin{aligned} Z_{SC} &= \left(\frac{n}{n-1}\right)^2 Z_{HM} \\ Z_{CT} &= Z_{ML} \\ Z_{TS} &= \frac{n}{n-1} Z_{HL} - \frac{1}{n-1} Z_{ML} + \frac{n}{(n-1)^2} Z_{HM} \end{aligned} \right\} \quad (96)$$

$$\left. \begin{aligned} Z_{HM} &= \left(\frac{n-1}{n}\right)^2 Z_{SC} \\ Z_{ML} &= Z_{CT} \\ Z_{HL} &= \frac{Z_{TC}}{n} + \frac{n-1}{n} Z_{TS} - \frac{n-1}{n^2} Z_{CS} \end{aligned} \right\} \quad (97)$$

$$\left. \begin{aligned} Z_S &= \left(\frac{n}{n-1}\right)^2 Z_H + \frac{n}{(n-1)^2} Z_M \\ Z_C &= \frac{n}{n-1} Z_M \\ Z_T &= Z_L - \frac{Z_M}{n-1} \end{aligned} \right\} \quad (98)$$

$$\left. \begin{aligned} Z_H &= \left(\frac{n-1}{n}\right)^2 Z_S - \frac{n-1}{n^2} Z_C \\ Z_M &= \frac{n-1}{n} Z_C \\ Z_L &= Z_T + \frac{Z_C}{n} \end{aligned} \right\} \quad (99)$$

Example.—An example will be worked out to show the application of these relations in converting from one group to the other.

The autotransformer is three phase with H , M , and L windings rated at 66, 33, and 12 kv., respectively. It will be assumed that the H , M , and L impedances are given. The resistance components will be neglected. The impedances between the various windings are:

Z_{HM} (66-33). 7.8% on 25,000-kv. base.

Z_{ML} (33-12). 9.0% on 10,000-kva. base, or 22.5% on 25,000 kva.

Z_{HL} (66-12). 10.5% on 10,000-kva. base, or 26.2% on 25,000 kva.

Reducing all impedances to ohms on 33-kv. base:

$$Z_{HM} = \frac{0.078 \times (33,000)^2}{25,000,000} = j3.4$$

$$Z_{ML} = j9.8$$

$$Z_{HL} = j11.4$$

The equivalent star impedances are formed by Eqs. (95):

$$Z_H = \frac{j}{2}(11.4 + 3.4 - 9.8) = j2.5 \text{ ohms on 33-kv. base}$$

$$Z_M = \frac{j}{2}(9.8 + 3.4 - 11.4) = j0.9$$

$$Z_L = \frac{j}{2}(9.8 + 11.4 - 3.4) = j8.9$$

The equivalent star impedances in terms of the S - C - T combination are found from Eqs. (98) where $n = \frac{66}{33} = 2$.

$$Z_S = \left(\frac{2}{2-1}\right)^2(j2.5) + \frac{2}{(2-1)^2}(j0.9) = j11.8$$

$$Z_C = \frac{2}{2-1}(j0.9) = j1.8$$

$$Z_T = j8.9 - \frac{j0.9}{(2-1)} = j8.0$$

The impedance between windings in the S - C - T combination can be found from the H - M - L group Eqs. (96)

$$Z_{SC} = \left(\frac{2}{2-1}\right)^2 j3.4 = j13.6$$

$$Z_{CT} = 9.8$$

$$Z_{TS} = \left(\frac{2}{2-1}\right) j11.4 - \frac{1}{(2-1)} j9.8 + \frac{2}{(2-1)^2} j3.4 = j19.8$$

AUTOTRANSFORMERS WITH EXTERNAL IMPEDANCE

The zero-sequence impedance of the autotransformer generally represents only a part of the total zero-sequence network. In addition to the series impedance of the line conductors, there may be neutral impedance. The external impedance can be included in the zero-sequence network of the autotransformer by means of the relations which have been developed, thus obtaining the complete zero-sequence network.

To be consistent, all external series impedance should be converted to the C - or M -winding base. An impedance in series with the T or L winding is merely added in series with the impedance representing that winding in the equivalent circuit after changing the series impedance from the base of the T or L winding to the C or M winding by multiplying by $(E_M/E_T)^2$. External impedance in series with the a terminal is treated in the same manner after reducing to the M - or C -winding base by dividing the actual ohms by n^2 . Impedance in series with the b or c conductors may offer some difficulty. If the autotransformer constants are expressed in the H - M - L combination, an impedance Z_{bb} in series with the b terminal can be added in series with the impedance representing the M winding in the equivalent circuit of the H - M - L combination. The actual ohms of this series impedance is already on the M -winding base, so no conversion is necessary. An impedance Z_{cc} in series with the c conductor can be added in series with the impedance representing the C winding in the equivalent circuit for the S - C - T combination. The actual ohms of this impedance is already on the C - or M -winding base. If the H - M - L combination is to be used in the calculations, it is necessary to apply the same coefficients to Z_{cc} as was applied to Z_c in Eqs. (99) to obtain the star impedances for the H - M - L combination. These coefficients for the Z_H , Z_M , and Z_L branches of the equivalent circuit are $-\frac{n-1}{n^2}$,

$\frac{n-1}{n}$, and $\frac{1}{n}$, respectively. In Fig. 21a is shown an autotransformer with external impedances in series with each conductor. The equivalent circuit containing both autotransformer and external impedances is shown in Fig. 21b. The relations between equivalent and actual currents and voltages are found by the same method that was used for the autotransformer above.

The principles which have been given are sufficient to determine the complete zero-sequence network for a system involving any combination of autotransformers and impedances. The most general cases will be treated, and from the general cases the solution can be found for practically any combinations that may be encountered. Consider the connections of Fig. 22a involving a generator supplying an autotransformer through a two-winding transformer. The neutrals of both the two-winding

transformer and the autotransformer are grounded through an impedance. The impedance of the system to zero-sequence currents will be determined as viewed from the high-voltage side

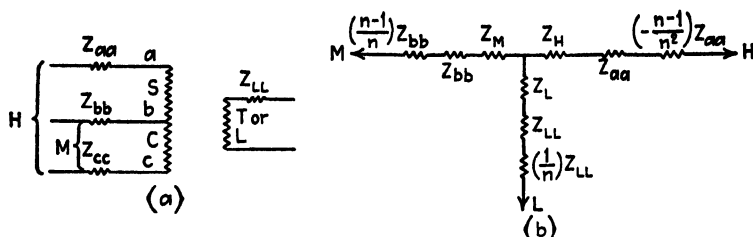


FIG. 21.—(a) Single-line diagram; (b) equivalent zero-sequence impedance circuit for a three-winding auto transformer with external impedance.

of the autotransformer. The current flow will be assumed as shown in Fig. 22a. The complete circuit reduced to a single-line basis is shown in Fig. 22b. All impedances are assumed to be on

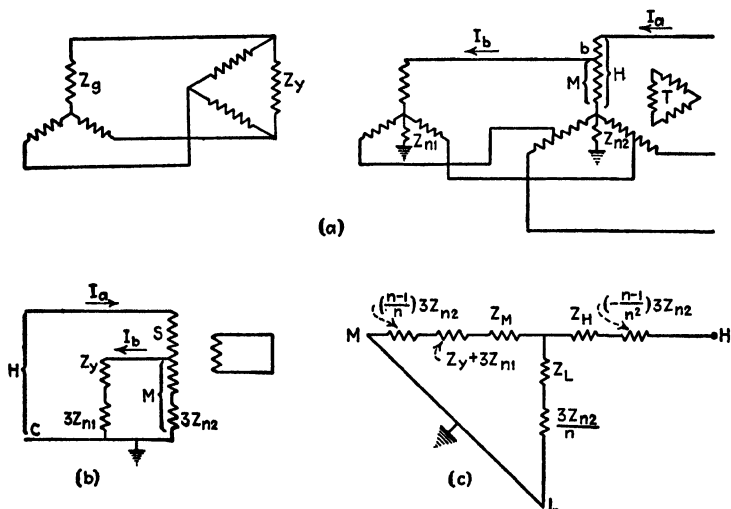


FIG. 22.—(a) Three-phase connections; (b) single-line diagram; (c) equivalent zero-sequence circuit.

the base of the M winding. The impedances Z_y and Z_{n1} are in series with the b conductor and thus combine directly. The impedance Z_{n2} is in series with the C winding of the S - C - T

constants, and must have the conversion coefficients applied, as in the preceding case. The complete equivalent circuit is given in Fig. 22c. The zero-sequence impedance to ground is simply the parallel impedance of the M and L branches in series with the impedance of the H branch of the equivalent circuit. The complete expression for the zero-sequence impedance of the circuit is then

$$Z_0 = Z_H - \frac{3(n-1)}{n^2} Z_{n2} + \frac{\left(Z_L + \frac{3Z_{n2}}{n}\right) \left(Z_M + Z_y + 3Z_{n1} + \frac{3(n-1)}{n} Z_{n2}\right)}{\left(Z_L + \frac{3Z_{n2}}{n}\right) + \left(Z_M + Z_y + 3Z_{n1} + \frac{3(n-1)}{n} Z_{n2}\right)} \quad (100)$$

The current relations are easily found. Let K be the denominator of the second term of Eq. (100), then

$$I_a = I_H = \frac{I'_H}{n} \quad (101)$$

$$I_{n1} = 3I_b = 3I'_M = \frac{3\left(Z_L + \frac{Z_{n2}}{n}\right)}{K} I'_H \quad (102)$$

$$I_{n2} = 3I_c = 3I'_M - 3\frac{I'_H}{n} = \frac{3[(n-1)Z_L - Z_M - Z_y - 3Z_{n1}]}{nK} \quad (103)$$

It will be noted in Eq. (103) that the neutral current may be positive, negative, or zero, depending upon the relations of the various constants in the zero-sequence network. There are two paths for the flow of zero-sequence currents in the autotransformer, and the current flow is in opposite directions through the two paths. It will later be seen that this is of importance in the application of relays to autotransformers and associated circuits.

The zero-sequence voltage at any point in the circuit may be found by calculating the voltage drop to that point in the equivalent circuit and then converting the equivalent drop to actual voltage.

CASE 1. Neutral of two-winding transformer solidly grounded. Neutral of autotransformer grounded through impedance. For this case Z_{n1} becomes zero, and Eq. (100) reduces to

$$Z_0 = Z_H - \frac{3(n-1)}{n^2}Z_{n2} + \frac{\left(Z_L + \frac{3Z_{n2}}{n}\right)\left(Z_M + Z_Y + \frac{3(n-1)}{n}Z_{n2}\right)}{\left(Z_L + \frac{3Z_{n2}}{n}\right) + \left(Z_M + Z_Y + \frac{3(n-1)}{n}Z_{n2}\right)} \quad (104)$$

CASE 2. Neutral of two-winding transformer grounded through impedance. Neutral of autotransformer solidly grounded.

$$Z_{n2} = 0$$

$$Z_0 = Z_H + \frac{Z_L(Z_M + Z_Y + 3Z_{n1})}{Z_L + (Z_M + Z_Y + 3Z_{n1})} \quad (105)$$

CASE 3. Neutrals of both transformers solidly grounded.

$$Z_{n1} = Z_{n2} = 0$$

$$Z_0 = Z_H + \frac{(Z_L)(Z_M + Z_Y)}{Z_L + (Z_M + Z_Y)} \quad (106)$$

CASE 4. Neutral of autotransformer grounded through impedance. Neutral of two-winding transformer isolated. For this case Z_{n1} becomes infinite; therefore no current of zero sequence can flow in the b conductor. The M branch of the equivalent circuit (Fig. 22c) should be open to represent this case. The zero-sequence impedance is then

$$Z_0 = Z_H - \frac{3(n-1)}{n^2}Z_{n2} + Z_L + \frac{3Z_{n2}}{n} \quad (107)$$

CASE 5. Neutral of autotransformer isolated. Neutral of two-winding transformer grounded through impedance. Z_{n2} becomes infinite.

An inspection of the equivalent circuit (Fig. 22c) shows that Z_{n2} appears in all three legs of the equivalent star; hence this case cannot be treated in the same manner as case 4 in which Z_{n1} is infinite. It is necessary to reduce Eq. (100) to a simple fraction and then differentiate both numerator and denominator in order to eliminate Z_{n2} which is infinite. The final expression obtained is

$$Z_0 = Z_H + \left(\frac{n-1}{n}\right)^2 Z_L + \frac{Z_M + Z_Y + 3Z_{n1}}{n^2}$$

or

$$Z_0 = \left(\frac{n-1}{n} \right)^2 Z_{ST} + \frac{Z_y + 3Z_{n1}}{n^2} \quad (108)$$

$$\frac{n-1}{n} = \frac{E_s}{E_R}$$

In any of the preceding cases if there is impedance in the lines between the autotransformer and the two-winding transformer, such impedance may be included in the equivalent circuit in exactly the same manner as Z_y . This should be the zero-sequence

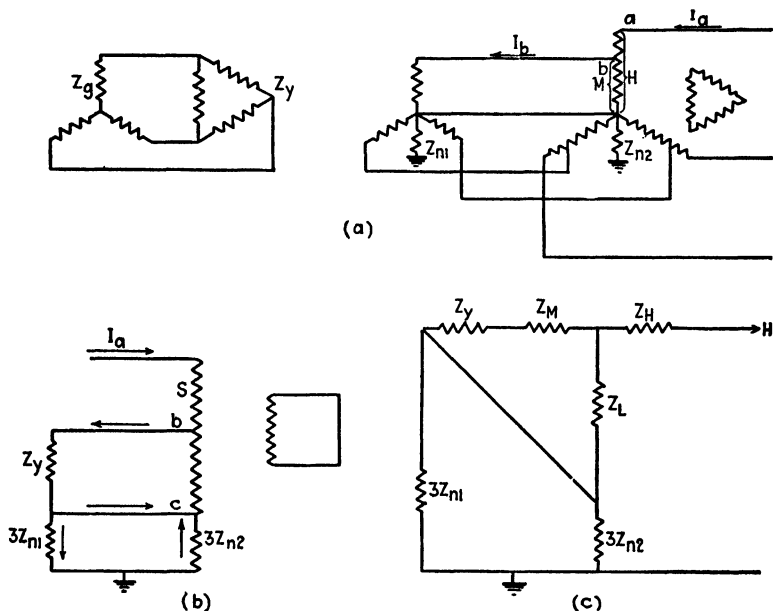


FIG. 23.—(a) Three-phase connections; (b) single-line diagram; (c) equivalent zero-sequence circuit.

impedance of the lines connecting the two transformers and not the ordinary line-to-neutral impedance. It is evident that the relations which have been developed may be applied equally well to the determination of the zero-sequence impedance of auto-transformers without delta windings by letting Z_L or Z_T become infinite in the equations.

If the two-winding transformer is removed in Fig. 22a, the zero-sequence impedance Z_{g0} of the generator takes the place of Z_y in the equations when the neutral of the generator is grounded.

The impedance Z_{n1} , if placed in the neutral of the generator, will appear in the equations exactly as it did when in the transformer neutral.

In Fig. 23a is shown an autotransformer connected the same as Fig. 22a except that the neutrals are interconnected. The single-line diagram is given in Fig. 23b and the equivalent circuit in Fig. 23c.

The zero-sequence impedance of the equivalent circuit is

$$Z_0 = Z_H + \frac{Z_L(Z_M + Z_Y)}{Z_L + (Z_M + Z_Y)} + \frac{3Z_{n1}Z_{n2}}{Z_{n1} + Z_{n2}} \quad (109)$$

The currents are:

$$\begin{aligned} I'_a &= I'_H = nI_a \\ I_a &= I_H = \frac{I'_H}{n} \\ I_{n1} &= \left(\frac{3Z_{n2}}{Z_{n1} + Z_{n2}} \right) \frac{I'_H}{n} \\ I_{n2} &= \left(\frac{3Z_{n1}}{Z_{n1} + Z_{n2}} \right) \frac{I'_H}{n} \end{aligned}$$

It will be observed that with this connection, if either Z_{n1} or Z_{n2} is equal to zero, the other neutral impedance no longer has any effect upon the zero-sequence network or zero-sequence current.

Example.—In Fig. 24 is shown a circuit involving a generator, transformer, autotransformer, and transmission lines. The autotransformer will be assumed to have the same characteristics as the one in the preceding example. The zero-sequence impedance of the transmission circuit is three times the positive. The fault is assumed to be single line-to-ground on the 66-kv. side of the autotransformer.

The impedance characteristics are:

Generator. 25,000 kva. $Z_1 = j35\%$; $Z_2 = j25\%$

Transformer. 25,000 kva. $Z_1 = j8.5\%$

Transmission line. $Z_1 = Z_2 = j10\%$; $Z_0 = j30\%$

Autotransformer. $Z_H = j2.50\%$; $Z_M = j0.90\%$; $Z_L = j8.9\%$

The equivalent positive- and negative-sequence systems are shown in Fig. 24b and c. All impedances are in ohms on a 33-kv. base.

The positive-sequence network consists of the positive-sequence impedance of generator, transformer, line, and autotransformer, all in series and on the common base of 33 kv. The autotransformer impedance is simply Z_{HM} . The negative-sequence network is the same as the positive, except for the generator impedance. The equivalent positive-sequence impedance is the sum of the series impedances or $j32.3$. Similarly, the equivalent negative-sequence impedance is $j28.95$.

The zero-sequence network is identical with that previously considered (Fig. 22). The complete zero-sequence network is shown in Fig. 24d. The zero-sequence network reduces to Fig. 24e and to a final equivalent impedance $Z_0 = j9.1$ Fig. 24f.

The phase-sequence currents at the point of fault are equal to the line-to-neutral voltage divided by the sum of the positive-, negative-, and zero-sequence equivalent impedances all on the 33-kv. base.

$$I_{a0} = I_{a1} = I_{a2} = \frac{j19,100}{j(9.1 + 32.3 + 28.95)} = \frac{j19,000}{j70.35} = 272 \text{ amp.}$$

The positive direction of current flow in the three networks was assumed to be in a direction from the generator toward the fault. In the develop-

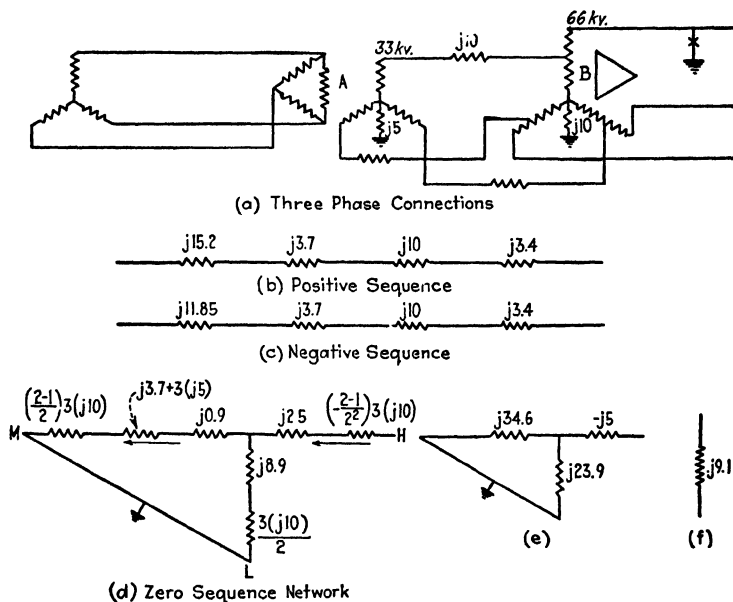


FIG. 24.

ment of the formulas for the zero-sequence circuits for autotransformers (Fig. 22), it was more convenient to adopt an opposite direction of current flow, that is from the H toward the M winding. These arbitrarily chosen directions of current flow of positive sense do not affect the zero-sequence impedance or the values of the sequence currents at the fault. The zero-sequence current I_H in Fig. 24d is the negative of the zero-sequence current at the fault or -272 amp. The currents at all points in the autotransformer can be found by using the formulas previously developed and the negative values of current.

Zero-sequence current in M winding:

$$I_{M0} = \frac{j23.9}{j(23.9 + 34.6)}(-272) = 111$$

Current in neutral A:

$$I_{n1} = 3(-111) = -333$$

Actual zero-sequence current in *H* winding:

$$I_{H0} = 27\frac{1}{2} = 136$$

Zero-sequence current in neutral B:

$$\begin{aligned} I_{n2} &= 3[I_M(\text{actual}) - I_H(\text{actual})] \\ &= 3(-111 + 136) = 75 \end{aligned}$$

Zero-sequence current in *L* winding:

$$I_{L0} = \frac{j34.6}{j(23.9 + 34.6)}(-272) = 161$$

$$\text{Actual} = (-161)^{3\frac{3}{4}}_2 = 442$$

The actual zero-sequence-current distribution is shown in Fig. 25. It should be remembered that the positive sense of current flow for the zero-sequence currents is the opposite or negative of the assumed current direction used in determining the distribution.

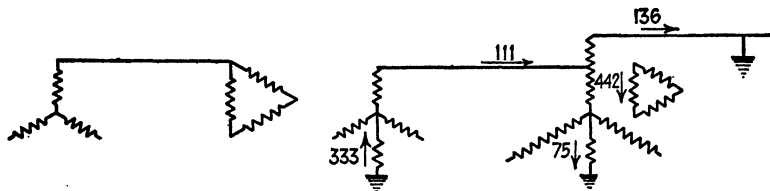


FIG. 25.—Actual zero-sequence current distribution.

After determining the zero-sequence distribution, the positive and negative currents may be combined with the zero-sequence currents in the usual manner to find the phase currents.

INTERCONNECTED-STAR-DELTA TRANSFORMER¹

The equivalent circuit for two-winding interconnected-star transformers was given under the heading of two-winding transformers. The interconnected-star-delta transformer is treated in a manner similar to other three-winding transformers. The vector representation of this type of transformer is given in Fig. 26a and the complete wiring diagram in Fig. 26b. The equivalent star impedances for the three windings of each transformer are Z_H , Z_M , and Z_L , where Z_H is the impedance of the star winding adjacent to the high-voltage terminals, Z_M the winding adjacent to the neutral, and Z_L the delta winding. The notation and current flow in the complete circuit are indicated in

¹ WAGNER, C. F., and R. D. EVANS, *op. cit.*, December, 1929, p. 580.

Fig. 26b. The equivalent circuit for the three windings of phase *a* transformer is shown in Fig. 26c. The current I_x flows out of branch Z_H , and $-I_x$ out of branch Z_M , consistent with Fig. 26b.

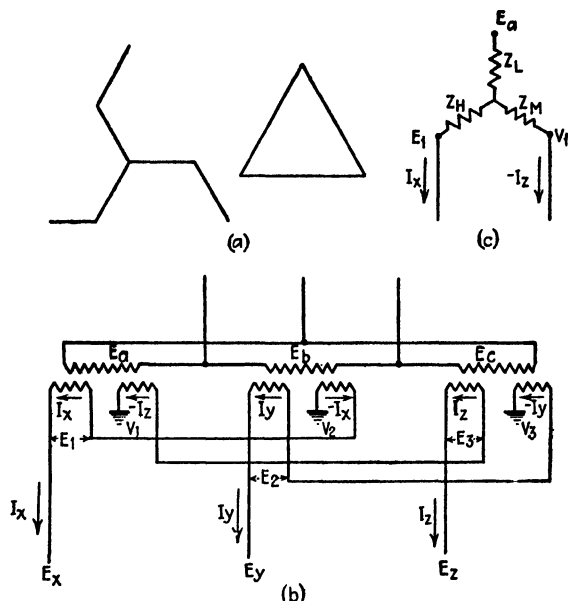


FIG. 26.—(a) Vector diagram for interconnected-star-delta transformer; (b) three-phase connections; (c) equivalent zero-sequence circuit.

With the assumed direction of current flow, the following equations may be written:

$$E_1 = E_a - Z_L(I_x - I_s) - Z_H I_s \quad (110)$$

$$V_1 = E_a - Z_L(I_x - I_s) + Z_M I_s \quad (111)$$

Similarly,

$$E_2 = E_b - Z_L(I_y - I_s) - Z_H I_s \quad (112)$$

$$V_2 = E_b - Z_L(I_y - I_s) + Z_M I_s \quad (113)$$

$$E_3 = E_c - Z_L(I_z - I_y) - Z_H I_s \quad (114)$$

$$V_3 = E_c - Z_L(I_z - I_y) - Z_M I_s \quad (115)$$

It is evident from an inspection of Fig. 26b that

$$E_s = E_1 - V_2 \quad (116)$$

Substituting the values of E_1 and V_1 from Eqs. (110) and (111) in (116),

$$E_s = E_a - E_b - Z_L(2I_s - I_s - I_y) - Z_H I_s - Z_M I_s \quad (117)$$

The positive-sequence impedance can be found from Eq. (117) by substituting the following relations in (117):

$$\left. \begin{aligned} E_b &= a^2 E_a \\ I_y &= a^2 I_x \\ I_z &= a I_x \end{aligned} \right\} \quad (118)$$

Substituting,

$$E_x = (1 - a^2)E_a - (3Z_L + Z_H + Z_M)I_x \quad (119)$$

If the number of turns on each winding is the same, for the no-load condition $I_x = 0$, then the line-to-neutral voltage on the star side is $(1 - a^2)$ times the line-to-line voltage on the delta side. Unit-positive sequence current flowing in the star side reduces the star voltage by the amount $(3Z_L + Z_H + Z_M)$. This quantity is therefore the series impedance in ohms per phase for the positive sequence when the voltage of the star side is used as the base. The impedances Z_H , Z_M and Z_L must be in ohms on the voltage base corresponding to the operating voltage across the individual windings of the two in series, or $0.50E_x/0.866$.

The negative-sequence impedance is the same as the positive.

The zero-sequence impedance can be determined by considering all currents and voltages to be of zero sequence:

$$\left. \begin{aligned} E_b &= E_a \\ I_x &= I_y = I_z \end{aligned} \right\} \quad (120)$$

Substituting Eq. (120) in (117),

$$E_x = -(Z_H + Z_M)I_x = -Z_{HM}I_x \quad (121)$$

The impedance Z_{HM} in ohms on the voltage base of the individual windings on the interconnected-star side is the zero-sequence impedance per phase. The delta impedance does not enter because the two windings on the star side have an equal number of turns and consequently the ampere turns balance. If the star windings were unequal, there would be current in the delta and hence that impedance would be involved in the circuit.

Example.—To illustrate the calculation of the impedances of an interconnected-star-delta transformer, assume a bank of three 2500-kva. single-phase transformers, connected on the interconnected-star side to a 132-kv. line. The delta windings are 13.2 kv.

The voltage across the individual windings E_1 and V_1 (Fig. 26) is 44 kv. corresponding to the line-to-line voltage of 132 kv.

The transformer reactances are:

$$x_{HL} = 15\% \text{ on 2500-kva. base}$$

$$x_{ML} = 15\% \text{ on 2500-kva. base}$$

$$x_{HM} = 18\% \text{ on 2500-kva. base}$$

The equivalent star-impedance branches, to represent the impedances of the three windings of the single-phase transformer, are as follows:

$$Z_L = j\left(\frac{15 + 15 - 18}{2}\right) = j6\%$$

$$Z_H = j\left(\frac{15 + 18 - 15}{2}\right) = j9\%$$

$$Z_M = j\left(\frac{18 + 15 - 15}{2}\right) = j9\%$$

Converting these impedances to ohms at 44 kv :

$$Z_L = \frac{j0.06(44,000)^2}{2,500,000} = j46.5 \quad \text{ohms on 44-kv. base}$$

$$Z_H = j69.7 \quad \text{ohms on 44-kv. base}$$

$$Z_M = j69.7 \quad \text{ohms on 44-kv. base}$$

The positive- and negative-sequence impedance can be found from the reactances of the single-phase transformer:

$$(3Z_L + Z_H + Z_M) = 3(j46.5) + j69.7 + j69.7 = j-78.9$$

This is the value of positive- and negative-sequence reactance to use when using 76,300 volts to neutral corresponding to 132 kv. line to line on the star side of the transformer for short-circuit current calculations.

The zero-sequence impedance of the bank of transformers is simply $Z_{HM} = j69.7 + j69.7 = j139.4$.

CHAPTER VI

CALCULATION OF TRANSIENT SHORT-CIRCUIT¹ CURRENTS

The advent of the use of high-speed relays and oil circuit breakers has required the extension of the methods of calculation to include the transient phenomena. In the past it has been the practice when using conventional relay systems to calculate the short-circuit quantities on the basis of sustained values determined by synchronous reactance, or some average between the initial and sustained values of current. In many cases it is necessary to calculate the transient currents when applying high-speed relays which operate in from $\frac{1}{2}$ to 12 cycles on a 60-cycle system. It may be necessary to determine the transient currents and voltages for the period from the instant of short circuit until the relays and circuit breakers operate. In the past few years, rapid strides have been made in the development of practical methods of calculating transient phenomena associated with synchronous apparatus. The work of Doherty, Nickle, Park, and Wagner has been particularly noteworthy. These methods are adequate for most cases and have been quite extensively used. It is not the intention of the authors to offer different methods but rather to present the essentials of the present methods that are necessary for the application of relays. Certain extensions will be made where necessary to treat special cases not covered by the published data.

PER-UNIT REPRESENTATION

The use of a per-unit or fractional system of notation in equations and calculations has come into widespread use in recent years, superseding the percentage system to a great extent. In the per-unit system, each quantity is expressed as a fraction

¹ DOHERTY, R. E., and C. A. NICKLE, Synchronous Machines, Pts. I and II, *Trans. A.I.E.E.*, vol. 45, 1926.

DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, Pt. III, vol. 46, 1927.

of some definite normal value.¹ Thus, armature reactances may be expressed as the ratio of the reactances in ohms to normal ohms, where normal ohms are the ratio of normal voltage and current. That is,

$$\begin{aligned}\text{Per-unit reactance} &= \frac{\text{reactance in ohms}}{\text{normal ohms}} \\ \text{Normal ohms} &= \frac{\text{normal line-to-neutral voltage}}{\text{normal line current}}\end{aligned}$$

This system is simply an outgrowth of the percentage system where unity is used as the base rather than 100. The use of per-unit quantities eliminates carrying the cumbersome factor 100 in calculations. All quantities may be expressed in per-unit. Thus armature current may be taken as numerically equal to the m.m.f. of armature reaction and per-unit linkages as equal to per-unit voltage or field current. With these quantities expressed as numeric ratios, conversion factors are unnecessary. Normal armature voltage may be taken as normal no-load voltage and all voltages then expressed as fractions of this normal value. Field current may be expressed as a fraction of the field current corresponding to open-circuit normal voltage. Armature current may be expressed as a fraction of normal-rated current.

The analysis of transients in synchronous machines is based principally upon the method of symmetrical components, Blondel's two-reaction theory, and the constant-linkage theorem. When balanced three-phase currents of positive sequence flow in the three phases of a synchronous machine, a flux wave is produced by the m.m.f., the fundamental of which rotates at synchronous speed and is thus stationary with respect to the rotor. The positive-sequence system of armature currents may be resolved into two component three-phase systems by Blondel's two-reaction theory. These two components are spaced 90° apart. One component is so chosen that the current in each individual phase reaches a maximum at the instant that the axis of the field poles coincides with the axis of the particular armature phase considered. This system of currents is known as the *direct-axis components* because an armature reaction flux is produced which tends to magnetize or demagnetize the rotor

¹ PARK, R. H., and B. L. ROBERTSON, The Reactances of Synchronous Machines, *Trans. A.I.E.E.*, vol. 47, p. 516, 1928.

in the axis of the field poles and is affected by a positive-sequence direct-axis reactance. In the other component system, the currents in each individual phase reach a maximum at the instant the axis of the phase considered coincides with an axis midway between the field poles. These currents are known as *quadrature-axis components* because armature reaction fluxes are produced which magnetize or demagnetize the rotor in the interpolar axis and are affected by a positive-sequence quadrature-axis reactance.

Positive-sequence armature currents may be classified as to effect, depending upon whether they are suddenly or continuously applied. When currents are suddenly applied, such as by short-circuiting all three phases of a synchronous machine from a condition of no load, there is a sudden increase in the armature m.m.f. Now, remembering that the fundamental component of armature m.m.f. is stationary with respect to the field poles, it is clear that a unidirectional current will be induced in the field. This action is adequately explained by the constant-linkage theorem,¹ which states that in a closed electric circuit without resistance or capacity the number of magnetic interlinkages must remain constant. This can be demonstrated as follows: Consider a closed rotor winding and one phase of the armature winding short-circuited at the terminals. The summation of voltages in the closed circuits must be zero. Let i_a be the armature current, i_f the field current, M the mutual inductance between the closed windings, and L_a the self-inductance of the armature winding.

$$i_a r_a + L_a \frac{di_a}{dt} + M \frac{di_f}{dt} = 0$$

Neglecting resistance this becomes

$$\frac{d}{dt}(L_a i_a + M i_f) = 0$$

The quantity in the parentheses is the number of magnetic interlinkages of the circuit. The equation shows that the rate of change is zero; therefore, the number of interlinkages is con-

¹ DOHERTY, R. E., and E. T. WILLIAMSON, Short-Circuit Current of Induction Motors and Generators, *Trans. A.I.E.E.*, vol. 40, 1921, discussion, p. 548.

stant. If the turns in the closed circuit are not concentrated, the number of interlinkages in each turn may not be constant, but the total linkages in the closed circuit will be. The number of interlinkages in any closed circuit cannot be changed by a change in magnetic reluctance or by the effect of any secondary coils. When the short circuit occurs, the flux linkages with each circuit on the rotor and armature must remain constant for the first instant. The currents suddenly appearing in the armature phases tend to demagnetize the field circuits, hence the currents in these circuits must increase in order that the condition of constant flux linkages in each circuit may be fulfilled. The initial value of short-circuit current is greater than the steady-state sustained value due to the increased rotor m.m.f. produced by the induced currents. The induced field currents have no voltage to sustain them, hence decay exponentially. It is convenient to account for the increase in rotor m.m.f. at the instant of short circuit by means of a fictitious reactance termed the *transient reactance*. Thus the initial short-circuit current is the ratio of the no-load voltage before short circuit to the transient reactance. All reactances are defined as the fundamental reactive component of armature voltage to the corresponding fundamental component of armature current. After the induced field currents have disappeared, the short-circuit current is limited by the synchronous reactance.

In order to define completely the currents resulting from short circuit, it is necessary to recognize two transient reactances in each axis. This is due to the presence of other closed rotor circuits in addition to the main field winding on many synchronous machines. The currents induced in these additional rotor circuits decay rapidly, hence there is a corresponding high-speed transient in the armature current. The reactances which limit the initial current when additional rotor circuits are present are termed the direct and quadrature subtransient reactances. The subtransient reactance consists of the effective leakage reactance between the armature and the solid rotor teeth and wedges in turbogenerators, and between the armature and damper winding in salient pole machines. If no rotor circuit other than the main field winding is present, then the subtransient and transient reactances are equal. The transient reactance is the effective leakage between the armature and

main field winding. The low value of the subtransient reactance is partly due to saturation in the leakage paths.

If there are no rotor windings, such as an amortisseur or solid rotor teeth and wedges, in the quadrature axis, then the quadrature subtransient and synchronous reactances are the same. For this case $x_q'' = x_q' = x_q$. If a damper winding is present, the quadrature subtransient reactance is different from the transient due to the effects of induced currents. However, since there is no really effective quadrature-axis winding, the quadrature transient and synchronous reactances are the same.

The transient short-circuit armature current may consist of symmetrical components and an asymmetrical or direct-current component. The symmetrical components are classified in three groups:

1. A rapidly decaying subtransient component
2. A slowly decaying transient component
3. A sustained or steady-state component

The symmetrical component of short-circuit current can be expressed as a series of terms consisting of a steady-state and several exponential terms. The maximum instantaneous symmetrical short-circuit current is $i'' = e/x_d''$; the maximum symmetrical current, neglecting the high-speed terms, is $i' = e/x_d'$; and the sustained current, after all decrement terms have disappeared, is $i = e/x_d$. The complete expression for the decrement curve of the symmetrical component of armature short-circuit current is

$$i = e \left[\left(\frac{1}{x_d''} - \frac{1}{x_d'} \right) e^{-\frac{t}{T_d''}} + \left(\frac{1}{x_d'} - \frac{1}{x_d} \right) e^{-\frac{t}{T_d'}} + \frac{1}{x_d} \right]$$

where T_d'' and T_d' are the machine short-circuit time constants applying to the subtransient and transient components of current, respectively.

Where a sudden three-phase short circuit occurs, the initial current is determined by the subtransient reactance. The rapidly decaying component, due to the induced currents in the additional rotor circuits, disappears after a few cycles. The slowly decaying current is affected by the transient reactance. The transient reactance is the reactance that would determine the initial current if the field winding was the only rotor circuit.

The initial transient current is determined by projecting the envelope of the current wave to the instant of short circuit, neglecting the first few cycles which include the high decrement quantities as shown in Fig. 1.

In addition to the symmetrical components of transient fault current, there may be a unidirectional component in the armature, the magnitude of which depends upon the angle between the axes of the field poles and the armature phases at the instant of short circuit. If the axis of a particular phase coincides with

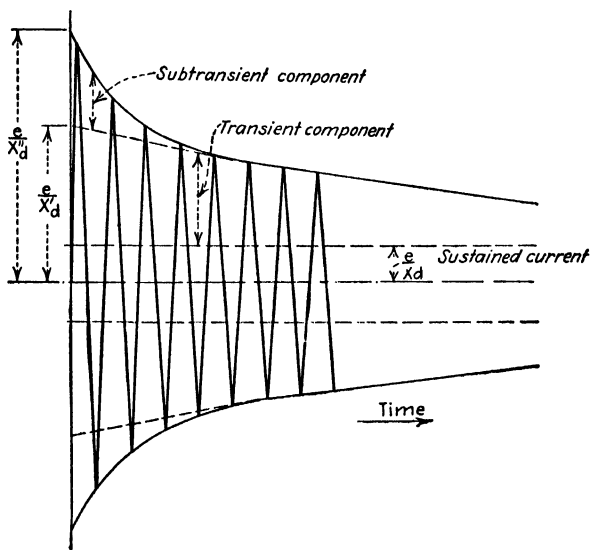


FIG. 1.—Symmetrical short-circuit current.

the axis of the field poles at the instant of fault at no load, the direct-current component in that phase will be a maximum. No direct-current component will appear if the angle between the two axes is 90° . The direct current flows in the armature in order that the flux entrapped at the instant of short circuit will be maintained constant for the first instant. There may be a second harmonic associated with the direct-current component depending upon the machine characteristics. The second-harmonic current appears, owing to the variation of the transient reactance between the direct- and quadrature-axis values and may be quite large in a salient-pole machine without damper windings.

In Fig. 2 is shown an asymmetrical current wave for a three-phase short circuit. The symmetrical component is determined

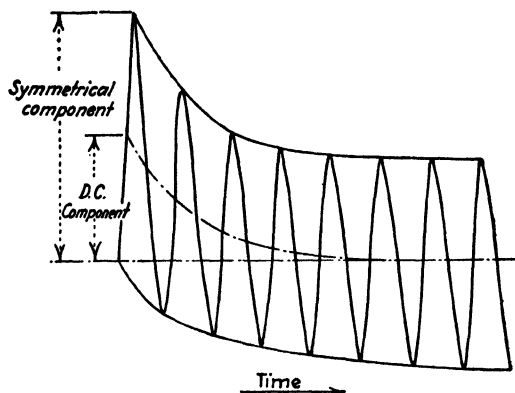


FIG. 2.—Asymmetrical current wave.

by drawing the two envelopes. The peak value of the symmetrical current at any instant is half the distance between envelopes.

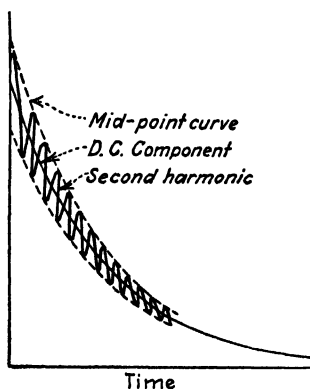


FIG. 3.—Second harmonic and direct-current component three-phase short circuit on salient-pole machine.

The asymmetrical component is determined by drawing a mid-point curve between envelopes, the distance from the zero current line to the mid-point line at any instant being the asymmetrical component. In case there is a second harmonic associated with the direct-current component, the second harmonic has positive peak values^{1,2} in phase with the positive and negative peak values of the symmetrical component. In this case the mid-point curve becomes tangent to the total asymmetrical component as shown in Fig. 3.

Certain interesting phenomena occur in connection with the transient armature currents. Since the field is rotating at

¹ DOHERTY, R. E., and C. A. NICKLE, *Trans. A.I.E.E.*, vol. 49, p. 700, 1930.

² WRIGHT, S. H., Determination of Synchronous Machine Constants by Test, *Trans. A.I.E.E.*, vol. 50, p. 1331, 1931.

synchronous speed with respect to the direct-current component of current in the armature, a current of fundamental frequency is induced in the field. The second harmonic, if present in the armature circuit, will induce a third-harmonic current in the field, etc.

The sustained value of armature short-circuit current, as previously mentioned, is determined by the direct-axis synchronous reactance. According to present accepted practice,¹ this reactance is determined from test as the unsaturated value for a sustained three-phase short circuit, the current being of normal-rated value. The reactance is equal to the ratio of the

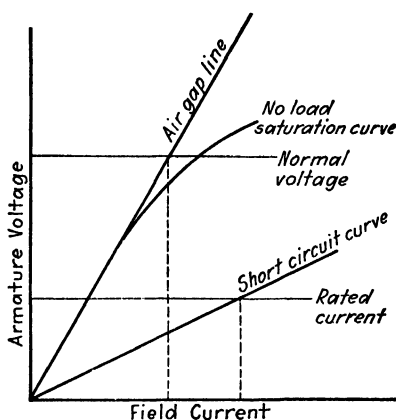


FIG. 4.—Determination of synchronous reactance.

field current required to circulate normal current on short circuit to the field current corresponding to rated voltage on the air-gap line as shown in Fig. 4.

The synchronous reactance is composed of the reactance due to armature leakage fluxes and armature reaction fluxes. The armature reaction, according to present practice² is assumed to refer only to the space fundamental or stationary component with respect to the rotor of armature m.m.f. Thus a distinction is made between m.m.fs. that produce fluxes which penetrate the rotor but do not induce eddy currents and those that have relative motion with respect to the poles and hence induce eddy currents. These eddy currents exert a demagnetizing influence

¹ WRIGHT, S. H., *op. cit.*

² PARK, R. H., and B. L. ROBERTSON, *op. cit.*

upon the producing m.m.f. The m.m.fs. that move relative to the rotor are considered as producing only leakage flux. The synchronous reactance is normally calculated as the sum of the reactance of armature reaction and the armature leakage reactance¹ $x_d = x_{da} + x_l$. Similarly, the quadrature-axis synchronous reactance is

$$x_q = x_{qa} + x_l$$

The armature leakage reactance x_l is due to the flux in the slots and the fluxes in the air gap or end windings which have relative motion with respect to the rotor. The reactance of armature reaction x_{da} is simply that due to the space fundamental of air-gap flux produced by the armature current acting alone.

PRINCIPAL REACTANCE COEFFICIENTS

x_d''	Direct-axis subtransient reactance
x_q''	Quadrature-axis subtransient reactance
x_d'	Direct-axis transient reactance
x_q'	Quadrature-axis transient reactance
x_d	Direct-axis synchronous reactance
x_q	Quadrature-axis synchronous reactance
x_2	Negative-sequence reactance
x_0	Zero-sequence reactance

The negative-sequence reactance x_2 and the zero-sequence reactance x_0 come into effect only during unbalanced short circuits. The negative-sequence reactance is the ratio of the fundamental component of armature voltage due to the fundamental component of applied negative-sequence armature current to that component of armature current at rated frequency. When negative-sequence currents appear in the armature of a machine running at synchronous speed, the armature m.m.f. due to these currents produces a flux which rotates at synchronous speed in a direction opposite to the field produced by positive-sequence currents. The negative-sequence flux field is thus traveling at twice synchronous speed with respect to the rotor, and, as it sweeps past the field poles, a second-harmonic current is induced in the field winding. This is illustrated in the oscillogram of Fig. 5 for a single line-to-line short circuit on a turbine generator. Since the m.m.f. due to the negative-

¹ KILGORE, L. A., Calculation of Synchronous Machine Constants, *Trans. A.I.E.E.*, vol. 50, p. 1201, 1931.

sequence current has motion relative to the field, the armature currents produce no demagnetizing effect upon the field.

The negative-sequence reactance is dependent upon the terminal conditions as to whether fundamental current or voltage is applied and whether external reactance is present or not. It has been shown by Park¹ that the application of fundamental negative-sequence voltage to the terminals of a synchronously

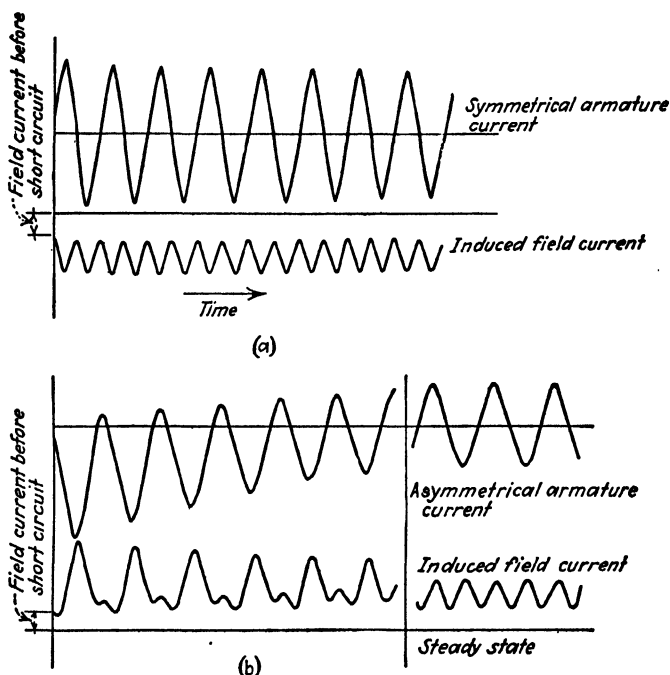


FIG. 5.—Asymmetrical and symmetrical current waves for single line-to-line short circuits on a 35,000-kw. 12,000-volt turbine generator.

rotating machine will give rise to a fundamental negative-sequence current and a positive-sequence third harmonic. The application of fundamental negative-sequence currents produces a fundamental and third-harmonic voltage. Reactance is normally considered as that existing with fundamental current impressed. For the case of a sustained single-phase line-to-line fault without external reactance, the fundamental component of current is affected by a negative-sequence reactance $x_2 = \sqrt{x_d'x_q''}$.

¹ PARK, R. H., and B. L. ROBERTSON, *op. cit.*

If the short circuit occurs through a large external reactance, the effective negative-sequence reactance can be shown to be

$$x_2 = \frac{1}{2}(x_d'' + x_q'').$$

This is the negative-sequence reactance for impressed fundamental current and is that normally used.

The zero-sequence reactance is the ratio of the fundamental reactive armature voltage to the fundamental impressed zero-sequence current. The zero-sequence currents are all in phase, hence the fundamental m.m.fs. of the three phases cancel and there is no appreciable fundamental air-gap flux. The zero-sequence currents produce a pulsating third-harmonic m.m.f. in the air gap and end windings and a slot flux. The slot flux varies with the coil pitch and is reduced by currents flowing in opposite directions in coil sides in the same slot for any pitch other than full pitch. The zero-phase sequence slot reactance is a minimum for $\frac{2}{3}$ pitch. The zero-sequence air-gap leakage varies similarly and is zero at $\frac{2}{3}$ pitch, since for this condition there is no air-gap m.m.f. due to the zero-sequence currents. Since there is no fundamental m.m.f. due to the presence of zero-sequence currents, there is no demagnetizing effect upon the rotor—in other words, no armature reaction.

TIME CONSTANTS

A time constant is generally defined as the time required for the current in a circuit to decay to $1/\epsilon$, or 0.368 of the initial value when the e.m.f. is removed. For example, consider a coil of constant resistance r ohms and constant inductance L henrys suddenly short-circuited on itself when carrying a current I . The differential equation expressing the relations in the circuit after short circuit is

$$ri + L\frac{di}{dt} = 0$$

The solution of this differential equation gives the well-known exponential expression $i = I/\epsilon^{-t/T}$, where i is the current at any time t , $T = L/r$ sec., and ϵ is the base of Napierian logarithms, or 2.718 numerically. If $t = T$, then $i = I/\epsilon = 0.368I$; therefore, the time constant is the time required for the current to decrease to 0.368 of the initial value and is the ratio L/r .

A general method proposed by Doherty¹ for the calculation of transient short-circuit currents in synchronous machines has proved to be the most practical and accurate method devised for general problems. Since the resistance of most circuits and machines is small compared with the reactance, it is neglected in his method when determining the initial and sustained values of current. The resistance of the various circuits, however, is introduced in the time constants which govern the decay of the transient components of the short-circuit current. If it were not for the resistance in the circuits, there would be no decrements and the initial currents would also be the sustained currents.

Calculations involving the use of constants in the direct and quadrature axes give accurate results for salient-pole machines where previous methods were inadequate. In a salient-pole machine, if the axis of the fundamental armature m.m.f. is not in phase with either the direct or quadrature axis, the flux wave produced by the m.m.f. is not in space phase with the m.m.f. wave and calculations involving the total wave would be extremely complicated. From symmetry of the machine, the flux wave is in space phase with the producing m.m.f. wave if the latter is in phase with either the direct or quadrature axis. By resolving the total m.m.f. wave into two components, one along the direct and the other along the quadrature axis, the component flux waves will be in space phase with their respective component m.m.f. waves. Thus, by determining the two components, calculation can be made in which the effect of windings in both the direct and quadrature axes is taken into account by applying the reactance constants associated with the particular axis.

DECREMENT FACTORS

It has previously been stated that when a short circuit occurs, the flux linkages in the field circuit cannot change in the first instant, but, due to the demagnetizing effect of the m.m.f. of the armature short-circuit current, the field flux is forced to pass through leakage paths of higher reluctance. An additional current arises in the field² to support the flux in the new paths.

¹ DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, Pt. IV, vol. 47, p. 457, 1928.

² PARK, R. H., and B. L. ROBERTSON, *op. cit.*

Thus the initial field current $I_{f(0)}$ under short circuit is

$$I_{f(0)} = I_f + \Delta I_f,$$

where I_f is the current before the short circuit and ΔI_f is the induced current. The component ΔI_f is purely transient and, as in any inductive circuit (assuming no saturation), decays exponentially at a rate determined by the time constant L/r , where r is the resistance of the circuit in which the transient flux is decaying and L is the total circuit inductance. The short-circuited armature is equivalent to an increased magnetic reluctance¹ in the field. Obviously, if the armature is short-circuited through an external inductance, the current, and hence the armature m.m.f., will be less than for a short circuit at the armature terminals; the induced field current will be less and the magnetic reluctance of the field is less, hence the effective field inductance depends upon the armature terminal conditions. The decrement is thus affected by external armature reactance. The armature resistance may also produce appreciable effect upon the field transient. With zero armature-circuit resistance all flux linkages are forced into the high-reluctance field leakage paths, and with infinite armature resistance (*i.e.*, open circuit) none of the magnetic flux is forced out of the normal paths. Thus the effective reluctance and corresponding inductance may have any value between the limits corresponding to zero and infinite armature resistance. It is evident, therefore, that the effect of armature resistance on the field decrement may be taken into account by modification of the effective field inductance. This is usually not necessary, unless the magnitude of the resistance is sufficient to affect appreciably the value of the direct-axis component of the armature current.

The transient component of armature current in the direct axis depends upon and is directly proportional in magnitude to the induced component of field current in the direct axis. The field decrement factor is, therefore, applied to the transient component of armature current in the direct axis, and the armature voltage due to this current as well as the induced direct component of current in the field.

The same argument applies to quadrature-axis transient-field-current decrement factors, except, of course, that normally no current exists in quadrature-axis windings.

¹ DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, Pt. IV.

It has been pointed out that a unidirectional current may arise in the armature due to the entrapped flux in the armature at the instant of short circuit. The magnitude of this current is a function of the angle between the axis of the field poles and the axis of the armature phases. The current is transient in character and is affected by a decrement depending upon the effective armature inductance and resistance. The field inductance and resistance affect the armature decrement in a manner analogous to the effect of the armature resistance and inductance on the field decrement factors. The armature decrement factor is applied to the direct current in the armature and to all phenomena which depend upon the current, such as the transient alternating current induced in the field.

The determination of the expression for the field decrement^{1,2} can readily be derived keeping in mind the principles outlined. The assumption will be made that the main field winding is the only winding in the direct axis. The time constant T'_{d0} of the field circuit with the armature open-circuited is equal to L_{f0}/R_f , where L_{f0} is the total self-inductance of the field circuit in henrys and R_f is the effective ohmic resistance of the field circuit. If a machine is running at rated speed, with the armature on open circuit, and the field is suddenly short-circuited, the field current and the armature voltage, being proportional, decay at the same rate, the time constant being T'_{d0} . More specifically this is the time constant of the slowly decreasing component present in the direct-axis component of symmetrical armature voltage as distinguished from high-speed transients. If, in the direct axis, per-unit field current is taken as numerically equal to per-unit nominal voltage, it can be shown that a per-unit m.m.f. suddenly appearing over the direct axis will induce a per-unit field current ΔI_f of magnitude

$$\Delta I_f = (x_d - x'_d)i'_d \quad (1)$$

where i'_d is the transient per-unit armature current, or m.m.f.

Since per-unit field current and armature nominal voltage are equal, $\Delta I_f = \Delta e_d$.

Before short circuit, $I_f = e_d$ in per-unit terms.

¹ DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, vol. 49, p. 700, 1930.

² *Ibid.*, Pt. IV, vol. 47, p. 457, 1928.

Immediately after short circuit the total field current is

$$I_{f(i)} = I_f + \Delta I_f = e_d + \Delta e_d \quad (2)$$

Substituting (1) in (2),

$$I_{f(i)} = e_d + (x_d - x'_d)i'_d \quad (3)$$

For the simple case of a three-phase short circuit at the armature terminals from an open-circuit condition, neglecting armature resistance and assuming only the main field winding in the direct axis, the symmetrical component of armature current in the direct axis is given by the expression

$$i'_d = \frac{e_d}{x'_d} \quad (4)$$

Substituting (4) in (3)

$$I_{f(i)} = \frac{x_d}{x'_d} e_d \quad (5)$$

The field current increases from a value e_d before short circuit to $\frac{x_d}{x'_d} e_d$ immediately after short circuit. This increase in field current occurs in order that the flux linkages of the field remain constant for the first instant; hence, with the armature short-circuited, the effective inductance of the direct-axis field is x'_d/x_d times the value when the armature circuit is open. Since the field and armature resistance is assumed the same for both conditions, it follows that the open-circuit and short-circuit time constants are proportional to the open-circuit and short-circuit effective inductances, respectively.¹

$$\frac{T'_{d0}}{T'_d} = \frac{L_{f0}}{L_{f0} \frac{x_d}{x'_d}}$$

or

$$T'_d = \frac{x'_d}{x_d} T'_{d0} \quad (6)$$

If there is appreciable resistance in the armature circuit, the time constant T'_d will be increased by the same proportion as the equivalent inductance is increased. An expression is

¹ PARK, R. H., and B. L. ROBERTSON, *op. cit.*

given by Wagner¹ for T'_d when appreciable resistance is present.

$$T'_d = \frac{r_a^2 + x'_d x_q}{r_a^2 + x_d x_q} T'_{d0}$$

where r_a is the armature resistance.

The time constant T'_d for a short circuit through an external reactance x is simply

$$T'_d = \frac{x'_d + x}{x_d + x} T'_{d0} \quad (7)$$

These time constants apply to unsymmetrical as well as symmetrical short circuits. In the case of single-phase short circuits the time constants are determined as for an equivalent three-phase fault.²

The time constants given are average factors and determine the envelope of odd harmonics of current in the armature, the direct component of transient current in the field, and the envelope of even harmonics in the field.

If the main field winding is not the only winding in the direct axis of the rotor, the induced field current ΔI_f or Δe_d will appear partly in the main and partly in the other windings. The time constant T'_d is applicable to the slowly decaying quantities neglecting the high-speed phenomena due to the current in the additional direct-axis rotor circuits. There is a time constant T''_{d0} and T''_d corresponding to the high-speed terms determined by the subtransient reactance. Thus the constant T''_d applies to the transient component of symmetrical current $\left(\frac{e_d}{x''_d} - \frac{e_d}{x'_d} \right)$ which is produced by the currents induced in additional direct-axis rotor circuits. These constants are most readily determined from short-circuit tests as described later.

In case of short circuits under load or through resistance, the time constants of the rotor circuits in the quadrature axis are involved.³ Corresponding to the direct-axis constants, there are quadrature-axis subtransient time constants T''_{q0} , T''_q and transient constants T'_{q0} , T'_q . The quadrature-axis constants are all so small, except for solid rotor turbine generators, as to

¹ WAGNER, C. F., Effect of Armature Resistance upon Hunting of Synchronous Machines, *Trans. A.I.E.E.*, vol. 49, p. 1011, 1930.

² PARK, R. H., and B. L. ROBERTSON, *op. cit.*

³ DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, vol. 49, p. 700, 1930.

be of no great importance in practical calculations. The constants T'_{q0} and T'_a are of no importance except in turbine generators. T''_{q0} and T''_a are present principally in salient-pole machines with amortisseur windings. With a short circuit through external resistance, the resistance displaces the armature m.m.f. from the rotor axes of symmetry when rotor circuits are present in the quadrature axes. This has the effect of furnishing coupling between the direct- and quadrature-axis windings on the rotor. There may be flux initially present in the quadrature axis when a short circuit occurs under load conditions. If more than one circuit is present in the direct or quadrature axis or both, these circuits will have mutual coupling which causes additional components to appear in the decrements. Solution of the complex equations for such circuits has been given by Park,^{1,2} but in practice it is seldom necessary to resort to these more accurate solutions.

The armature time constant T_a applies to the direct-current component of armature current produced by the flux linkages trapped in the armature circuit at the instant of short circuit. This time constant on dead short circuit is given by the expression

$$T_a = \frac{x_2}{2\pi f r_a} \quad (8)$$

Assuming the resistance of the rotor circuit to be negligible, the time constant of the armature for direct current depends upon the armature inductance, in the same manner as the time constant for direct current in the field depends upon the field inductance. Since the direct and quadrature axes of the rotor sweep past the m.m.f. due to the direct current in the armature at synchronous speed, the effective inductance of the armature to direct current is the average of the direct and quadrature sub-transient reactance x''_d and x''_q which is equal to the negative-sequence reactance x_2 . If the machine is short-circuited through an external reactance x , the time constant becomes

$$T_a = \frac{x_2 + x}{2\pi f r_a} \quad (9)$$

¹ PARK, R. H., and B. L. ROBERTSON, *op. cit.*

² PARK, R. H., Two Reaction Theory of Synchronous Machines, *Trans. A.I.E.E.*, vol. 48, p. 716, 1929.

CALCULATION OF TRANSIENT CURRENT

The calculation of the transient current in a single machine, using the methods described, is quite simple if the machine constants are known. In most practical problems arising in connection with relaying, more than one machine is involved and the machines may have entirely different constants and characteristics and, furthermore, will be operating under load at the instant of fault. Under fault conditions, the current supplied by the various machines will decay at different rates depending upon the short-circuit time constants. Another factor that may change the characteristics of the short-circuit current is the change in excitation where quick-acting voltage regulators and exciters are used. Machines having similar characteristics, when operating in parallel with the same excitation, can be combined into a single equivalent machine, since the rate of decay of current for each machine will be the same. However, for the general case of a network supplied by dissimilar units, it is necessary to devise a method of calculation whereby the instantaneous action of each individual unit can be determined when a fault occurs.

The method of calculating the initial current and applying decrement factors is not well suited to the solution of a network involving more than one machine, particularly where a calculating board is used. A more satisfactory method is to determine the instantaneous variation of internal voltages in the various machines, and for any particular instant the current in any branch of the network can be determined by applying the voltages of the various units to the network at the values existing at the instant considered. This can be shown more clearly from the following argument, considering only the transient or slowly decaying component of symmetrical current in the direct axis for a fault at the terminals of a machine at no load. As previously shown, the expression for the transient component of current in terms of direct-axis excitation voltage and the armature reactances is

$$\Delta i' = \left[\frac{e_d}{x_d'} - \frac{e_d}{x_d} \right] e^{-\frac{t}{T_d'}} = e_d \left(\frac{1}{x_d' - x_d} \right) e^{-\frac{t}{T_d'}} \quad (10)$$

The decrement is applied to the initial transient component of current, and it is assumed that the current decays exponentially.

Now, if a fictitious transient internal voltage could be determined, such that, with the same decrement factor applied, the current at each instant (determined by dividing the transient voltage existing at that instant by the transient reactance) would be the same as given in Eq. (10) for the same instant, then the transient variation could be applied to voltage rather than current.¹ This follows from the fact that the current in the direct axis of the armature and the total excitation in the direct axis are proportional at each instant. Therefore, an equation for the current may be written in terms of the fictitious transient internal voltage $\Delta e'_d$ which, as here considered, decays to zero

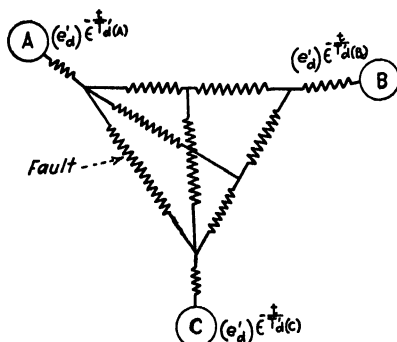


FIG. 6.—Network supplied by dissimilar generators.

and hence does not include the component which exists after steady-state conditions have been reached.

$$\Delta i' = \frac{(\Delta e'_d) \epsilon^{-\frac{t}{T'_d}}}{x'_d} \quad (11)$$

Equating (10) and (11) and solving for $\Delta e'_d$ in terms of e_d gives

$$\Delta e'_d = e_d \left(\frac{x'_d}{x'_d - x_d} \right) \quad (12)$$

Thus the transient internal voltage with appropriate decrement factors for each machine can be applied to the network (Fig. 6)

¹ The essential material for the internal voltage method of calculating transients here described was taken from Wagner's Works. C. F. Wagner, *op. cit.*, vol. 49, p. 1011, 1930; also Decrement of Short-circuit Currents, *Elec. Jour.*, May, 1933, p. 205.) A considerable number of extensions and derivations have been added by the authors.

and the resulting current determined for each instant. It will be recognized that this is in accordance with the principle of superposition. That is, the current distribution due to a given distribution of voltage may be found by superposing the current distributions due to each voltage or component of voltage taken individually.

Assumptions.—In the calculation of transient currents it is necessary to make certain simplifying assumptions in order to make the methods practical for general use. In accordance with generally accepted practice the following assumptions are made.

1. The open-circuit voltage of the synchronous machines is a sine wave. This is essentially true for all modern machines.

2. Saturation in the magnetic circuits is assumed to be negligible. If appreciable saturation exists, corrections can be made by shading the constants or using constants determined under saturated conditions.

3. The resistance of the armature and field circuits is neglected in determining the magnitude of fault currents but is taken into account in the decrements.

4. The speed of all machines remains constant under short circuit. If this assumption were not made, the principle of superposition could be applied only for the first instant of fault. Additional transient phenomena occur with change of speed which complicates the analysis greatly. A change of speed occurs in the case of a short circuit with fault resistance. The decrease in speed is due to the kinetic energy of the machines having to supply the increase in load due to the fault before the governor can operate. The speed may increase when a reactive fault occurs due to the dropping of load from low voltage. For all practical purposes the assumption of zero speed change is well within the accuracy requirements for all but very special cases.

TRANSIENT VOLTAGES

The relations between internal voltages and reactances can be demonstrated very simply.¹ In Fig. 7 are shown the no-load excitation relations for a synchronous machine with no saturation. Per-unit field current $I_{f(0)}$ is taken as that required to produce per-unit armature terminal voltage at no load $e_{d(0)}$. Unit air-gap flux is taken as that required to produce unit voltage, so $e_{d(0)}$ also represents air-gap flux. The component of armature current in the direct axis i_d produces a demagnetizing effect $i_d x_a$ where x_a is a factor converting per-unit armature current

¹ WAGNER, C. F., *op. cit.*, *Trans. A.I.E.E.*, vol. 49, p. 1011, 1930.

into terms of equivalent field current. In order to overcome this demagnetization of armature reaction, the field current must be increased by the amount $i_d x_a$ to maintain the same terminal voltage. The total field current is then $e_{d(0)} + i_d x_a$ and this is numerically equal to e_d the nominal voltage due to excitation in the direct axis.

$$e_d = e_{d(0)} + i_d x_a \quad (13)$$

The field windings are also linked with a field leakage flux ψ_{fl} (i.e., equivalent flux linking all turns) which is directly proportional to the total field current e_d if saturation effects are

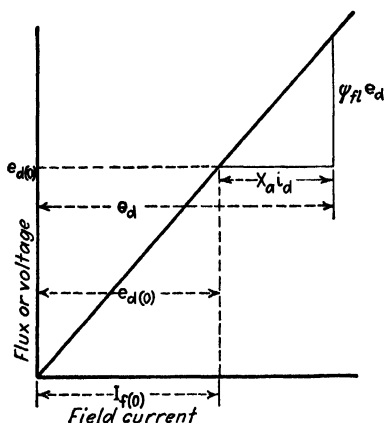


FIG. 7.—Flux and current relations.

neglected. If this field leakage flux is expressed as a fraction of the no-load air-gap flux, its value is $\psi_{fl} e_{d(0)}$. The total flux linkages with the field is $e_{d(0)} + \psi_{fl} e_{d(0)}$. The flux linking the field winding at no-load normal voltage is $(1 + \psi_{fl})$. If unit flux linkages are taken as the flux linking the field under this condition, then the per-unit flux linkages under load are

$$\psi_d = \frac{e_{d(0)} + \psi_{fl} e_d}{1 + \psi_{fl}} \quad (14)$$

or substituting Eq. (13) in (14)

$$\psi_d = e_d - \frac{x_a}{1 + \psi_{fl}} i_d \quad (15)$$

With unit armature current flowing on a three-phase short circuit, the total field current required to maintain the flux and unit

armature current is x_d , the synchronous reactance of which is equal to $(x_l + x_a)$ in which the air-gap voltage is equal to x_l and the armature reaction is x_d in terms of field current. The corresponding field leakage is $\psi_{fl}x_d$. The total field linkages with unit armature current flowing, expressed in per-unit, is then $\frac{x_l + \psi_{fl}x_d}{1 + \psi_{fl}}$. According to the constant-linkage theorem, if the circuit conditions suddenly change, the flux linkages with the field at the first instant must remain constant. The flux linkages with the field winding for no-load normal voltage is defined as unity; therefore, if the armature is suddenly short-circuited, the flux linkages for the first instant must remain at this value. The total linkages at no load divided by the total flux linkages per-unit armature current must equal the armature short-circuit current or $\frac{1 + \psi_{fl}}{x_l + \psi_{fl}x_d}$.

Transient reactance has been defined as the reactance which determines the initial symmetrical current, neglecting high-speed components, when a three-phase short circuit is applied to the terminals of a machine operating at normal voltage and no load. According to this definition it is evident that the transient reactance x'_d is the reciprocal of the expression for armature current, or

$$x'_d = \frac{x_l + \psi_{fl}x_d}{1 + \psi_{fl}} \quad (16)$$

Since $x_d = x_l + x_a$, this may be written

$$x'_d = x_l + \frac{\psi_{fl}}{1 + \psi_{fl}}x_a \quad (17)$$

$$x'_d = x_d - \frac{1}{1 + \psi_{fl}}x_a \quad (18)$$

Solving for $\frac{x_a}{1 + \psi_{fl}}$ in (18) and substituting in (15) gives

$$e_d - \psi_d = (x_d - x'_d)i_d \quad (19)$$

The difference between e_d and ψ_d in per-unit terms is equal to the difference between synchronous and transient reactance. It is evident that ψ_d can be used as a fictitious voltage. This is designated as the transient internal voltage e'_d and can be calculated the same as e_d under steady-state conditions except

that x'_d is used instead of x_d . Equation (19) may be written

$$e_d - e'_d = (x_d - x'_d)i_d \quad \text{steady state} \quad (20)$$

or

$$e'_d = e_d - (x_d - x'_d)i_d \quad (21)$$

It will be seen from Eq. (21) that for constant excitation the transient internal voltage decreases as the direct component of armature current increases, the term in the parentheses being in the nature of an armature reaction. If $i_d = 0$, then $e_d = e'_d$.

Similar relations exist in the quadrature axis:

$$e_q - e'_q = (x_q - x'_q)i_q \quad (22)$$

In the quadrature axis, however, no normal excitation is present so $e_q = 0$ and Eq. (22) reduces to

$$-e'_q = (x_q - x'_q)i_q \quad (23)$$

If amortisseurs or any auxiliary windings are present in the direct and quadrature axes, then for steady-state conditions

$$e'_d - e''_d = (x'_d - x''_d)i_d \quad (24)$$

$$-e'_q = (x'_q - x''_q)i_q \quad (25)$$

When an armature current i_d is present and the field is short-circuited, the effect in field terms is $x_a i_d$. The linkages with the field become $\psi_f = x_a i_d$. Substituting the value of x_a from Eq. (18) gives

$$\psi_f = (x_d - x'_d)(1 + \psi_{fl})i_d \quad (26)$$

Now by definition per-unit normal flux linkages is $(1 + \psi_{fl})$ times greater than per-unit normal field current; therefore, dividing ψ_f by normal flux linkages $(1 + \psi_{fl})$ converts the expression to terms of per-unit field current.

$$I_f = (x_d - x'_d)i_d \quad (27)$$

If the current is suddenly applied, it follows that the induced increment of field current is related to the increment change of armature current by the expression

$$\Delta I_f = (x_d - x'_d)\Delta i_d = \Delta e_d \quad (28)$$

The vector diagram (Fig. 8) shows the relations for steady-state load conditions in which the synchronous, transient, and subtransient voltages are $e_{(s)}$, $e'_{(s)}$, $e''_{(s)}$, and the respective components in the direct and quadrature axis corresponding to each

internal voltage are $e_d e_q$, $e'_d e'_q$, and $e''_d e''_q$. A similar notation with i substituted for e designates corresponding components of current.

When the machine is short-circuited, a new current appears in the direct axis. The difference between the direct-axis component of current i'_d after short circuit and the direct component of the steady-state current existing before short circuit

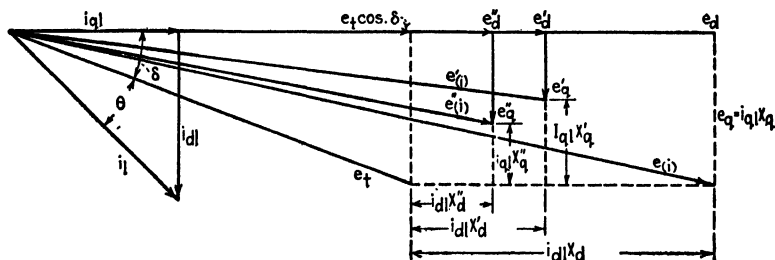


FIG. 8.—Internal-voltage relations for steady-load condition.

$$\begin{aligned}
 e''_d &= e_t \cos \delta + i_{dl} x''_d = e_d - (x_d - x''_d) i_{dl} \\
 e'_d &= e_t \cos \delta + i_{dl} x'_d = e_d - (x_d - x'_d) i_{dl} \\
 e_d &= e_t \cos \delta + i_{dl} x_d \\
 e''_q &= e_t \sin \delta - i_{ql} x''_q = -(x_q - x''_q) i_{ql} \\
 e'_q &= e_t \sin \delta - i_{ql} x'_q = -(x_q - x'_q) i_{ql} \\
 e_q &= e_t \sin \delta = i_{ql} x_q \\
 e_{(i)} &= e_d - j e_q \\
 e'_{(i)} &= e'_d - j e'_q \\
 e''_{(i)} &= e''_d - j e''_q
 \end{aligned}$$

i_{dl} arises suddenly over the direct axis and induces a current in the field corresponding to the increment in nominal voltage.

$$\Delta e_d = \Delta(i'_d - i_{dl})(x_d - x'_d) \quad (29)$$

The initial symmetrical armature current i'_d is

$$i'_d = \frac{e_d + \Delta e_d}{x_d} \quad (30)$$

$$= \frac{e_d + (i'_d - i_{dl})(x_d - x'_d)}{x_d} \quad (31)$$

In terms of transient reactance this may be written

$$i'_d = \frac{e_d - i_{dl}(x_d - x'_d)}{x'_d} \quad (32)$$

$$= \frac{e_t \cos \delta + i_{dl} x'_d}{x'_d} \quad (33)$$

$$= \frac{e'_d}{x'_d} \quad (34)$$

It is evident that with a fault at the machine terminals, the load current before fault becomes part of the total current after fault.

Under load conditions, since flux initially exists in the quadrature axis if an amortisseur winding is present in this axis, a current i_q'' will arise inducing a rotor current ΔI_q or Δe_q

$$\Delta e_q = (i_q'' - i_{ql})(x_q - x_q'') \quad (35)$$

$$i_q'' = -\frac{i_{ql}(x_q - x_q'')}{x_q''} = -\frac{e_q'}{x_q''} \quad (36)$$

When $x_q = x_q''$, the armature current in the quadrature axis is zero. For solid rotor turbine generators, the quadrature-axis transient reactance may produce appreciable effect, if there is considerable initial quadrature-axis flux at the instant of fault due to load current. In this case

$$i_q' = -\frac{i_{ql}(x_q - x_q')}{x_q'} = -\frac{e_q'}{x_q'} \quad (37)$$

The direct- and quadrature-axis values of armature current can be solved for independently and then combined to obtain the total fault current by taking the square root of the sum of their squares. The time constant T_q' must be applied to the quadrature-axis component of current and T_d' to the direct-axis component. The turbine generator is the only common type having an effective quadrature-axis circuit. The salient-pole machine has no effective quadrature-axis winding; therefore, $x_q' = x_q$ and no transient armature current can arise in the direct axis as shown by Eq. (35); however, a subtransient quadrature-axis current may arise, as previously shown. In most practical cases of turbine generators the saturated values of x_d' and x_q' are practically equal and for short-circuit calculations this permits simplification of methods. Thus, in general,

$$i_d' = \frac{e_d'}{x_d'} \quad (38)$$

$$i_q' = -\frac{e_q'}{x_q'} \quad (39)$$

$$i' = i_d' + i_q' \quad (40)$$

$$e_{(0)}' = e_d' - je_q' \quad (41)$$

But, if $x'_d = x'_q$, then,

$$i'_d + i'_q = \frac{e'_d - j e'_q}{x'_d} \quad (42)$$

Therefore,

$$i' = \frac{e'_{(i)}}{x'_d} \quad (43)$$

The direct-axis time constant T'_d is applied to the initial current to determine the decay, the assumption being made that $T'_d = T'_q$. In turbine generators the subtransient reactance (saturated values) in the direct and quadrature axes are practically equal; therefore,

$$i'' = \frac{e''}{x'_d}$$

It should be carefully observed that this applies only to round rotor machines in which the transient and subtransient reactances in the two axes are respectively equal. For salient-pole machines, if considerable initial quadrature-axis flux exists, it is necessary to calculate the quadrature-axis subtransient current independently and apply the proper time constant

TRANSIENT SYMMETRICAL COMPONENT OF ARMATURE CURRENT

The simple case of a three-phase short circuit on a single machine will be considered to demonstrate the internal-voltage method of calculating transient current. Let $e_{(i)}$, $e'_{(i)}$, $e''_{(i)}$ be the voltages before short circuit. It is assumed that the machine is a turbine generator and the subtransient terms are neglected. The variation of transient internal voltage is calculated and used as a basis to determine the transient component of current. Under short circuit this voltage varies from the value before short circuit $e_{(i)}$ to a final value of e'_∞ under sustained fault condition. The value of this voltage at any instant is e' . The assumption is made that e' varies exponentially between $e'_{(i)}$ and e'_∞ . The transient internal voltage under sustained fault conditions is from Eq. (21).

$$e'_\infty = e_{(i)} - (x_d - x'_d)i \quad (44)$$

where i is the sustained short-circuit current.

The total transient internal voltage at any instant is then

$$e' = e_{\infty}' + (e_{(i)}' - e_{\infty}')e^{-\frac{t}{T_d'}} \quad (45)$$

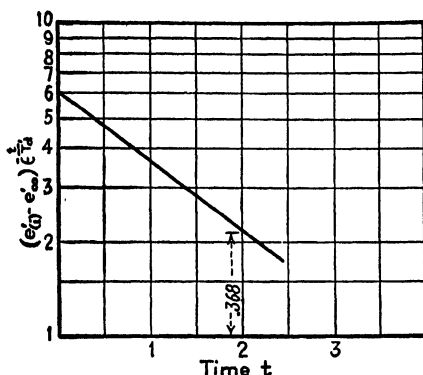


FIG. 9.—Graphical determination of exponential term.

The total armature short-circuit current at any instant, neglecting subtransient terms, is

$$i' = \frac{e'}{x_d'} = \frac{e_{\infty}' + (e_{(i)}' - e_{\infty}')e^{-\frac{t}{T_d'}}}{x_d'} \quad (46)$$

The value of the exponential term may be determined most readily by a graphical method (Fig. 9). For sustained fault conditions the exponential term in Eq. (46) disappears and the sustained current is

$$i = \frac{e_{\infty}'}{x_d'} = \frac{e_{(i)}}{x_d'} \quad (47)$$

The initial total current is

$$i' = \frac{e_{(i)}}{x_d'} \quad (48)$$

FIG. 10.— e_t , i_t , and θ Known—To locate e_d erect perpendicular to i_t which will touch e_t . A distance $x_q i_t$ on this line from e_t locates point "b" on line e_d .

In determining the direct- and quadrature-axis internal voltages before short circuit, only the terminal voltage, line current, and the power-factor angle of each machine are generally known. In Fig. 10 is shown the vector diagram of a machine under load in which e_t , i_t , and θ are known. The angle δ and the location of the direct-axis line along

which e_d lies are to be found. Taking e_d as the reference line for all quantities, the expression for e_d is

$$e_d = e_t + x_d i_{dl} + j x_q i_{ql} \quad (49)$$

Now

$$x_q i_{ql} = j x_q (i_{ql} - j i_{dl}) = x_q i_{dl} + j x_q i_{ql} \quad (50)$$

The quadrature term $x_q i_{ql}$ of Eq. (49) is the quadrature component of $x_q i_{ql}$ in Eq. (50). If the quadrature component $x_q i_{ql}$ of the voltage $x_q i_{ql}$ is drawn in the proper direction from the end of the terminal voltage vector e_t , the direct component $x_q i_{dl}$ will lie along the vector e_d . It follows, therefore, that if the voltage vector $x_q i_{ql}$ is drawn in the proper direction from the end e_t , its terminal will lie on e_d . But $x_q i_{ql}$ is a reactive drop and the vector representing this voltage will be perpendicular to the vector i_{ql} . Therefore, the direction of e_d may be determined by drawing $x_q i_{ql}$ from the end of vector e_t , perpendicular to the current vector i_{ql} . A line through point O and the end of $x_q i_{ql}$ is coincident with e_d .

DIRECT-CURRENT COMPONENT OF ARMATURE CURRENT

The maximum value of the direct-current component of current is assumed to be equal to the crest value of the initial symmetrical short-circuit current.¹ This is theoretically true for a short circuit at no load when the axis of an armature phase coincides with the direct axis of the field poles. The direct-current component may have any value between the maximum and zero, depending upon the angle of the axes and the excitation at short circuit. For the maximum condition the wave is completely offset with respect to the normal zero line, and for the minimum condition the positive and negative crests of the wave are symmetrical with respect to the zero line. A clear physical conception of the nature of the direct-current component may be obtained from an examination of the equation for the current wave for a short circuit at no load. Assume

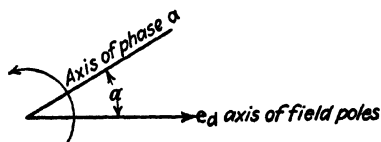


FIG. 11.

¹ WAGNER, C. F., Decrement of Short-Circuit Currents, *Elec. Jour.*, May, 1933, p. 205.

that the axis of phase a is ahead of the axis of the poles by an angle α at the instant of short circuit (see vector diagram, Fig. 11). The instantaneous current in phase a after short circuit is

$$i_a = \sqrt{2} \frac{e_d}{x_d'''} \cos (t - \alpha) \quad (51)$$

Counting time t from the instant of short circuit, for $\alpha = \pi/2$, $i_a = 0$, and the wave will be symmetrical since it starts from zero at $t = 0$, but for $\alpha = 0$, $i_a = \sqrt{2} \frac{e_d}{x_d'''}$ which is a maximum value.

The equation $i_a = \sqrt{2} \frac{e_d}{x_d'''} \cos t$ is plotted in Fig. 12a. Since i_a

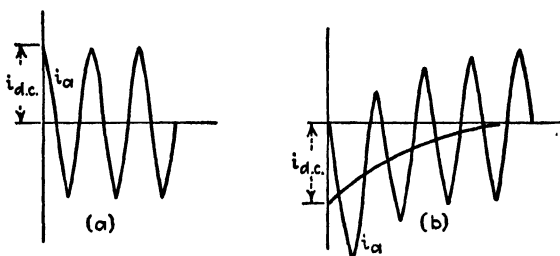


FIG. 12.

obviously cannot rise to a maximum value in zero time, the equation simply means that the zero axis has shifted from its normal position to a new position displaced by an amount $-\sqrt{2} \frac{e_d}{x_d'''}$ as shown in Fig. 12b. The equation for $i_{d.c.}$ as a function of the angle α at the instant of fault is

$$i_{d.c.} = -\sqrt{2} \frac{e_d}{x_d'''} \cos \alpha \quad (52)$$

The total r.m.s. current is equal to the square root of the sum of the squares of the r.m.s. values of the direct-current and symmetrical components. Thus, the maximum value of the total r.m.s. current is

$$\sqrt{\left(\frac{e_d}{x_d'''}\right)^2 + \left(\frac{\sqrt{2}e_d}{x_d'''}\right)^2} = 1.73 \frac{e_d}{x_d'''}$$

This is the maximum value of the total r.m.s. current at zero time and is a value which is never actually reached as some of the high-speed components in the subtransient current

decrease before the first current peak is reached. At any time other than zero, the symmetrical and direct-current components are decaying at different rates; therefore, the total current at any instant must be determined from the components existing at that instant.

When a short circuit occurs under load, a new current arises in the armature. The difference between this new current and the load current before short circuit rises suddenly. It is

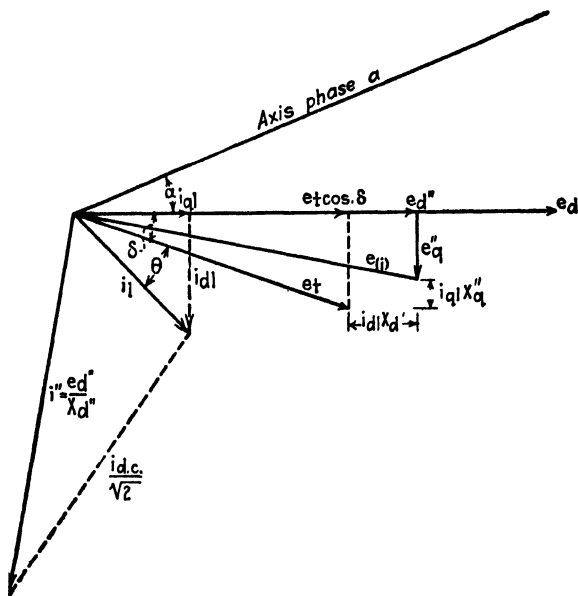


FIG. 13.—Determination of maximum direct-current component of fault current.

this component of current that determines the direct-current component of armature current for a fault under load. Thus the initial value of the direct-current component is calculated as the crest value of the difference between the load current before fault and the subtransient current i'' immediately after fault. The relations for a three-phase short circuit under load are shown in Fig. 13.

THREE-PHASE SHORT CIRCUIT WITH ANY NUMBER OF MACHINES

1. From the known load conditions existing before fault on each machine, calculate the internal voltages $e_{(0)}$, $e'_{(0)}$, $e''_{(0)}$, of each

machine using the formulas (assuming solid rotor turbine generators in which $x'_d = x'_q$ and $T'_d = T'_q$)

$$e_{(i)} = e_i \cos \delta + i_d x_d - j i_q x_q \quad (53)$$

$$e'_{(i)} = e_{(i)} - (x_d - x'_d) i_l \quad (54)$$

$$e''_{(i)} = e_{(i)} - (x_d - x''_d) i_l \quad (55)$$

2. Set up the complete network (positive sequence) for fault condition with $e_{(i)}$ for each machine applied at the proper point in the network, using synchronous reactance in each machine. The sustained short-circuit current i for each machine may be determined by simultaneous equations or by setting up the

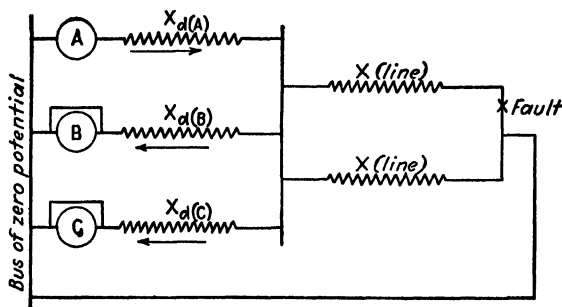


FIG. 14.—Positive-sequence network set up to determine time constant T'_d for machine A.

complete network on an alternating-current calculating board and applying the voltages $e_{(i)}$ of each machine at the proper points and measuring the current. It should be emphasized that the current i of each machine must be calculated for the condition of all voltages, $e_{(i)}$, being applied to the network simultaneously.

The transient internal voltage under sustained fault condition is then calculated for each machine from the formula

$$e'_{\infty} = e_{(i)} - (x_d - x'_d) i \quad (56)$$

3. The short-circuit time constant for each machine is next determined. To find the time constant of machine A, set up the synchronous reactance x_d in each machine and apply any convenient internal voltage to machine A only, the internal voltage of the other machines being zero. This means that the point of application of internal voltage in all other machines must be connected to the zero-potential bus as shown in Fig. 14.

The ratio of the applied voltage to the current in machine *A* is the total circuit reactance to the fault, as viewed from *A*, and is designated as $(x_{d(A)} + x)$ where $x_{d(A)}$ is the synchronous reactance of machine *A*, and x is the total external reactance as viewed from *A*. The time constant for machine *A* is then calculated from Eq. (7):

$$T'_{d(A)} = \frac{(x + x'_{d(A)})}{(x + x_{d(A)})} T'_{d0(A)}$$

Obviously x is calculated from $(x_{d(A)} + x)$, as determined above, simply by subtracting the value of $x_{d(A)}$, or

$$x = (x_{d(A)} + x) - x_{d(A)}.$$

The short-circuit time constants for all other machines are calculated in a similar manner. The method of determining the short-circuit time constant follows from the fact that in the classical differential equations for a closed inductive circuit, the rate of decay of the current produced by a particular voltage is not influenced by any other voltage acting on the same closed circuit. The time constant of each machine is then independent of the time constants of the other machines. The effective reactance of other machines when determining the time constant of a particular machine is taken as the synchronous reactance.

4. The internal voltage e' of each machine at any time t is calculated from Eq. (45),

$$e' = e'_{\infty} + (e'_{(0)} - e'_{\infty})e^{-\frac{t}{T'_d}}$$

The transient reactance x'_d of each machine is set up in the network, and the calculated value e' of each machine for any time t is applied to the network simultaneously and the current i' in any branch is calculated or measured. Another interval of time t is chosen and the calculations or measurements repeated. Using this step-by-step process, the variation of the current in any machine or branch can be determined. The final steady-state short-circuit value is, of course, the same as determined in item 2.

5. The subtransient component (present only when there are additional rotor circuits) i'' is determined by setting up $e''_{(0)}$ and x''_d in each machine and calculating or measuring i'' in any

branch. The difference between this current and the current determined for e' when $t = 0$ in item 4 for each machine is the increment of current due to the subtransient phenomena. This is

$$\Delta i'' = \frac{e''_{(i)}}{x'_d} - \frac{e'_{(i)}}{x'_d} \quad (57)$$

The current decays to zero, and the value at any instant in any machine is determined from the exponential expression

$$\left(\frac{e''_{(i)}}{x'_d} - \frac{e'_{(i)}}{x'_d} \right) e^{-\frac{t}{T''_d}} \quad (58)$$

The time constant T''_d is $T''_d = \frac{x'_d + x}{x'_d + x} T''_{d0}$, where x is the external reactance as found in item 3. If the fault is far removed from the terminals of a particular machine, the subtransient component of current will be small and may be neglected. If the fault is close to the terminals of any machine, the value of x may be taken as the reactance between the terminals and point of fault with little error.

The variation of the subtransient current can be calculated using the internal voltages as follows:

Set up x'_d and $e'_{(i)}$ in each machine and determine i' . Calculate the steady-state value of the transient internal voltage from $e''_{\infty} = e'_{(i)} - (x'_d - x'')i'$. The increment of subtransient voltage is $\Delta e'' = (e''_{(i)} - e''_{\infty})e^{-\frac{t}{T''_d}}$. The subtransient component of current is then determined by applying $\Delta e''$ for each machine for any time t with x'_d set up in each machine. For $t = 0$ this becomes $(i'' - i')$, and for $t = \infty$ the increment approaches 0.

6. The maximum initial value of the direct-current component $i_{d.o.(i)}$ for each machine is calculated as the difference between the initial value of the subtransient current i'' determined in item 5 and the load current existing before short circuit. This is found using the vector diagram (Fig. 13). The vector expression is $i_{d.o.(i)} = \sqrt{2}(i'' - i_l)$. The value of $i_{d.o.}$ for any value between the initial and zero is calculated from

$$i_{d.o.} = i_{d.o.(i)} e^{-\frac{t}{T_d}}$$

The direct-current time constant for any machine is

$$T_a = \frac{x_2 + x}{2\pi f(r_a + r)},$$

where x and r are the equivalent external reactance and resistance, respectively, between the terminals and point of fault. Theoretically x and r should be determined as described in items 3 and 5. If the fault is far removed from the terminals, the decrement is so rapid that the direct-current component may be neglected.

7. The total current in any branch is found by the expression $i_{\text{total}} = \sqrt{(i' + \Delta i'')^2 + (i_{\text{d.c.}})^2}$ using the individual values of each component determined for the time t . For $t = 0$ the maximum possible value is $\sqrt{(i'')^2 + (i_{\text{d.c.}(i)})^2}$, and for $t = \infty$ the value is, of course, the sustained value i .

SINGLE LINE-TO-GROUND FAULT ON CONDUCTOR a

1. The internal voltages $e_{(i)}$, $e'_{(i)}$, $e''_{(i)}$, before short circuit, are calculated as for the three-phase fault.

2. Lay out the complete positive-, negative-, and zero-sequence networks connected in series for single line-to-ground fault. Set up $e_{(i)}$ and x_d in each machine in the positive-sequence network and determine i_1 for each machine in the positive-sequence network. Then remembering that only positive-sequence currents produce demagnetization in the machines,

$$e'_{\infty} = e_{(i)} - (x_d - x'_d)i_1$$

3. The short-circuit time constant T'_d for each machine is determined as for the three-phase fault except that all three sequence networks are set up and connected in series. The ratio of the applied voltage to the resulting current when any convenient value of $e_{(i)}$ is applied to machine A with all other machine potentials zero, is the sum of the equivalent positive-, negative-, and zero-sequence reactances from the bus of zero potential to the fault as viewed from machine A . Using the same designation as for three-phase short circuits, the total reactance is $(x_{d(A)} + x)$. In this case the total external reactance x is $x = [x_{d(A)} + x_{1(eq)} + x_{2(eq)} + x_{0(eq)}] - x_{d(A)}$, where $x_{1(eq)}$ is the reactance from the machine terminals to the fault in the equivalent positive-sequence network; $x_{2(eq)}$ is the total equivalent

reactance in the negative-sequence network to the fault; and $x_{0(eq)}$ is the total equivalent reactance in the zero-sequence network to the point of fault. The time constant $T'_{d(A)}$ is then given by the same expression as for the three-phase fault, but it should be carefully observed that x has a different value.

$$T'_{d(A)} = \frac{x'_{d(A)} + x}{x_{d(A)} + x} T'_{d0(A)}$$

It will be noted that $(x_{d(A)} + x)$ is the reactance for an equivalent three-phase fault; *i.e.*, a three-phase fault having the same positive-sequence current as the single-line fault.

4. The internal voltage e' for each machine at any time t is calculated following the procedure as for a three-phase fault:

$$e' = e'_{\infty} + (e'_{(i)} - e'_{\infty})e^{-\frac{t}{T'_d}}$$

The transient reactance x'_d of each machine is set up in the positive-sequence network and the calculated value of e' for each machine for any time t is applied simultaneously to the networks in series and the current i'_1 , i'_2 and i'_0 determined in the positive-, negative-, and zero-sequence networks, respectively. The current in the faulted phase of any branch at time t is then simply the sum of the sequence currents in that branch:

$$i' = i'_1 + i'_2 + i'_0$$

5. For the initial value of the subtransient current, set up $e''_{(i)}$ and x''_d in each machine and determine i''_1 , i''_2 and i''_0 in the positive-, negative-, and zero-sequence networks, respectively. The initial value of the subtransient current is equal to the sum of the three components,

$$i''_{(i)} = i''_1 + i''_2 + i''_0$$

The subtransient component of current at any time t is calculated from the difference of the initial subtransient current and the initial transient current $i'_{(i)}$:

$$\Delta i'' = (i''_{(i)} - i'_{(i)})e^{-\frac{t}{T''_d}}$$

$$T''_d = \frac{x''_d + x}{x'_d + x} T'_{d0}$$

where x is the value of reactance found in item 3.

6. The maximum initial value of the direct-current component is found from $i''_{(i)}$ and the load current before fault as for the three-phase fault condition.

The direct-current time constant T_a for any machine is

$$T_a = \frac{(x_2 + x)}{2\pi f(r_a + r)}$$

The value of the reactance x is the same as given in item 3. But

$$(x_2 + x) = x_2 + x_{1(eq)} + x_{2(eq)} + x_{0(eq)} = 2x_{2(eq)} + x_{0(eq)}$$

Similarly,

$$(r_a + r) = 2r_{1(eq)} + r_{0(eq)}$$

$r_{1(eq)}$ is the direct-current resistance in the positive- or negative-sequence network between the machine internal voltage and fault.

$r_{0(eq)}$ is the direct-current resistance in the zero-sequence network from the zero-potential bus to the fault. Therefore,

$$T_a = \frac{2x_{2(eq)} + x_{0(eq)}}{2r_{1(eq)} + r_{0(eq)}}$$

The value of $i_{d.e.}$ for any time t is calculated from

$$i_{d.e.} = i_{d.e.(i)} e^{-\frac{t}{T_a}}$$

as for a three-phase fault.

7. The total current is calculated from the components as for a three-phase fault.

LINE-TO-LINE SHORT CIRCUIT BETWEEN CONDUCTORS b AND c

1. Compute internal voltages $e_{(i)}$, $e'_{(i)}$ and $e''_{(i)}$ before short circuit, as for three-phase faults.

2. Lay out the complete positive- and negative-sequence networks and interconnect properly for line-to-line fault. Set up $e_{(i)}$ and x_d in each machine in the positive-sequence network and determine i_1 for each machine. Compute e'_∞ for each generator from

$$e'_\infty = e_{(i)} - (x_d - x'_d)i_1$$

3. With x_d set up in each machine, apply any convenient voltage to a machine with the potential of all others set at zero. The ratio of the voltage and current for the particular machine is $(x_d + x)$. The external reactance x is

$$x = (x_d + x_{1(eq)} + x_{2(eq)}) - x_d = x_{1(eq)} + x_{2(eq)}$$

$x_{1(eq)}$ is the reactance from the machine terminals to the fault in the positive-sequence network, and $x_{2(eq)}$ is the total equivalent reactance in the negative-sequence network from the machine internal voltages to the fault.

The short-circuit time constant $T'_{d(A)}$ is determined using the value of x given above in the formula

$$T'_d = \frac{x'_d + x}{x_d + x} T'_{d0}$$

4. The internal voltage e' for each machine at any time t is calculated following the same procedure as for a three-phase fault:

$$e' = e'_\infty + (e'_{(i)} - e'_\infty)e^{-\frac{t}{T'_d}}$$

The transient reactance x'_d of each machine is set up in the positive-sequence network and the calculated value e' of each machine for any time t is applied simultaneously to the networks interconnected for a line-to-line fault and the currents i'_1 and i'_2 determined in the positive- and negative-sequence networks, respectively. It will be noted that the currents i'_1 and i'_2 are for unfaulted phase a and at the fault $i'_2 = -i'_1$, and the total current in phase a is zero. The current in phase b and c at the fault is

$$\begin{aligned} i''_b &= a^2 i'_1 + a i'_2 = a^2 i'_1 - a i'_1 = -j \frac{\sqrt{3}}{2} i''_{a1} \\ i''_c &= -i''_b \end{aligned}$$

These relations are true at the point of fault; however, the values of the positive- and negative-sequence currents in a particular branch are not necessarily equal and, in general, are not when more than one machine is connected to the network. The phase- and sequence-current relations for a line-to-line fault are shown in the vector diagram, Fig. 15, for any branch. The positive- and negative-sequence currents in the reference

phase a are in opposition. The sequence currents in phases b and c are 60° out of phase.

The total value of the current in b or c phase of any branch of the network can be found in terms of the positive- and negative-sequence currents i'_1 and i'_2 for the corresponding branch in reference phase a . Expressing i'_2 for phase a in terms of i'_1 , $i'_2 = -(i'_1 K)$ where K is a proportionality factor which when

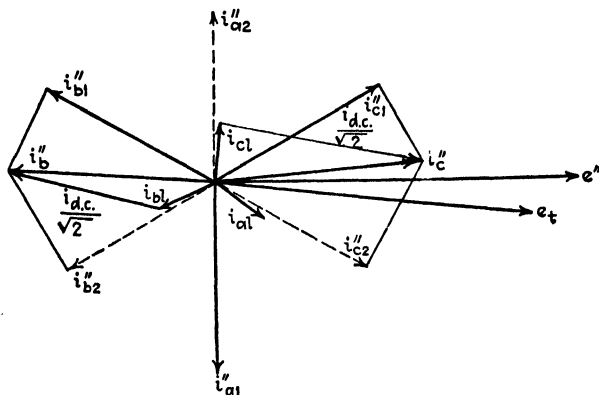


FIG. 15.—Line- and sequence-current relations for a line-to-line fault.

multiplied by i'_1 gives the value of i'_2 in the corresponding branch and phase. At the fault $K = 1$. Then,

$$\begin{aligned} i'_b &= a^2 i'_1 - a(i'_1 K) \\ &= \left(-0.5 - j\frac{\sqrt{3}}{2}\right)i'_1 - \left(-0.5 + j\frac{\sqrt{3}}{2}\right)(i'_1 K) \end{aligned}$$

The absolute value of i'_b is found by taking the square root of the sum of the squares of the terms in the above equation.

$$\begin{aligned} i'_b &= \sqrt{\left(-\frac{i'_1}{2} + \frac{i'_1 K}{2}\right)^2 + \left(-\frac{\sqrt{3}i'_1}{2} - \frac{\sqrt{3}i'_1 K}{2}\right)^2} \\ &= \sqrt{(i'_1)^2 + (i'_1 K)^2 + (i'_1 K)(i'_1)} \end{aligned}$$

Since the values of i'_2 and $(i'_1 K)$ are the same, the former may be substituted above, giving

$$i'_b = \sqrt{(i'_1)^2 + (i'_2)^2 + i'_1 i'_2} \quad (59)$$

The current i'_b for any time t is found from this equation after determining i'_1 and i'_2 for time t with e' set for the calculated value in each machine.

5. For the initial value of subtransient current, set up e''_0 and x''_{d0} in each machine and determine i''_1 and i''_2 in the positive- and negative-sequence networks in any branch. The initial current is

$$i''_b = \sqrt{(i''_1)^2 + (i''_2)^2 + (i''_1 i''_2)}$$

The subtransient component of current at any time t is

$$\Delta i''_b = (i''_b - i'_b) e^{-\frac{t}{T''}}$$

T''_d is calculated from $T''_d = \frac{x''_d + x}{x'_d + x} T''_{d0}$ where x has the value found in item 3.

6. The maximum initial value of the direct-current component is found from i''_{c0} in the faulted phases and the load current before fault. An examination of the vector diagram (Fig. 15), shows the relations of load current before fault and resulting current after fault for all phases. It will be observed that because of the relative positions of the load current in the three phases, the resulting direct-current component is largest in phase c . The load current is practically in quadrature with the fault current i''_c ; therefore, the direct-current component is approximately $\sqrt{2}i''_c$.

The direct-current time constant T_a for any machine is

$$T_a = \frac{x_2 + x}{2\pi f(r_{1(eq)} + r_{2(eq)})}$$

where x has the value given in item 3. Now $x = x_{1(eq)} + x_{2(eq)}$, but $x_{1(eq)} + x_2 = x_{2(eq)}$ so the numerator $x_2 + x = 2x_{2(eq)}$. The total direct-current resistance $r_{1(eq)}$ in the positive-sequence network between the machine internal voltage and the point of fault is equal to the total direct-current resistance $r_{2(eq)}$ in the negative-sequence network between the same points. The direct-current time constant is then

$$T_a = \frac{2x_{2(eq)}}{2\pi f(2r_{2(eq)})} = \frac{x_{2(eq)}}{2\pi f(r_{2(eq)})}$$

This is exactly the same value as for a three-phase fault at the same location; therefore, the direct-current time constants for a three-phase and line-to-line fault are the same for the same fault location.

The value of the direct-current component in phase c for any time t is

$$i_{d.c.} = -\sqrt{2}i_c'' e^{-\frac{t}{T_d}}$$

7. The total current in each phase is calculated from the components as for a three-phase fault.

DOUBLE LINE-TO-GROUND SHORT-CIRCUIT CONDUCTORS b AND c

1. Compute internal voltages $e_{(i)}$, $e'_{(i)}$, $e''_{(i)}$ before short circuit as for a three-phase fault.

2. Lay out the complete positive-, negative-, and zero-sequence networks and interconnect properly for double line-to-ground fault. Phase a is used as the reference. Set up $e_{(i)}$ and x_d in each machine in the positive sequence network and determine i_1 for each machine. Compute e'_∞ for each generator from

$$e'_\infty = e_{(i)} - (x_d - x'_d)i_1$$

3. With x_d set up in each machine apply any convenient voltage to a machine with the potential of all others set at zero. The ratio of the voltage and current for the particular machine is $(x_d + x)$. The external reactance x is

$$x = x_d + x_{1(eq)} + \frac{x_{2(eq)} \cdot x_{0(eq)}}{x_{2(eq)} + x_{0(eq)}} - x_d$$

The short-circuit time constant T'_d for the particular machine is determined using the value of x given above in the formula

$$T'_d = \frac{x'_d + x}{x_d + x} T'_{d0}$$

4. The internal voltage e' for each machine at any time t is calculated following the same procedure as for a three-phase fault:

$$e' = e'_\infty + (e'_{(i)} - e'_\infty)e^{-\frac{t}{T'_d}}$$

The transient reactance x'_d of each machine is set up in the positive-sequence network and the calculated values of e' of each machine for any time t are applied simultaneously with the networks interconnected for a double line-to-ground fault and the currents i'_1 , i'_2 , and i'_0 determined in the positive-, negative-,

value in phase *b* is

$$i_b'' = a^2 i_1'' + a i_2'' + i_0''$$

The subtransient component of current at any time *t* is

$$\Delta i_b'' = (i_b'' - i_b') e^{-\frac{t}{T_d''}}$$

T_d'' is calculated from

$$T_d'' = \frac{x_d'' + x}{x_d' + x} T_{d0}''$$

where *x* has the value found in item 3.

6. The maximum initial value of the direct-current component is found from i_1'' in the faulted phases and the load current before fault. The vector diagram, Fig. 16, shows the relations of load current before fault and resulting current after fault in all phases. It is evident from the diagram that because of the relative positions of the load currents in the three phases, the resulting direct-current component will generally be larger in phase *b*.

The direct-current time constant for any machine is

$$T_a = \frac{x_2 + x}{2\pi f \left[r_{1(eq)} + \frac{r_{2(eq)} r_{0(eq)}}{r_{2(eq)} + r_{0(eq)}} \right]}$$

where *x* has the value given in item 3.

$$T_a = \frac{x_{2(eq)} + \frac{x_{2(eq)} x_{0(eq)}}{x_{2(eq)} + x_{0(eq)}}}{2\pi f \left[r_{2(eq)} + \frac{r_{2(eq)} r_{0(eq)}}{r_{2(eq)} + r_{0(eq)}} \right]}$$

The value of the direct-current component at any time *t* is

$$i_{d.s.} = i_{d.s.(i)} e^{-\frac{t}{T_a}}$$

7. The total current in any branch is calculated from the components as for other types of faults.

GENERAL CASE INVOLVING SALIENT-POLE MACHINES

In case the problem involves round-rotor and salient-pole machines, the required accuracy in the mathematical treatment may necessitate consideration of the phenomena in the direct

and quadrature axes individually. In such cases exactly the same procedure may be followed as described for the round-rotor case except that now direct- and quadrature-axis quantities are calculated separately. Thus for a three-phase short circuit the procedure is as follows.

1. Calculate the internal voltages in the direct and quadrature axes of each machine from the known load conditions existing before fault using the formula

$$e_{d(i)} = e_i \cos \delta + i_{di} x_d \quad (60)$$

$$e'_{d(i)} = e_i \cos \delta + i_{di} x'_d = e_d - (x_d - x'_d) i_{di} \quad (61)$$

$$e''_{d(i)} = e_i \cos \delta + i_{di} x''_d = e_d - (x_d - x''_d) i_{di} \quad (62)$$

$$e_{q(i)} = e_i \sin \delta = i_{qi} x_q \quad (63)$$

$$e'_{q(i)} = e_i \sin \delta - i_{qi} x'_q = -(x_q - x'_q) i_{qi} \quad (64)$$

$$e''_{q(i)} = e_i \sin \delta - i_{qi} x''_q = -(x_q - x''_q) i_{qi} \quad (65)$$

2. Set up the complete network for the fault condition with $e_{d(i)}$ for each machine applied, using direct-axis synchronous reactance. The sustained short-circuit current i_d is determined for each machine.

The sustained transient internal voltage in the direct axis of each machine is calculated from

$$e'_{d\infty} = e_{d(i)} - (x_d - x'_d) i_d \quad (66)$$

3. The transient direct-axis short-circuit time constant T'_d is determined as described previously under three-phase short circuits.

4. The direct-axis internal voltage e'_d of each machine at any time t is calculated from

$$e'_d = e'_{d\infty} + (e'_{d(i)} - e'_{d\infty}) e^{-\frac{t}{T'_d}} \quad (67)$$

The reactance x'_d is set up in each machine and the calculated value of e'_d of each machine for any time t is applied to the network simultaneously and the current i'_d in any branch determined. This process is continued to find the complete variation of the transient direct-axis component.

5a. The direct-axis subtransient component i''_d is determined by setting up $e''_{d(i)}$ and x''_d in each machine and finding i''_d in any branch. The difference between this current and the initial transient current determined for e'_d when $t = 0$ in item 4 for each

machine is the increment due to the subtransient phenomena in the direct axis. The current at any instant in any machine is

$$(i''_{d(s)} - i'_{d(s)})e^{-\frac{t}{T''_d}}$$

The time constant T''_d is found exactly as described in item 5 under three-phase short circuits.

b. The quadrature-axis subtransient current for machines with damper windings is found by setting up $e''_{q(s)}$ and x''_q in each machine and finding i''_q . This follows from Eq. (25) in which $e''_{q(s)} = -i_q(x_q - x''_q)$. If no effective winding is present in the quadrature axis, then the quadrature-axis component of current i''_q decays to zero according to the time constant T''_q . The value of T''_q for any machine is

$$T''_q = \frac{x''_q + x}{x'_q + x} T''_{q0} \quad (68)$$

The value of x is the same as determined in item 3 under three-phase short circuits.

In case there are transient, but no subtransient, phenomena all subtransient terms in item 5b are replaced by corresponding transient terms. If both subtransient and transient quantities are present, then the subtransient component is found as the difference between subtransient and transient initial values and the time constant T''_q applied to this component. The time constant T'_q is applied to the transient component. It should be remembered that the sustained current in the quadrature axis is zero. The quantities in the quadrature axis are handled by the same procedure as described for those in the direct axis.

6. The maximum initial value of the direct-current component is calculated as the difference between the initial value of the total subtransient current i'' and the load current before short circuit using the same procedure as described under three-phase short circuits. The total initial current i'' is equal to $\sqrt{(i''_d)^2 + (i''_q)^2}$.

7. The total symmetrical current at any time t is found by summing up the symmetrical terms in the direct and quadrature axes and taking the square root of the sum of the squares to obtain the total symmetrical current at that instant. The total asymmetrical current is then found by taking the square root of

the sum of the squares of the total symmetrical current and the direct-current component for any time t .

The solution for any other type of fault where quadrature-axis quantities are involved is obtained by following the same procedure as described for round-rotor machines and using the principles outlined for three-phase faults for the general case.

INCLUSION OF FAULT OR CIRCUIT RESISTANCE

Problems sometimes arise where the circuit resistance is of sufficient magnitude to produce appreciable effect upon the fault current, or it may be necessary to consider the effect of

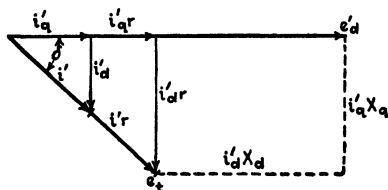


FIG. 17.

fault resistance. The fault or circuit resistance is set up in the sequence networks for different types of faults as outlined in Chap. III.

The presence of resistance of such magnitude as to affect appreciably the fault current

will cause the direct-current component to decay so rapidly that, for all practical purposes, it may usually be neglected. The general method of setting up a solution for circuits and apparatus involving resistance follows readily from the following consideration.¹ Assuming all resistance external to the machine, the vector diagram for a three-phase resistance fault is shown in Fig. 17. For simplicity it is assumed that the fault occurs at no load and only one rotor circuit is present. The diagram is drawn for external resistance and all reactances internal, *i.e.*, a resistance load shunted across the armature terminals. For the case of external reactance, the vector diagram is the same except the reactance terms are increased by the amount of the external reactance. In this case the resistance is applied beyond the external reactance, so the terminal voltage e_t becomes a fictitious one being the terminal voltage at the resistance thus including the external reactance with the machine reactance. The analysis, however, is the same. From the vector diagram the following relations may be written

¹ DOHERTY, R. E., and C. A. NICKLE, *op. cit.*, *Trans. A.I.E.E.*, vol. 49, p. 700, 1930.

$$i'_d = i' \sin \delta = \frac{e_t}{r_a} \sin \delta \quad (69)$$

$$i'_q = i' \cos \delta = \frac{e_t}{r_a} \cos \delta \quad (70)$$

$$e'_d = e_t \cos \delta + i'_d x_d = e_t \cos \delta + \frac{e_t x_d}{r_a} \sin \delta \quad (71)$$

$$i'_q x_q = e_t \sin \delta = \frac{e_t x_q}{r_a} \cos \delta \quad (72)$$

The direct-axis synchronous voltage before short circuit e_d is equal to the transient internal voltage e'_d at the instant after short circuit, when the short circuit occurs under an open-circuit condition:

$$e'_d = e_d \quad (73)$$

From Eq. (72)

$$\tan \delta = \frac{x_q}{r_a} \quad (74)$$

Equating (71) and (73) and substituting for $\cos \delta$, $\frac{r_a}{\sqrt{r_a^2 + x_q^2}}$ and for $\sin \delta$, $\frac{x_q}{\sqrt{r_a^2 + x_q^2}}$, which follows from the relations in Eq. (74), give

$$e_t = e'_d \frac{r_a \sqrt{r_a^2 + x_q^2}}{r_a^2 + x'_d x_q} \quad (75)$$

$$i' = e'_d \frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x'_d x_q} \quad (76)$$

The sustained short-circuit current is

$$i = e'_d \frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x_d x_q} \quad (77)$$

The transient current in the direct axis may be found by substituting the value of e_t from Eq. (75) in (69):

$$\begin{aligned} i'_d &= e'_d \frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x'_d x_q} \sin \delta \\ &= e'_d \frac{x_q}{r_a^2 + x'_d x_q} \end{aligned} \quad (78)$$

The sustained direct-axis component is

$$i_d = e'_d \frac{x_q}{r_a^2 + x_d x_q} \quad (79)$$

The field current immediately after short circuit from (3) is

$$I'_d = e'_d + (x_d - x'_d)i'_d \quad (80)$$

Substituting i'_d from Eq. (78) in (80),

$$I'_d = e'_d + e'_d \frac{x_q(x_d - x'_d)}{r_a^2 + x'_d x_q} = e'_d \left[1 + \frac{x_q(x_d - x'_d)}{r_a^2 + x'_d x_q} \right] \quad (81)$$

The field current has increased from e'_d before short circuit to $\left[1 + \frac{x_q(x_d - x'_d)}{r_a^2 + x'_d x_q} \right]$ times e'_d immediately after short circuit. The short-circuit time constant T'_d is the open-circuit time constant T'_{d0} multiplied by the ratio of the field current before short circuit to the current immediately after. This reduces to

$$T'_d = \frac{r_a^2 + x'_d x_q}{r_a^2 + x_d x_q} T'_{d0} \quad (82)$$

This is the same expression as given by Wagner. A solution involving i''_q reduces to the same expression for one rotor circuit.

In case the resistance fault occurs under a load condition then e'_d before short circuit must be calculated from the known load conditions as previously described. If a rotor circuit is present in the quadrature axis, the solution must involve the quadrature-axis subtransient or transient reactance, and the derivation is obtained using Eqs. (29) to (37), inclusive. It must be remembered that under these conditions there is a mutual coupling between the rotor circuits which causes additional components to appear in the decrement. An exact solution may be obtained by solving the differential equations of the various circuits.¹ For most practical cases it is unnecessary to resort to these more accurate methods.

The equation for the total current from (76) and (77) is

$$i' = e'_d \left\{ \left[\frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x'_d x_q} - \frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x_d x_q} \right] e^{-\frac{t}{T'_d}} + \frac{\sqrt{r_a^2 + x_q^2}}{r_a^2 + x_d x_q} \right\} \quad (83)$$

EFFECT OF VARIABLE EXCITER VOLTAGE ON TRANSIENT CURRENT

Thus far it has been assumed that the field excitation and hence the exciter voltage has remained constant during the short circuit. This will not be the case if any generators are equipped

¹ PARK, R. H., *op. cit.*

with voltage regulators, since the regulators will tend to increase the excitation during the short circuit. The magnitude of the effect will depend upon the machine characteristics and the rate of building up the exciter voltage.¹ The effect may be great on machines equipped with high-speed excitation systems or negligible for very low rates of building up of voltage.

The rate of decay of the subtransient and direct-current components in the short-circuit current is generally so rapid as not to be appreciably affected by any change of excitation. It is, therefore, necessary to take into account the effect of variable exciter voltage only on the slowly decaying or transient component.

The procedure in calculating transient current with variable exciter voltage is first to calculate the voltage time characteristic of the exciters from the instant the regulator contacts close and then to determine e'_ω as a function of time. It will be observed that as the exciter voltage increases, the corresponding short-circuit current under sustained conditions is increased. To determine e'_ω , convert the instantaneous values of exciter voltage for each machine to per-unit values, using as unit value of exciter voltage that required to produce normal or unit voltage at no load on the generator. The value of generator voltage corresponding to each value of exciter voltage should be taken from the generator no-load saturation curve in order to take into account initial saturation. Set up the synchronous reactance of each machine in the complete network, and determine the sustained short-circuit current corresponding to the instantaneous values of exciter voltage by simultaneously adjusting the applied voltage of each machine to the value of exciter voltage as a function of time. For example, designating the instantaneous exciter voltage of the various machines as $e_{X(A)}$, $e_{X(B)}$, $e_{X(C)}$, etc., set up the value of e_X in each machine for $t = 0$ and measure or calculate the current i ; then set up the value of e_X in each machine for say $t = 0.1$ again determining i for each machine, continuing in this manner over the desired period of time. The factor e'_ω for each machine can then be calculated as a function of time from the equation $e'_\omega = e_X - (x_d - x'_d)i$, where e_X and i are given the values previously found for each interval of time. A con-

¹ ROBINSON, P. H., Practical Considerations Affecting Quick-Response Excitation of Synchronous Machines, *Elec. Jour.*, February, 1928, p. 65.

venient method of determining e'_∞ , when using a calculating board, is to set up x_d of each machine in two sections, one section $(x_d - x'_d)$ adjacent to the applied voltage and another section x'_d . The sum of these is, of course, $(x_d - x'_d) + x'_d = x_d$. The value of e'_∞ for each machine can be measured between the two sections as the voltage e_x of each machine is varied simultaneously.

After calculating or measuring e'_∞ for each machine, the variation of the transient internal voltage can be determined, taking into account the instantaneous variation of e'_∞ . The time constant T'_d for each machine is found exactly as described for the condition of constant exciter voltage. Now e'_∞ has been determined using instantaneous values of e_x as though the value of the generator voltage instantly corresponds to the exciter voltage as the exciter voltage changes. This, of course, is not true because the rate of building up of the generator field current with a given change in exciter voltage depends upon the effective field inductance and resistance just as the rate of decay of the induced field current under short circuit depends upon the same inductance and resistance. Therefore, when the generator armature is short-circuited, the time constant which applies to the current increase in the generator field under the influence of the increasing exciter voltage is the short-circuit time constant T'_d .

The instantaneous value of e' can be calculated by a step-by-step process from the formula

$$e' = (e'_\infty + \Delta e'_\infty) + [e'_{(i)} - e'_\infty + \Delta e'_\infty]e^{-\frac{t + \Delta t}{T'_d}} \quad (84)$$

or by a graphical method¹ which is much simpler. The derivation of the graphical method is most easily described by considering the case of a coil carrying a current which is suddenly short-circuited upon itself. The instantaneous value of the current in the coil after short circuit is

$$i = I_{(i)}e^{-\frac{t}{T}} \quad (85)$$

The slope of the curve at any instant is

$$\frac{di}{dt} = I_{(i)}e^{-\frac{t}{T}}\left(-\frac{1}{T}\right) = -\frac{i}{T} \quad (86)$$

¹ WAGNER, C. F., *op. cit.*, *Elec. Jour.*, May, 1933, p. 205.

The slope or tangent to the exponential curve for $t = 0$, substituting in Eq. (86), is $-I_{(0)}/T$. The tangent intersects the horizontal axis at point T as shown in Fig. 18a. If some source of potential is present in the short-circuited coil, then the current

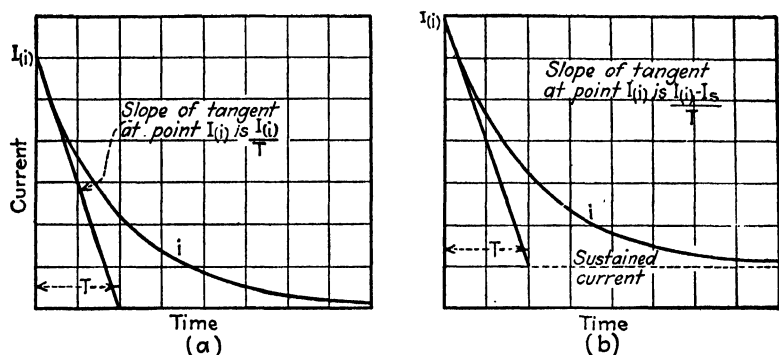


FIG. 18.

will decay to the sustained value i_s rather than zero as shown in Fig. 18b. Now the tangent to the curve for $t = 0$ intersects the line i at point T . In applying these principles to the determination of e' , the slope of the tangent is equal to the difference between the instantaneous value of e' and the steady-state value that e'_∞ would reach if the exciter voltage at the same instant remained constant, divided by the time constant. Thus, to calculate e' for a particular machine plot e'_∞ is a function of time, offset on the time axis a distance corresponding to the time constant T_d as shown in Fig. 19. The tangent to e' for $t = 0$ intersects e'_∞ at the initial point b . It is assumed that e' follows the tangent ab for a small increment of time Δt , measured horizontally, which locates point c . Point d is then located by measuring the same horizontal distance Δt between b and d as between a and c . The tangent line cd is then drawn. The curve e' is assumed to follow the tangent cd for the interval of

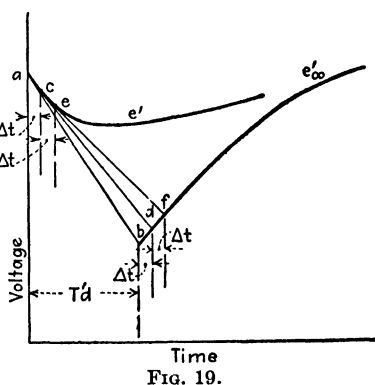


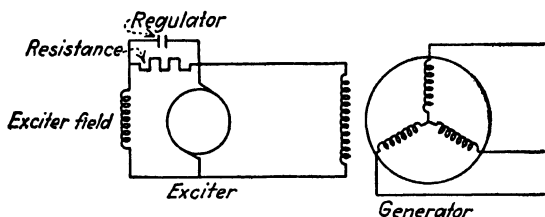
FIG. 19.

time Δt , thus locating ef , etc., until the complete curve is determined. The smaller the increment of time Δt is chosen, the more accurate will be the approximation. It is evident that this method of construction follows the fundamental principles previously outlined.

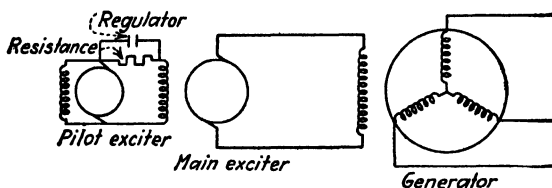
After finding e' as a function of time for each machine, exactly the same procedure for finding the transient, subtransient, and direct-current components is followed as described for constant exciter voltages.

CALCULATION OF EXCITER VOLTAGE AS A FUNCTION OF TIME

It is necessary to determine the exciter voltage as a function of time when including the effect of variable exciter voltage.



(a) Self Excitation



(b) Separate Excitation

FIG. 20.

The rate at which the exciter voltage builds up depends upon the rate at which the current in the exciter field can be increased. The excitation supply is usually provided by a self-excited or separately excited exciter as shown in Fig. 20a and b, respectively. Self-excited exciters build up more slowly than separately excited ones because the armature voltage and rate of increase are mutually dependent. High rates of increase in voltage are generally obtained by dividing the exciter field into a number of

parallel paths and placing resistance in series with each path, by using a high-voltage exciter with voltage-limiting resistance, and by laminating the exciter field poles and yoke.

A simple step-by-step method for the calculation of e_x is given in the following derivation:

N = number of turns per field pole

n = number of poles

n' = number of parallel field circuits

ϕ = total flux per pole

E_F = applied voltage to exciter field

r_x = resistance per field circuit with regulator contacts closed

i = current per circuit in the exciter field

e_x = exciter-armature terminal voltage

ϕ_n = total flux per pole at no-load normal voltage

e_N = exciter no-load normal voltage

$$E_F - ir_x = \frac{n}{n'} N \frac{d\phi}{dt} 10^{-8} \quad (87)$$

$$\phi = \phi_n \frac{e_x}{e_N} \quad (88)$$

$$\frac{d\phi}{dt} = \frac{\phi_n}{e_N} \frac{de_x}{dt} \quad (89)$$

Substituting (89) in (87)

$$E_F - ir_x = \frac{nN\phi_n}{e_N 10^8} \frac{de_x}{dt} \quad (90)$$

$$\begin{aligned} \frac{de_x}{dt} &= \frac{(E_F - ir_x)e_N 10^8}{\frac{n}{n'} N \phi_n} \\ &= K(E_F - ir_x) \\ K &= \frac{e_N 10^8}{\frac{n}{n'} N \phi_n} \end{aligned} \quad (91)$$

The voltage e_x is calculated from Eq. (91) by a step-by-step process using columns with the following headings:

$$t, \quad i, \quad ir_x, \quad (E_F - ir_x), \quad \frac{de_x}{dt}, \quad \Delta t \frac{de_x}{dt}, \quad e_x$$

For separate excitation, assuming the regulator contacts remain closed, the voltage E_F applied to the field is constant

throughout the calculations. For self-excitation the value of E_f changes for each interval of time Δt . The value of E_f at any instant should be that value of e_x at the end of the preceding interval of time. The value of e_x at the instant of fault $t = 0$ depends upon the position of the regulator contacts. Some average value should be assumed for $t = 0$, and then the length of time the contacts are closed and open must be considered. The value of i at each interval of time is that corresponding to e_x of the preceding interval of time and is obtained from the no-load saturation curve. This is not exact since there is an increasing voltage drop in the armature as the current builds up, so some curve between the no-load and full-load saturation curves should be used. Tests show that, due to the high inductance of the generator field, the exciter armature current builds up so slowly that there is no appreciable difference in the rate of building up of voltage when the exciter is on open circuit and when connected to the generator field. The calculated values of e_x should be shaded somewhat because eddy currents induced in the pole structure tend to prevent the flux from changing and hence retard the rate of building up.

SYNCHRONOUS MACHINE CONSTANTS

The constants of synchronous machines are generally given by the manufacturers as determined either by calculations from design data or by test. In order to give the reader a comprehensive understanding of the constants, a brief description will be given of the methods of determination by test.¹ Unless otherwise specified, no distinction will be made between reactance and impedance.

The reactances and time constants of synchronous machines are affected by saturation in the magnetic circuits during fault conditions. Saturation being a function of voltage and current will necessarily be affected by the terminal conditions. The constants, therefore, are really not constant under fault conditions when saturation exists to some degree, as is usually the case. Saturation causes sufficient changes in machine-reactance values that it must be taken into consideration. The effects will be

¹ The basic material for this section was taken from S. H. Wright's Determination of Synchronous Machine Constants by Test, *Trans. A.I.E.E.*, vol. 50, p. 1331, 1931.

different for round-rotor and salient-pole machines, and it has been found that tests suitable for determining the constants of one-type machine will not be suitable for another type. In general only one value of each constant is necessary for practical calculation of fault currents, and that is the value existing under dead short-circuit. Thus direct-axis subtransient and transient reactances and short-circuit time constants are determined as the saturated values for a dead three-phase short circuit applied at no-load normal voltage. Negative-sequence reactance is the saturated value effective for a line-to-line short circuit at the terminals of the machine under similar conditions. The effect of saturation on zero-sequence currents is not generally large enough but that the reactance can be determined by circulating rated current in the armature. This gives an unsaturated or rated current value of reactance. Quadrature-axis subtransient and transient reactances and time constants are taken as the saturated values existing when the machine is short-circuited from a slip test. The direct- and quadrature-axis synchronous reactance is usually taken as the unsaturated value as defined by the A.I.E.E. When saturated values of these reactances are used, they are defined as those effective at rated load.

DIRECT-AXIS CONSTANTS

x''_d and x'_d from Short Circuit.—The machine driven as a generator at no-load normal voltage and frequency is suddenly short-circuited. An oscillograph record is taken of the current in the three phases. The envelopes and mid-point curves are drawn for the wave of each phase. From the known rated r.m.s. current of the machine and the oscillograph calibration, factors are computed for converting measured values of symmetrical and asymmetrical current from the oscillogram to per-unit values. The per-unit values of the asymmetrical component are plotted on semilogarithmic paper, *i.e.*, time scale uniform and current scale logarithmic. Since the transient components have a rate of decay essentially exponential, when plotted on semilogarithmic paper they appear as straight lines and can be more accurately projected to zero time. The plotted values are shown in Fig. 21. The current during the first half-cycle of short circuit cannot be accurately determined so the curves are projected to zero time, as shown by the dotted lines

in Fig. 21, to obtain the initial values in each phase at zero time. The magnitude of the direct-current component in each phase

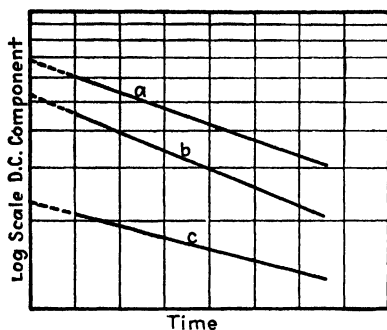


FIG. 21.—Direct-current component plotted on semi-log scale.

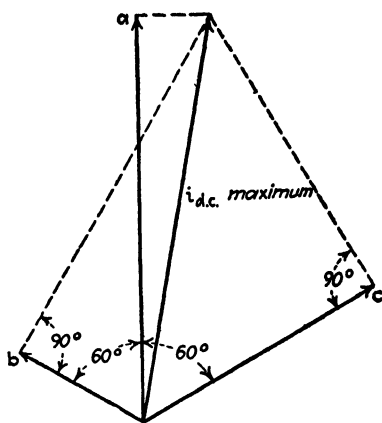


FIG. 22.—Graphical method of determining maximum possible direct-current component from three-phase short circuit.

depends upon the instant at which the fault is applied so that it is a matter of chance whether the maximum possible component appears in any particular phase.

The maximum possible direct-current component can be determined from a graphical construction using the values in Fig. 21 for zero time. The values of a , b , and c are laid off 60° apart, the largest value being the middle line as shown in Fig. 22. Perpendiculars are drawn to each line and the intersections of the three perpendiculars locate the termination of $i_{d.c.}$ which is the maximum possible direct-current component.

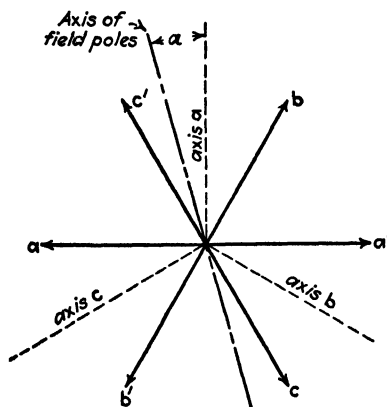


FIG. 23.—Relation between armature phases and field axes.

The method applies to machines having a sine-wave symmetrical component of short-circuit current. The method of construction follows from the fact that when, for example, the axis of phase a and the axis of the field coincide,

the axes of phase b and c are 60° and -60° , respectively, out of space phase with the axis of the field poles. These relations are shown in Fig. 23. The equations for the direct-current component in each phase for a short circuit from no load are:

$$\begin{aligned} i_{d.c.(a)} &= k \cos \alpha \\ i_{d.c.(b)} &= k \cos (\alpha + 120^\circ) \\ i_{d.c.(c)} &= k \cos (\alpha - 120^\circ) \end{aligned}$$

where α is the angle between the axis of phase a and the pole axis. For a fault occurring when $\alpha = 0$,

$$\begin{aligned} i_{d.c.(a)} &= k \\ i_{d.c.(b)} &= k \cos (+120^\circ) = -0.866k \\ i_{d.c.(c)} &= k \cos (-120^\circ) = -0.866k \end{aligned}$$

Under this condition the maximum possible direct-current component appears in phase a . Using the construction shown in Fig. 22 it will be seen that the resultant of b - and c -phase components intersects a or is simply k . Similar relations exist for any other value of α . The average of the per-unit r.m.s. values of the symmetrical component in the three phases is plotted as shown in Fig. 24. The envelope cannot be drawn for the first half-cycle of fault. The subtransient reactance depends upon the initial zero-time current, hence must be accurately determined. With sinusoidal current, the maximum initial symmetrical component is equal to $1/\sqrt{2}$

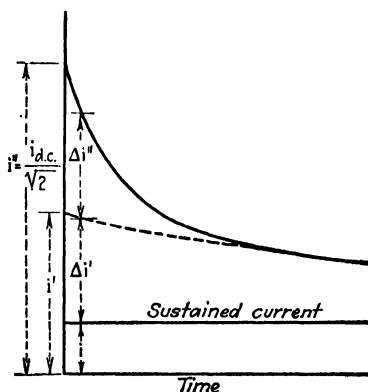


FIG. 24.—Average r.m.s. symmetrical component of current in the three phases for a three-phase fault.

times the maximum initial direct-current component, or $i'' = 0.707i_{d.c.}$, where $i_{d.c.}$ is the value determined in Fig. 22. The curve of Fig. 24 can now be extended to the zero time value of i'' . The subtransient reactance is the voltage before short circuit divided by i'' , normal voltage being unity in the per-unit system, i.e., $x_d'' = 1/i''$.

The transient reactance x'_d is found as the ratio of voltage to current, neglecting the subtransient component. The difference between the total current, excluding $\Delta i''$, and the sustained current or $\Delta i'$ is plotted on semilogarithmic paper and projected to find the value at zero time just as was done for the direct-current component. The $\Delta i'$ component can then be replotted in Fig. 24. The transient reactance is the voltage divided by the initial transient current, exclusive of the subtransient component.

x''_d from Locked-rotor Test.—The rotor is short-circuited and locked in some position. A single-phase voltage of such value as to circulate about 25 per cent normal current is applied between

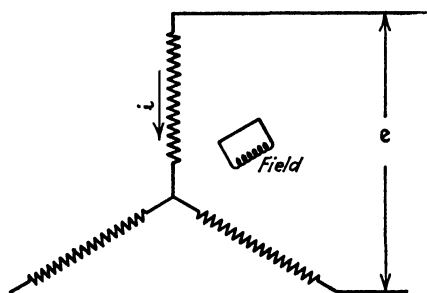


FIG. 25.—Locked-rotor test for determining x'_d and x''_d .

any two terminals, as shown in Fig. 25. Readings of armature and field current and armature voltage are taken for various positions of the rotor over a pole pitch. The maximum value of the ratio of field current to armature voltage is the direct-axis position and the minimum is the quadrature-axis position. With the rotor set in the direct-axis position, rated current is circulated and x''_d calculated as one-half the ratio of applied voltage to line current.

This method of test is applicable to salient-pole machines with dampers, as x'_d determined in this way is very nearly the saturated value; however, it is not suitable for determining saturated values for round-rotor machines.

x'_d from Locked-rotor Test.—The same type of locked-rotor test can be applied to salient-pole machines without damper windings to determine the unsaturated value of transient reactance. A high value of circulated current will not give the saturated value because saturation is produced by armature

current and field excitation and the latter is not present. The saturated value may be determined approximately by multiplying the unsaturated value measured at low current by an empirical factor 0.88. This factor was determined from a large number of tests by Wright.¹

x'_d from Other Constants.—The saturated value of transient reactance can be found from the relation

$$x'_d = \frac{T'_d}{T'_{d0}} x_d$$

where T'_d is the saturated value and x_d the unsaturated value.

Synchronous Reactance x_d from Slip Test.—The synchronous reactance is usually determined as the ratio of the field current

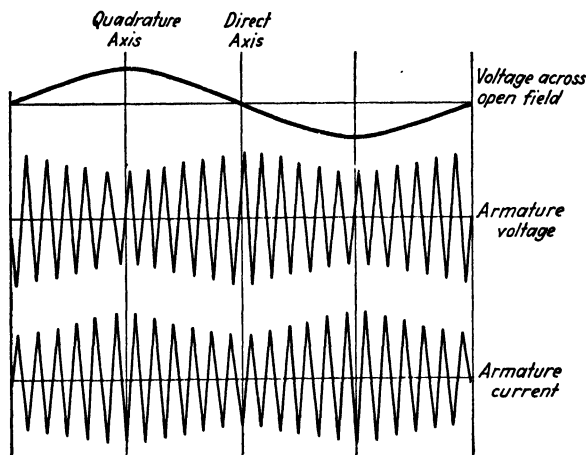


FIG. 26.—Relations during slip test for determining x_d and x_q .

required to circulate rated armature current on a dead three-phase fault to the field current required to produce normal voltage at no load if there is no saturation. x_d can also be measured by a slip test in which the machine is driven with the field open and a balanced three-phase voltage of about 25 per cent normal value applied to the armature. The speed is adjusted for a low value of slip. A voltage of slip frequency is induced in the field. An oscillograph record of the field and armature voltage and armature current is shown in Fig. 26. The ratio of applied voltage to current is x_d when the field is magnetized in the direct

¹ WRIGHT, S. H., *op. cit.*

axis as indicated by zero voltage. The slip must be very low, or induced rotor currents will reduce the measured value of x_d .

QUADRATURE-AXIS CONSTANTS

x_q'' and x_q' from Short Circuit.—The saturated values of x_q'' and x_q' are determined by applying a short circuit during a slip test. Rated current is circulated in the armature and a dead three-phase fault applied when the rotor is magnetized in the quadrature axis. An oscillograph record of the current in all three phases is taken and analyzed following the same procedure as for x_d'' and x_d' . The total symmetrical current is projected to zero time to obtain the initial subtransient current. If e is the applied voltage, i_l the current before short circuit, and i_q'' the initial subtransient symmetrical current, then

$$e'' = i_l(x_q - x_q'') = e - i_l x_q'' \quad (92)$$

$$i_q'' = \frac{e''}{x_q''} \quad (93)$$

$$e'' = i_q'' x_q'' = i_l(x_q - x_q'') \quad (94)$$

Substituting the value of e'' from Eq. (93) in (94) and solving for x_q''

$$x_q'' = \frac{i_l x_q}{i_l + i_q''} \quad (95)$$

$$i_l x_q = e \quad (96)$$

$$x_q'' = \frac{e}{i_l + i_q''} \quad (97)$$

The values of x_q'' before and after short circuit are different so it is necessary to determine for the same current, the low current value existing during the modified slip test. This value is used in calculating e'' in Eq. (92) and then the saturated value is determined from Eq. (93). The saturated value of x_q'' is generally equal to x_d'' saturated, x_q' is found from the same test as x_q'' . The unsaturated value of x_q' is found from the modified slip test with normal current flowing and is used for calculating the transient internal voltage before short circuit. The oscillogram is analyzed for x_q' as described for x_d' , and the initial symmetrical transient current is determined neglecting the subtransient component. The equation for the transient current is of the same form as the subtransient in Eqs. (92) to (97) except

that a transient term appears in place of the corresponding subtransient term:

$$e' = i_l(x_q - x'_q) \quad (98)$$

$$i'_q = \frac{e'}{x'_q} \quad (99)$$

$$x'_q = \frac{e}{i_l + i'_q} \quad (100)$$

Salient-pole machines have no effective quadrature-axis rotor circuit, so $x'_q = x_q$. For turbine generators the saturated value of x'_q can usually be taken as equal to the saturated value of x'_d .

x''_q from Locked-rotor Test.—This test is made with the rotor in the quadrature-axis position and near normal current circulated, following the same procedure as for the corresponding test for x''_d (Fig. 25). The subtransient reactance is equal to one-half the ratio of applied voltage to current when the rotor is in the direct-axis position.

This test gives approximately the saturated value for salient-pole machines with dampers and the unsaturated value for turbine generators.

x'_q Unsaturated from Modified Slip Test.—The applied voltage is suddenly disconnected when the rotor is magnetized in the quadrature axis during a slip test. An oscillograph record is taken of the terminal voltage. The terminal voltage, excluding the rapidly decaying subtransient component is projected to zero time using a semilogarithmic scale. The terminal voltage e , the armature current i_l before short circuit, and the initial transient internal voltage determined from the oscillogram are related by the expression

$$\begin{aligned} e - e' &= i_l x'_q \\ x'_q &= \frac{e - e'}{i_l} \end{aligned} \quad (101)$$

Synchronous Reactance x_q from Slip Test.—The unsaturated value of x_q can be determined from the same slip test as x_d . With rotor magnetized in the quadrature axis, x_q is the ratio of applied voltage and current.

x_2 Negative-sequence Reactance.—The negative-sequence reactance is taken as the arithmetical average of the saturated values of x''_d and x''_q :

$$x_2 = \frac{1}{2}(x''_d + x''_q)$$

The negative-sequence reactance can be determined from a sustained line-to-line short circuit as shown in Fig. 27. With the fault on b and c phases,

$$i_b = -j \frac{\sqrt{3}e_a}{Z_1 + Z_2} \quad (102)$$

e_a is generated voltage in phase a ,

$$e_{ab} = \frac{3e_a Z_2}{Z_1 + Z_2} \quad (103)$$

Dividing (103) by (102) gives

$$Z_2 = -j \frac{e_{ab}}{\sqrt{3}i_b} \quad (104)$$

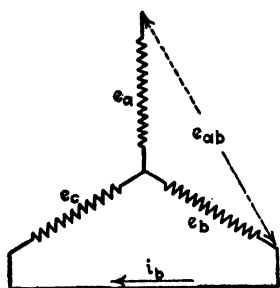


FIG. 27.—Line-to-line short circuit for determining x_2 .

x_0 Zero-sequence Reactance.—The zero-sequence reactance is not appreciably affected by saturation or by rotation of the rotor; therefore, the locked-rotor test may be used as shown in Fig. 28. The rotor is short-circuited and the armature windings

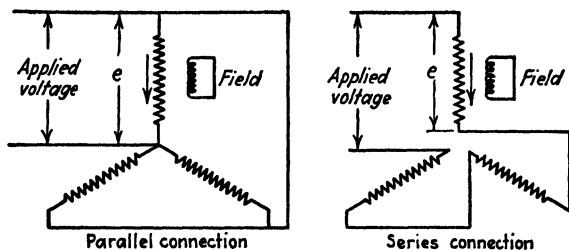


FIG. 28.—Locked-rotor test for determining Z_0 .

connected in either series or parallel and about normal current circulated in each phase. The zero-sequence impedance is the ratio of the applied voltage per phase to the current per phase.

x_0 may also be determined from a sustained double line-to-neutral short circuit. With the fault on phases b and c the voltage of the open phase is (see Chap. III for derivation).

$$e_a = \frac{3Z_0 Z_2 e}{Z_0 Z_1 + Z_0 Z_2 + Z_1 Z_2}$$

where e_a is the generated voltage in phase a . The neutral current i_n is equal to $3i_0$. Dividing e_a by i_n

$$Z_0 = e_a \quad (105)$$

Sequence Resistance.—The armature resistance to positive-, negative-, and zero-sequence current is the resistance component of corresponding impedances. The positive- and zero-sequence resistances are readily determined by taking a wattmeter measurement in the tests described for determining impedance. The negative-sequence resistance may be considerably different from the positive because the flux associated with negative-sequence currents rotates at twice the synchronous speed with respect to the rotor. The negative-sequence loss may not vary as the square of the current and it is sometimes necessary to determine R_2 as a loss coefficient rather than a simple resistance.

The effective negative-sequence resistance R_2 is most accurately measured as the energy loss for a sustained line-to-line fault. The measurements are made without external resistance or reactance in the field circuit. A method given by Wagner is to measure the shaft input at no load and with the machine short-circuited line-to-line. The value of R_2 is given as

$$R_2 = \frac{\text{net shaft input}}{2i_2^2} = 1.5 \frac{\text{net shaft input}}{i_b^2} \quad (106)$$

The net shaft input is the input with the machine short-circuited less the prime-mover input at no load, the prime-mover losses, and the friction and windage losses.

The value of R_2 can be determined from a wattmeter measurement during a sustained line-to-line fault with a current 1.73 times rated current. The resistance R_2 by this method is (see Fig. 27)

$$R_2 = \frac{e_{ab}}{\sqrt{3}i_b} \sqrt{1 - \frac{W_2}{e_{ab}i_b}} \quad (107)$$

Direct-axis Time Constants.—The variation of unsustained currents and voltages may not be exponential if saturation exists in the circuit. In most cases it is found that an exponential curve can be fitted to the actual curve using plots on semi-logarithmic paper. More weight should be given to the initial values between zero time and the time corresponding to the time constant than for the other values.

T_d'' Subtransient Time Constant.—The constant is present only in machines having additional rotor circuits. Its value is

found from a semilogarithmic plot of the subtransient component of current $\Delta i''$ with a dead three-phase short circuit at no load. T_d'' is the time in seconds for the $\Delta i''$ component of current to decay to 0.368 of the initial value. If considerable saturation is associated with the subtransient current, as is usually the case, only an approximate exponential curve can be fitted to the actual.

T_{d0}'' Open-circuit Subtransient Time Constant.—This constant can be found from a modified slip test or suddenly removed symmetrical short circuit. In the slip test, with the field short-circuited, the applied armature voltage is suddenly disconnected when the rotor is magnetized in the direct axis. An oscillograph record of the armature voltage is obtained. The slowly decaying component of voltage is found by a semilogarithmic plot of the envelope of the wave. The difference between this component and the total wave is the subtransient component. This difference is plotted on semilogarithmic paper and T_{d0}'' determined.

The other method for finding T_{d0}'' from test is to obtain an oscillograph record of the armature voltage when a symmetrical short circuit is suddenly removed. The subtransient component is determined as the difference between the slowly decaying component and the total wave from a semilogarithmic plot.

The subtransient open-circuit and short-circuit time constants are related by the simple expression $T_{d0}'' = \frac{x_d'}{x_d''} T_d''$ similar to that for transient open-circuit and short-circuit time constants.

T_d' Transient Short-circuit Time Constant.—This is the time constant of the transient component $\Delta i'$ and is found by plotting the values for a dead three-phase fault on semilogarithmic paper. Since T_d' is the time constant of the field with the armature short-circuited, any external resistance or inductance in the field circuit will affect the decrement. The test should be made without external-field constants. For calculations involving T_d' under practical operating conditions, T_d' should be corrected for the value of external resistance or inductance (rheostats, etc.) present at the instant of short circuit.

The unsaturated value of T_d' for salient-pole machines may be determined from an oscillograph record of armature and field current taken when the field winding is suddenly short-circuited with about 50 per cent normal current flowing in the short-cir-

cuted armature. T'_a is determined from a semilogarithmic plot of current. The value, if multiplied by the empirical factor 0.88, gives the approximate saturated value. The test is not satisfactory for turbine generators.

T'_{d0} Transient Open-circuit Field Time Constant.— T'_{d0} is the time constant of the main field circuit with the armature on open circuit. It is determined from an oscillograph record of field current and armature voltage taken when the field winding is suddenly short-circuited on itself while the machine is running with about 50 per cent normal voltage at no load. The value of T'_{d0} is found from a semilogarithmic plot. This test is not satisfactory for turbine generators. For turbine generators the modified slip test is used as described for determining T''_{d0} .

T_a Armature Short-circuit Time Constant.— T_a is determined from a semilogarithmic plot of the asymmetrical component of armature current for a dead three-phase fault. The direct-current component may decay with different time constants in the three phases. Generally T_a should be determined from one of the two largest components having the most gradual initial slope.

The time constant can be calculated, if the direct-current armature resistance and the saturated value of x_2 are known, from the expression

$$T_a = \frac{x_2}{2\pi f r_a}$$

QUADRATURE-AXIS TIME CONSTANTS

T''_q Subtransient Time Constant.—This time constant is determined from the slip short-circuit test used to determine x''_q and x'_q . It is the time in seconds for the difference between the total current and the slowly decaying component to decrease to 0.368 times the initial difference.

T''_{q0} Open-circuit Subtransient Time Constant.—This constant is determined from the modified slip test used for finding the unsaturated value of x'_q . The value of T''_{q0} is the time in seconds for the rapidly decreasing component of armature voltage to decrease to 0.368 of the initial value.

T'_q Transient Time Constant.—The value of T'_q is obtained from the short-circuit test used for determining x''_q and x'_q as the

TABLE I.—CONSTANTS OF THREE-PHASE SYNCHRONOUS MACHINES¹
(Values below lines give range of values and those above give average)

	x_d	x_q	x'_d	x''_d	x_s	x_0^*	T'_{d0}	T'_d	T_d	T_e
Two-pole turbine generators	1.10 0.95-1.45	1.07 0.92-1.42	0.15 0.12-0.21	0.09 0.07-0.14	($= x'_d$)	0.01-0.08	4.4 2.8-6.2	0.6 0.35-0.90	0.035 0.02-0.05	0.09 0.04-0.15
Four-pole turbine generators	1.10 1.00-1.45	1.08 0.97-1.42	0.23 0.20-0.28	0.14 0.12-0.17	($= x'_d$)	0.015-0.14	6.2 4.0-9.2	1.3 0.8-1.8	0.035 0.02-0.05	0.2 0.15-0.35
Salient-pole machines with dampers	1.15 0.60-1.45	0.75 0.40-1.00	0.37 0.20-0.50	0.24 0.13-0.35	0.24 0.13-0.35	0.02-0.20	5.6 1.5-9.5	1.8 0.5-3.3	0.035 0.01-0.05	0.15 0.03-0.25
Salient-pole machines without dampers	1.15 0.60-1.45 1.80 1.50-2.20	0.75 0.40-0.95 1.15 0.95-1.40	0.35 0.20-0.45 0.40 0.30-0.60	($= x'_d$) 0.25 0.18-0.38	0.55 0.30-0.70 0.24 0.17-0.37	0.04-0.25	6.6 3.0-10.5 9.0 6.0-11.5	2.0 1.0-3.3 2.0 1.2-2.8	 0.035 0.02-0.05	0.30 0.1-0.5 0.17 0.1-0.3
Condensers						0.02-0.15				

¹ These data were taken from S. H. Wright, Determination of Synchronous Machine Constants by Test, *Trans. A.I.E.E.*, vol. 50, p. 1331, 1931.

* x_0 varies so critically with winding pitch that no average is given.

time for the slowly decaying component of symmetrical quadrature-axis current to decay to 0.368 of its initial value.

T'_{q0} Transient Open-circuit Time Constant.—This constant is determined from the voltage-decrement test used for determining x'_q . Its value is the time for the slowly decreasing component of armature voltage to decay to 0.368, its initial value neglecting the rapidly decaying components.

Typical average values of constants for synchronous machines are given in Table I. These values may be used in the absence of specific data on machines.

CHAPTER VII

TRANSMISSION-CIRCUIT FORMULAS AND EQUIVALENT CIRCUITS¹

The degree of refinement necessary in the calculation of faults for relay applications and settings depends upon the type of relays used and the characteristics of the system. The capacitance effects can be neglected on short lines but should be taken into account on long lines and cables in the positive-, negative- and zero-sequence networks where accuracy is required. The effect of ground wires must be considered in the calculation of zero-sequence constants. The accurate determination of transmission-circuit constants represents one of the most important phases of the entire problem. Therefore, the general formulas and procedure bear emphasis.

INDUCTANCE FORMULAS

The self-inductance of a single conductor is due to the flux within the conductor and that exterior to the conductor resulting from the current flowing in the conductor. The general formula² for self-inductance of a single, solid, circular, non-magnetic conductor carrying a current which is uniformly distributed over the cross section of the conductor and with a return circuit at some great distance D is

¹ Many of the basic data contained in this chapter have been obtained from the following sources:

WAGNER, C. F., and R. D. EVANS, Symmetrical Components, *Elec. Jour.*, April, 1931, pp. 239-244; October, 1931, pp. 586-590; May, 1931, pp. 308-312; November, 1931, pp. 624-630.

"Symmetrical Components as Applied to Advanced Alternating Currents," Westinghouse Electric and Mfg. Co., Educational Department, East Pittsburgh, Pa.

SIMMONS, D. M., Calculation of the Electrical Problems of Underground Cables, *Elec. Jour.*, May, 1932, pp. 283-287; July, 1932, pp. 336-340.

² See WOODRUFF, L. F., "Electric Power Transmission and Distribution," John Wiley & Sons, Inc., New York.

$$L = 2 \log_e \frac{D}{r} + \frac{1}{2} \text{ abhenrys per centimeter } (r = \text{radius of conductor}) \quad (1)$$

The second term of this equation represents the flux linkages due to the flux within the conductor. It has been found convenient to eliminate this term by finding an equivalent value of r such that the value of Eq. (1) is unchanged but reduced to one term. This artifice represents the determination of the radius of a hollow cylinder with infinitesimally thin walls and having the same self-inductance as the original conductor. The current in the equivalent conductor flows very near the surface and the internal flux is therefore zero. If an expression is set up involving all the elements of the first term of the right-hand member of Eq. (1), except with r_{eq} substituted for r , and then equated to the right-hand member of Eq. (1), the solution for r_{eq} is

$$r_{eq} = 0.779r \quad (2)$$

Thus the radius of the equivalent conductor r_{eq} replacing a solid round conductor is 77.9 per cent of the radius of the actual conductor. The equivalent radius r_{eq} is commonly called the "geometric mean radius" (G.M.R.) of the solid conductor. Carrying this theory a step farther, the G.M.R. of a stranded conductor can be found by considering self-inductance of the individual strands involving the strand G.M.R. and the mutual inductance between each strand and every other strand assuming a common return at some large distance D . The mutual inductance between two parallel strands 1 and 2 is

$$M_{12} = 2 \log_e \frac{D}{d_{12}} \quad (3)$$

where d_{12} is the geometric mean distance (G.M.D.) between strands. The G.M.D. between two round parallel conductors is equal to the distance between centers. The procedure is the same whether determining the G.M.R. of a stranded conductor or the G.M.R. of two or more parallel composite or solid conductors. In either case the radius r_{eq} of an equivalent conductor is found having the same inductance as the several conductors which it replaces.

For example, with three conductors in parallel it can be shown that

$$L = 2 \log_e \frac{D}{\sqrt[9]{r_{eq} d_{12} d_{13} r_{eq} d_{21} d_{23} r_{eq} d_{31} d_{32}}} = 2 \log_e \frac{D}{r_{eq}} \quad (4)$$

where r_{eq} is the ninth root of the nine product terms or the G.M.R. of the three parallel conductors.

The mutual inductance between three conductors 1, 2, 3 in one group a and a fourth conductor x is

$$M_{ax} = 2 \log_e \frac{D}{\sqrt[3]{d_{1x} d_{2x} d_{3x}}} \quad (5)$$

The conductor group a can be replaced by an equivalent conductor at a distance d_{ax} from x . Thus $d_{ax} = \sqrt[3]{d_{1x} d_{2x} d_{3x}}$, and we may write $M_{ax} = 2 \log_e \frac{D}{d_{ax}}$. The distance d_{ax} is the G.M.D. between group a and x conductors.

In general the G.M.R. of a group of n parallel conductors $n_1, n_2, \dots, n_n$, is the n^2 root of the product of n^2 terms formed by multiplying the G.M.R. of the individual conductors by the G.M.D. from each conductor to every other conductor, and all terms being multiplied, *i.e.*,

$$\text{G.M.R.} = \sqrt[n]{(r_{eq1} d_{12} d_{13} \dots d_{1n})(r_{eq2} d_{21} d_{23} \dots d_{2n}) \dots (r_{eqn} d_{n1} d_{n2} \dots d_{n-1})} \quad (6)$$

in which d_{12} is the G.M.D. from conductor 1 to 2, etc. The G.M.D. from a group of n paralleled conductors to a group of m paralleled conductors is defined as mn root of the product of mn terms formed by multiplying the G.M.Ds. from each of the n conductors to each of the m conductors.

The G.M.R. of individual conductors not of circular section can be found by dividing the section into n equal areas and applying the methods outlined assuming uniform current distribution in each area. The same method can be applied to determining the G.M.D. between non-circular conductors. In Table I is given the G.M.R. of typical conductors in terms of the actual radius.

TABLE I.—GEOMETRIC MEAN RADIUS OF TYPICAL CONDUCTORS¹

Copper	Aluminum (steel core)
Solid..... 0.779r	Single layer.. 0.55r–0.70r
7-strand..... 0.726r	30-strand, 2-layer..... 0.826r
19-strand..... 0.758r	26-strand, 2-layer..... 0.809r
37-strand..... 0.768r	54-strand, 3-layer..... 0.810r
61-strand..... 0.772r	(Steel strand neglected)
91-strand..... 0.774r	Copper-clad steel wire..... 0.17r
127-strand..... 0.776r	

¹ WAGNER, C. F., and R. D. EVANS, "Symmetrical Components," p. 138, McGraw-Hill Book Company, Inc., New York.

POSITIVE- AND NEGATIVE-SEQUENCE CONSTANTS

Reactance.—The positive- and negative-sequence impedance of transmission circuits are the same. A certain amount of phase dissymmetry exists in any transmission line, but usually this effect is so small that it may be neglected. The effect of proper transposition of conductors is to produce symmetry. Ground wires have very little effect upon the positive-sequence reactance of transposed lines. The positive- and negative-sequence mutual impedance between parallel transposed circuits is generally so small that it can be neglected.

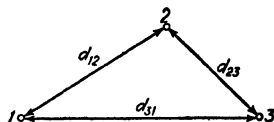


FIG. 1.—Unsymmetrical configuration.

The general equation for the reactance per phase of a single circuit line having any configuration, such as Fig. 1, is

$$x = j4.657f10^{-3} \log_{10} \frac{\sqrt[3]{d_{12}d_{23}d_{31}}}{\text{G.M.R.}} \text{ ohms per mile} \quad (7)$$

The G.M.R. and G.M.D. in this equation may be in any units so long as both are in the same units.

Resistance.—The positive- and negative-sequence resistance values to be used are the values per phase properly corrected for temperature and skin effect. Accurate data have been prepared on all types of conductors by the manufacturers.

Capacitance.—The positive- or negative-sequence capacitance of a three-phase line having any configuration such as Fig. 1 is given by the equation

$$C = \frac{0.3883 \times 10^{-7}}{\log_{10} \frac{\sqrt[3]{d_{12}d_{23}d_{31}}}{r}} \text{ farads per mile} \quad (8)$$

$$\text{Capacitive reactance} = -j \frac{0.4098 \times 10^7}{f} \log_{10} \frac{\sqrt[3]{d_{12}d_{23}d_{31}}}{r} \text{ ohms} \\ \text{per mile to neutral} \quad (9)$$

In making calculations it is frequently more convenient to use susceptance than reactance. The susceptance is the positive reciprocal of the reactance and is given by the equation

$$jb = \frac{2.44f \cdot 10^{-7}}{\log_{10} \frac{\sqrt[3]{d_{12}d_{23}d_{31}}}{r}} \text{ mhos per mile to neutral} \quad (10)$$

The value of r in these equations is the actual radius of the conductors rather than the G.M.R. as appeared in formulas for inductive reactance. This is due to the charge being distributed over the surface of the conductor while the current is distributed throughout the conductor.

The presence of the earth and of ground wires has some effect upon the capacitance, but with a properly transposed line this effect is so small that it may be neglected.

ZERO-SEQUENCE CONSTANTS

The zero-sequence impedance of transmission circuits is the impedance to currents in phase in the conductors. The zero-sequence currents divide equally between the three conductors of a single-circuit three-phase line and have a common return through earth or earth and ground wires. The current returning through the earth follows the irregularities of the line rather than taking any short cuts that may exist.

The earth is a conductor of enormous dimensions and non-uniform conductivity, and the earth-current distribution is not uniform as a consequence. It is necessary to know the distribution of the earth current in order to calculate the impedance of conductors with earth return. This problem has been investigated by a number of engineers and solutions obtained using different assumptions. The work of Carson has found more favor than others in this country. He started with the assumption that the earth is a solid with plane surface, infinite in extent,

and of uniform conductivity. In general his results check experimental results rather closely. On short lines, error is introduced due to end effects.¹

With certain simplifications, the Carson formula for the self-impedance of a single conductor a with earth return is

$$Z_{aa} = R_c + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_s}{r_{eq}} \text{ ohms per mile} \quad (11)$$

where R_c is resistance of conductor per mile.

The mutual impedance between two parallel conductors a and x with common earth return is

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_s}{d_{ax}} \quad (12)$$

The term D_s in these two equations is the equivalent depth of earth-return current and is given by the expression

$$D_s = \frac{6.843 \times 10^{-3}}{\sqrt{\lambda f}} \quad (13)$$

where λ is conductivity of earth in abmhos per centimeter cube. The formulas include both the impedance of the overhead conductors and the earth-return circuit. The value of λ is a variable factor depending upon soil conditions. The following are typical values:

	λ	D_s in feet, 60 cycles
Dry earth.....	10^{-14}	8840
Damp earth.....	10^{-13}	2800
Sea water.....	10^{-11}	280

Single Circuit—No Ground Wires.—The basic formulas given above can be applied to obtain the zero-sequence impedance of lines. Assuming a single-circuit line with transposed conductors, the zero-sequence current divides equally between the three conductors and the voltage drop in each conductor is equal. Since the currents divide equally, the three conductors can be

¹ For a more complete discussion see C. F. Wagner and R. D. Evans, *op. cit.*, *Elec. Jour.*, April, 1931, pp. 239-244 and J. E. Clem, *Reactance of Transmission Lines with Ground Return*, *Trans. A.I.E.E.*, vol. 50, pp. 901-918, 1931.

replaced by a single equivalent conductor of equivalent radius

$$\text{G.M.R.} = \sqrt[3]{r_{eq1}^2 d_{12}^2 d_{23}^2 d_{31}^2} \quad (14)$$

and resistance one-third the resistance of each conductor and carrying the total current. Substituting in Eq. (11) and multiplying by 3, since this equation is based on a single conductor, gives

$$Z_0 = R_c + 4.764f10^{-3} + j13.97f10^{-3} \log_{10} \frac{D_e}{\text{G.M.R.}} \text{ ohms per mile per phase} \quad (15)$$

where G.M.R. is as given in Eq. (14) and R_c is the resistance of one conductor.

Single Circuit—One Ground Wire.—When ground wires are present the zero-sequence return current divides between the

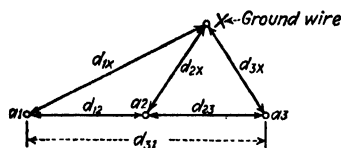


FIG. 2.—Single-circuit, one-ground wire. *a*, Group line wires; *x*, group ground wire.

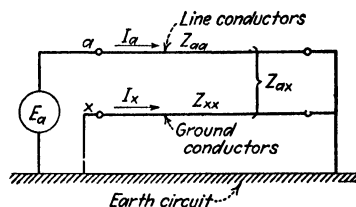


FIG. 3.—Equivalent circuit for single circuit with ground wires.

earth and the ground wires. The ground wires have considerable influence upon the zero-sequence impedance and must be taken into consideration. The impedance of such a circuit as Fig. 2 can be determined by setting up an equivalent circuit, Fig. 3, and applying Eqs. (11) and (12). In Fig. 3, Z_{aa} is the self-impedance of the three line conductors in parallel with earth return, Z_{xx} is the self-impedance of the ground wire with earth return and Z_{ax} is the mutual impedance between line and ground wires. We may write:

$$E_a = I_a Z_{aa} + I_x Z_{ax} \quad (16)$$

$$0 = I_a Z_{ax} + I_x Z_{xx} \quad (17)$$

Solving for I_x in Eq. (17) and substituting the result in (16) gives

$$E = I_a \left(Z_{aa} - \frac{Z_{ax}^2}{Z_{xx}} \right) \quad (18)$$

The zero-sequence impedance is the voltage drop with 1 amp. in each of the line conductors or 3 amp. in the a group of three parallel conductors as in the equivalent circuit.

$$Z_a = 3 \left(Z_{aa} - \frac{Z_{ax} Z_{ax}}{Z_{xx}} \right) \quad (19)$$

where

$$Z_{aa} = \frac{R_c}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(a)}} \quad (20)$$

$$Z_{xx} = R_x + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(x)}} \quad (21)$$

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{d_{ax}} \quad (22)$$

The terms have the following significance:

R_c = resistance of one line conductor.

R_x = resistance of ground wire.

$r_{eq(a)}$ = G.M.R. of three line conductors = $\sqrt[9]{r_{c1}^2 d_{12}^2 d_{23}^2 d_{31}^2}$.

$r_{eq(x)}$ = G.M.R. of ground wire.

d_{ax} = G.M.D. from line conductors to ground conductor
 $= \sqrt[3]{d_{1x} d_{2x} d_{3x}}$

The current returning in the ground wire from Eq. (17) is

$$-I_x = \frac{Z_{ax}}{Z_{xx}} I_a.$$

Single Circuit—Two Ground Wires.

The zero-sequence impedance of a single-circuit line with two ground wires can be determined in exactly the same manner as the preceding case.

The configuration for a typical circuit is shown in Fig. 4. The circuit Z_{xx} , Fig. 3, is now the impedance of two ground wires in parallel. Equations (19) to (22), inclusive, apply, except that the values of terms R_x , $r_{eq(x)}$, and d_{ax} have the following values:

R_x = resistance of two ground wires in parallel.

$r_{eq(x)}$ = G.M.R. of ground wires = $\sqrt{r_{eq1} d_{1'2'}}$

d_{ax} = G.M.D. from line to ground conductors
 $= \sqrt[6]{d_{11'} d_{12'} d_{21'} d_{22'} d_{31'} d_{32'}}$

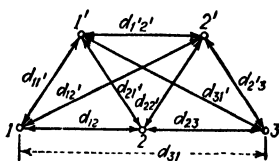


FIG. 4.—Single circuit with two ground wires. Ground wires group x ; line wires group a .

Double Circuit—No Ground Wires.—When parallel transmission circuits are closely spaced, it is necessary to take into consideration the mutual coupling for zero-sequence currents. The mutual impedance between two parallel lines with common earth return and no ground wires follows directly from the application of Eq. (12). In Fig. 5a is shown the configuration

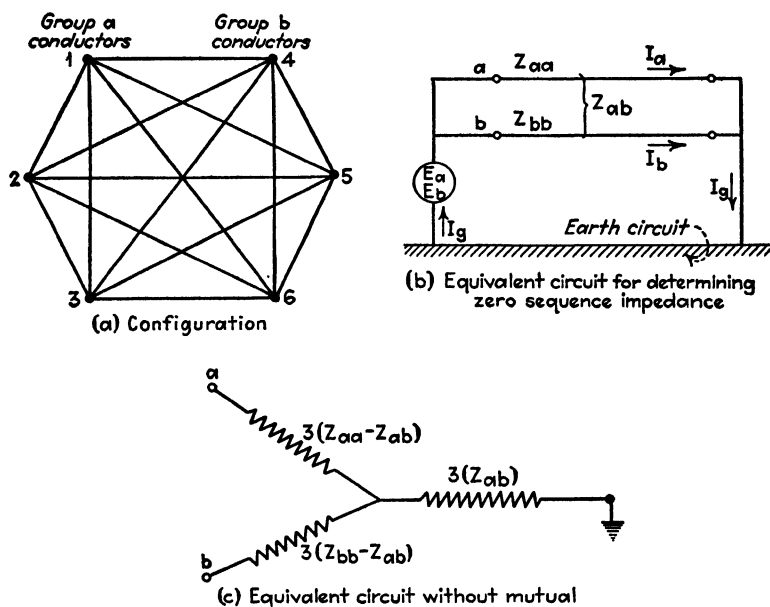


FIG. 5.—Double-circuit line, no ground wires.

for two circuits *a* and *b*. The zero-sequence mutual impedance is

$$Z_{ab} = 4.764f \cdot 10^{-3} + j13.97f \cdot 10^{-3} \log_{10} \frac{D_e}{d_{ab}} \text{ ohms per mile per phase} \quad (23)$$

where d_{ab} = G.M.D. between circuits = $\sqrt[3]{d_{14}d_{15}d_{16}d_{24}d_{25}d_{26}d_{34}d_{35}d_{36}}$

The equivalent circuit for determining the combined zero-sequence impedance of two dissimilar parallel circuits is shown in Fig. 5b and is of the same type as Fig. 3. The following equations may be written:

$$E_a = I_a Z_{aa} + I_b Z_{ab} \quad (24)$$

$$E_b = I_b Z_{bb} + I_a Z_{ab} \quad (25)$$

$$I_g = I_a + I_b \quad (26)$$

Equating (24) and (25) and solving for I_a gives

$$I_a = I_b \frac{Z_{bb} - Z_{ab}}{Z_{aa} - Z_{ab}} \quad (27)$$

Also

$$I_g = I_b K \quad \text{or} \quad I_b = \frac{I_g}{K} \quad (28)$$

where

$$K = \left(\frac{Z_{bb} - Z_{ab}}{Z_{aa} - Z_{ab}} + 1 \right)$$

Substituting Eq. (27) in (25) and substituting I_g from Eq. (28) for I_b gives

$$E_a = E_b = \frac{I_g}{K} \left(Z_{ab} + Z_{aa} \frac{Z_{bb} - Z_{ab}}{Z_{aa} - Z_{ab}} \right) \quad (29)$$

The zero-sequence impedance is the voltage drop with $I_g = 3$; therefore,

$$Z_0 = 3 \left(\frac{Z_{aa} Z_{bb} - Z_{ab}^2}{Z_{aa} + Z_{bb} - 2Z_{ab}} \right) \quad (30)$$

The terms in Eq. (30) have the following values:

$$Z_{aa} = \frac{R_{ca}}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(a)}}$$

where R_{ca} is resistance of one conductor in circuit a and

$$r_{eq(a)} = \sqrt[3]{r_{eq1}^3 d_{12}^2 d_{23}^2 d_{31}^2}$$

$$Z_{bb} = \frac{R_{cb}}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(b)}}$$

where R_{cb} is resistance of one conductor in circuit b and

$$r_{eq(b)} = \sqrt[3]{r_{eq4}^3 d_{45}^2 d_{56}^2 d_{64}^2}$$

Z_{ab} = one-third the value given in Eq. (23).

If the two parallel circuits are identical, then $Z_{aa} = Z_{bb}$ and Eq. (30) reduces to

$$Z_0 = 3 \left(\frac{Z_{aa} + Z_{ab}}{2} \right) \quad (31)$$

An equivalent circuit without mutual impedance can be found from inspection of Eqs. (24) and (25). This circuit is shown in Fig. 5c.

Double Circuit—One Ground Wire.—The configuration and equivalent circuit for this case are shown in Fig. 6a and b, respectively. In order to reduce the circuit to a convenient form for use in the zero-sequence-system network it is desirable to obtain an equivalent circuit with the ground wire eliminated. Following the same method as in the previous cases,

$$E_a = I_a Z_{aa} + I_b Z_{ab} + I_x Z_{ax} \quad (32)$$

$$E_b = I_b Z_{bb} + I_a Z_{ab} + I_x Z_{bx} \quad (33)$$

$$0 = I_x Z_{xx} + I_a Z_{ax} + I_b Z_{bx} \quad (34)$$

Solving Eq. (34) for I_x and substituting the result in Eqs. (32) and (33),

$$E_a = I_a \left(Z_{aa} - \frac{Z_{ax}^2}{Z_{xx}} \right) + I_b \left(Z_{ab} - \frac{Z_{ax} Z_{bx}}{Z_{xx}} \right) \quad (35)$$

$$E_b = I_b \left(Z_{bb} - \frac{Z_{bx}^2}{Z_{xx}} \right) + I_a \left(Z_{ab} - \frac{Z_{ax} Z_{bx}}{Z_{xx}} \right) \quad (36)$$

$$I_g = I_a + I_b$$

where I_g is the total return current.

These equations may be handled in the same manner as Eqs. (24) to (30), inclusive, the only change being the self and mutual coefficients of I_a and I_b . The equivalent circuit without mutual or ground wires (Fig. 6c) can be obtained from inspection of Eqs. (35) and (36). The values of the elements of this circuit are:

$$Z'_{aa} = \left[\left(Z_{aa} - \frac{Z_{ax}^2}{Z_{xx}} \right) - \left(Z_{ab} - \frac{Z_{ax} Z_{bx}}{Z_{xx}} \right) \right] 3 \quad (37)$$

$$Z'_{bb} = \left[\left(Z_{bb} - \frac{Z_{bx}^2}{Z_{xx}} \right) - \left(Z_{ab} - \frac{Z_{ax} Z_{bx}}{Z_{xx}} \right) \right] 3 \quad (38)$$

$$Z'_{ab} = \left(Z_{ab} - \frac{Z_{ax} Z_{bx}}{Z_{xx}} \right) 3 \quad (39)$$

The terms in these equations have the following values:

$$Z_{aa} = \frac{R_{ca}}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(a)}}$$

where

$$r_{eq(a)} = \sqrt[3]{r_{eq1}^3 d_{12}^2 d_{23}^2 d_{31}^2}$$

$$Z_{bb} = \frac{R_{ab}}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(b)}}$$

where

$$r_{eq(b)} = \sqrt[3]{r_{eq4}^3 d_{45}^2 d_{56}^2 d_{61}^2}$$

$$Z_{xx} = R_x + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{r_{eq(x)}}$$

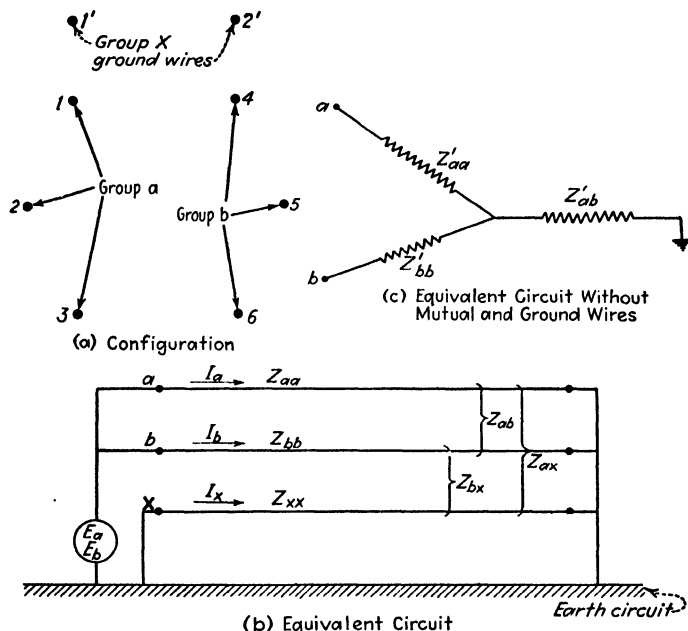


FIG. 6.—Double circuit with ground wires.

where $r_{eq(x)}$ is the G.M.R. of ground wire.

$$Z_{ab} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{d_{ab}}$$

where $d_{ab} = \sqrt[3]{d_{14}d_{15}d_{16}d_{24}d_{25}d_{26}d_{34}d_{35}d_{36}}$.

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{d_{ax}}$$

where $d_{ax} = \sqrt[3]{d_{11'}d_{21'}d_{31'}}$.

$$Z_{bz} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{d_{bz}}$$

where $d_{bz} = \sqrt[3]{d_{41}d_{51}d_{61}}$.

Double Circuit—Two Ground Wires.—The constants for a double-circuit line with two ground wires is the same as for the case with one ground wire, except that $r_{eq(x)}$, R_x , d_{ax} , and d_{bx} have the following values:

R_x = resistance of two ground wires in parallel.

$r_{eq(x)} = \text{G.M.R. of ground conductors } x = \sqrt{r_{eq1}d_{12}'}$

$d_{ax} = \sqrt[6]{d_{11}d_{12}d_{21}d_{22}d_{31}d_{32}'}$

$d_{bx} = \sqrt[6]{d_{41}d_{42}d_{51}d_{52}d_{61}d_{62}'}$

m Circuits—n Ground Wires.—The procedure demonstrated for the solution of the preceding cases can readily be extended to cover the general case of any number of line and ground wires. Sometimes cases occur where more than two circuits are involved such as two double-circuit tower lines on the same right of way with one ground wire per tower line. Each line circuit may be replaced by a single equivalent conductor, but the ground wires may be separated by such a distance that the current division is unequal for a fault at any point other than a bus; therefore, it is desirable to retain the identity of each ground wire.

For the general case let $a, b, c, \dots, m$ be the equivalent conductors replacing each line circuit and $x, y, \dots, n$ the equivalent conductors for the individual ground wires. The currents and voltages to ground in the line groups are $I_a,$

$I_b, \dots, I_m$ and $E_a, E_b, \dots, E_m$.

$$E_a = (I_a Z_{aa} + I_b Z_{ab} + \dots + I_m Z_{am}) + (I_x Z_{ax} + I_y Z_{ay} + \dots + I_n Z_{an}) \quad (40)$$

$$E_b = (I_b Z_{bb} + I_a Z_{ab} + \dots + I_m Z_{bm}) + (I_x Z_{bx} + I_y Z_{by} + \dots + I_n Z_{bn}) \quad (41)$$

$$E_m = (I_a Z_{am} + I_b Z_{bm} + \dots + I_m Z_{mm}) + (I_x Z_{mx} + I_y Z_{my} + \dots + I_n Z_{nm}) \quad (42)$$

For the ground conductors:

$$0_x = (I_x Z_{xx} + I_y Z_{yx} + \dots + I_n Z_{nx}) + (I_a Z_{ax} + I_b Z_{bx} + \dots + I_m Z_{mx}) \quad (43)$$

$$0_y = (I_y Z_{yy} + I_x Z_{yx} + \dots + I_n Z_{ny}) + (I_a Z_{ay} + I_b Z_{by} + \dots + I_m Z_{my}) \quad (44)$$

$$0_n = (I_x Z_{nx} + I_y Z_{ny} + \dots + I_n Z_{nn}) + (I_a Z_{an} + I_b Z_{bn} + \dots + I_m Z_{mn}) \quad (45)$$

The right-hand member of these equations should be multiplied by 3 to convert to a zero-sequence per-phase basis if all impedance terms are based on Eqs. (20) to (22).

As an example of the application of this method consider the case of a three-circuit line having line-conductor groups *a*, *b*, and *c* and ground wires *x* and *y*. It is assumed that the ground wires must not be replaced by an equivalent conductor because of the difference in their distances from the faulted circuit.

$$E_a = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac} + I_x Z_{ax} + I_y Z_{ay} \quad (46)$$

$$E_b = I_b Z_{bb} + I_a Z_{ab} + I_c Z_{bc} + I_x Z_{bx} + I_y Z_{by} \quad (47)$$

$$E_c = I_c Z_{cc} + I_a Z_{ca} + I_b Z_{bc} + I_x Z_{cx} + I_y Z_{cy} \quad (48)$$

$$E_x = 0 = I_x Z_{xx} + I_y Z_{xy} + I_a Z_{ax} + I_b Z_{bx} + I_c Z_{cx} \quad (49)$$

$$E_y = 0 = I_y Z_{yy} + I_x Z_{yx} + I_a Z_{ay} + I_b Z_{by} + I_c Z_{cy} \quad (50)$$

Solving Eqs. (49) and (50) for I_x and I_y in terms of I_a , I_b , and I_c and substituting the result in Eqs. (46) to (48) gives three new voltage equations involving only currents I_a , I_b , and I_c . An equivalent network is thus derived which may be substituted in the system zero-sequence network. The procedure is the same as for the case of a double-circuit line with one ground wire.

It is possible when reducing multiple circuits to a simpler equivalent to replace two or more circuits with an equivalent conductor. For example, assume that it is desired to make calculations for a fault on circuit 1 of a four-circuit line. If circuits 1 and 2 are on one tower line and 3 and 4 on the other, the latter two may be replaced by a single equivalent conductor and the former by individual equivalent conductors. The ground wires may be replaced by a single equivalent conductor or by individual equivalent conductors depending upon whether the current division is the same in the ground wires. The circuits may then be analyzed by the methods described.

ZERO-SEQUENCE CAPACITANCE

Single Circuit.—On lines of considerable length, in order to obtain an accurate analysis it is necessary to consider the zero-sequence capacitance. No definite rules can be given as to

the length of line where capacitance must be considered, as other characteristics, such as coupling, shunt reactive loads, and operating voltage, affect this. Furthermore, the particular relay application being investigated generally dictates the accuracy refinements.

The zero-sequence capacitance can be calculated¹ by assuming the earth to be an equipotential plane between the charge on the conductors and the opposite charge on the images of the conductors which are at the same

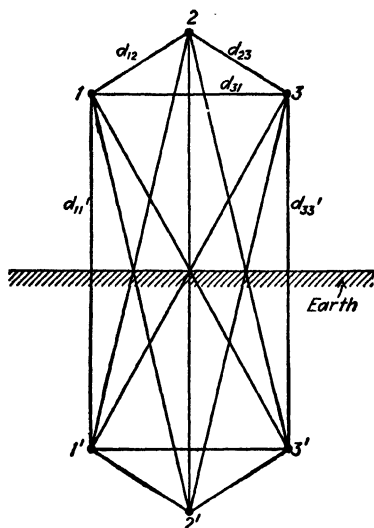


FIG. 7.—Transmission line and images.

distance below the plane as the real conductors are above, as shown in Fig. 7. The capacitance is given by the expressions

$$C_0 = \frac{0.1294 \times 10^{-7}}{\log_{10} \frac{\text{G.M.D.}}{\text{G.M.R.}}} \text{ farads per mile (51)}$$

$$x_0 = -j \frac{1}{2\pi f C_0} = -j \frac{1.229 \times 10^7}{f \log_{10} \frac{\text{G.M.D.}}{\text{G.M.R.}}} \text{ ohms per mile to neutral (52)}$$

$$b_0 = j2\pi f C_0 = \frac{0.8131f \cdot 10^{-7}}{\log_{10} \frac{\text{G.M.D.}}{\text{G.M.R.}}} \text{ mhos per mile (53)}$$

$$\text{G.M.D.} = \sqrt[3]{d_{11'}d_{12'}d_{13'}d_{21'}d_{22'}d_{23'}d_{31'}d_{32'}d_{33'}}$$

$$\text{G.M.R.} = \sqrt[3]{r^3 d_{12}^2 d_{23}^2 d_{31}^2}$$

It is generally sufficiently exact to calculate the G.M.D. from the following formula in which h_1 , h_2 , and h_3 are the conductor heights above the earth plane:

$$\text{G.M.D.} = \frac{2(h_1 + h_2 + h_3)}{3}$$

In determining the G.M.R., the value of r to use is the actual physical radius of the conductor since the charge is distributed over the surface.

¹ For a more complete discussion see C. F. Wagner and R. D. Evans, *op. cit.*, *Elec. Jour.*, May, 1931, pp. 308-309.

Double Circuit.—If two or more parallel circuits are closely spaced, it is generally convenient and necessary to calculate the self and mutual susceptance for use in the equivalent network. The charging current for two parallel circuits a and b can be expressed by the following equations:

$$I_a = E_a j b_{aa} - E_b j b_{ab} \quad (54)$$

$$I_b = E_b j b_{bb} - E_a j b_{ab} \quad (55)$$

In these expressions, b_{aa} and b_{bb} are self-susceptance or the susceptance due to the charge on a and b , respectively, while

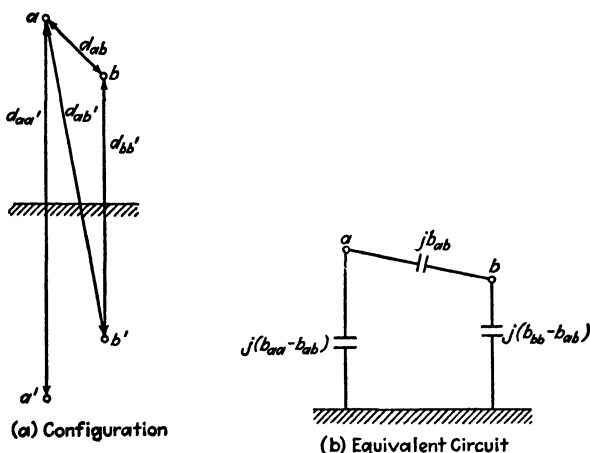


FIG. 8.—Susceptance of parallel circuits.

b_{bb} is the mutual susceptance or that due to the charge on one circuit resulting from potential on the other circuit. In Fig. 8 is shown a schematic representation of two circuits a and b with their images. Circuits a and b may be multiconductor circuits but are represented by single equivalent circuits. With a charge q_a in absolute units on a and no charge on b , the potentials above ground of a and b , respectively, are

$$V_a = 2q_a \log_e \frac{d_{aa'}}{r_{eq(a)}} \quad (56)$$

$$V_b = 2q_a \log_e \frac{d_{ab'}}{d_{ab}} \quad (57)$$

When circuit b has a charge q_b at the same time as a has a charge q_a , then

$$V_a = 2q_a \log_e \frac{d_{aa'}}{r_{eq(a)}} + 2q_b \log_e \frac{d_{ab'}}{d_{ab}} \quad (58)$$

$$V_b = 2q_b \log_e \frac{d_{bb'}}{r_{eq(b)}} + 2q_a \log_e \frac{d_{ab'}}{d_{ab}} \quad (59)$$

Solving for q_a and q_b in terms of V_a and V_b and converting to susceptance in mhos per mile per phase gives

$$b_{aa} = j \frac{0.8131 \times 10^{-7} \log_{10} \frac{d_{bb'}}{r_{eq(b)}}}{K_1} \quad (60)$$

$$b_{bb} = j \frac{0.8131 \times 10^{-7} \log_{10} \frac{d_{aa'}}{r_{eq(a)}}}{K_1} \quad (61)$$

$$b_{ab} = j \frac{0.8131 \times 10^{-7} \log_{10} \frac{d_{ab'}}{d_{ab}}}{K} \quad (62)$$

where

$$K_1 = \log_{10} \frac{d_{aa'}}{r_{eq(a)}} \log_{10} \frac{d_{bb'}}{r_{eq(b)}} - \left(\log_{10} \frac{d_{ab'}}{d_{ab}} \right)$$

The equivalent circuit, Fig. 8b for the parallel lines of Fig. 8a, can be determined from Eqs. (54) and (55). With unit voltage on a and one end of b grounded, the current flowing in b is jb_{ab} , from Eq. (55). The susceptance between a and b in Fig. 8b is therefore b_{ab} . For this same condition the current flowing in a is jb_{aa} , from Eq. (54). Since jb_{ab} is connected between a and b , it is necessary to connect a susceptance $j(b_{aa} - b_{ab})$ between a and ground in order that current jb_{aa} may flow in a . Similar relations exist for the circuit connecting b and earth.

This method of attack can be applied to any number of parallel, closely linked circuits, but the equations become quite cumbersome when more than two circuits are involved.

EFFECT OF GROUND WIRES

The effect of ground wires on the positive- and negative-sequence capacitance is generally negligible. The effect on the zero-sequence constant may be of such magnitude that it is desirable to include the effect. The ground wires should be handled as any other conductors, as described, and the equivalent circuit set up including all conductor groups. For example,

in Fig. 8a, if conductor b is the ground-wire group and is connected to earth, then, in the equivalent circuit, Fig. 8b, the same connection is made and the circuit reduces simply to b_{aa} connected from circuit a to earth.

CONSTANTS OF CABLES

The types of cables most generally encountered in practice are single conductor, belted three conductor, and shielded three conductor, all lead-covered. Owing to the close proximity of conductors, the equations applicable to the calculation of overhead-line constants are not sufficiently accurate for cables. The problem is further complicated by the presence of sheath currents which affect the apparent values of both resistance and reactance of conductors.

POSITIVE- AND NEGATIVE-SEQUENCE CONSTANTS

Resistance.—In cables the positive- and negative-sequence resistance is increased over the direct-current resistance due to spiraling of the strands, twisting of the three conductors about each other in three-conductor cables, skin effect, and proximity effect. The resistance of various cable conductors can be obtained from data published by cable manufacturers.

Voltages induced in the sheaths of single-conductor cables that are bonded and grounded produce sheath currents and hence sheath losses. These additional losses can be taken into consideration most conveniently by increasing the resistance assigned to the conductor and then neglecting the sheath currents entirely.

It can be shown that with three single-conductor cables with equilateral spacing and negligible sheath-proximity effect the additional resistance due to sheath losses¹ to be assigned to the conductor is given by the expression

$$\Delta R = \frac{X_m^2 R_s}{X_m^2 + R_s} \quad (63)$$

where

$$R_s = \text{sheath resistance} = \frac{0.200}{(r_s + r_4)(r_s - r_4)} \text{ ohms per mile} \\ \text{per phase at } 50^\circ\text{C.} \quad (64)$$

¹ SIMMONS, D. M., *op. cit.*, *Elec. Jour.*, June, 1932, p. 284.

$$X_m = \text{mutual reactance between conductors and sheaths} \\ = j4.657f10^{-3} \log_{10} \frac{2S}{r_4 + r_5} \text{ ohms per mile per phase (65)}$$

S = distance between conductor centers in inches.

r_4 = inner radius of lead sheath in inches.

r_5 = outer radius of lead sheath in inches.

$$\frac{r_4 + r_5}{2} = \text{mean radius of sheath in Eq. (65).}$$

R_c = alternating-current resistance of one conductor.

The total positive- or negative-sequence resistance is

$$R = R_c + \frac{X_m^2 R_c}{X_m^2 R_c^2} \text{ ohms per mile per phase (66)}$$

If the conductors are arranged in any form other than triangular it is sufficiently accurate to use the G.M.D. between conductors for S .

In general the sheath losses can be neglected in three-conductor cables in determining positive- and negative-sequence resistances. This discussion has been premised on flow of sheath currents. If devices are used to eliminate sheath currents, then the corrections can be eliminated.

Reactance.—The expression for the positive- and negative-sequence reactance of single-conductor or three-conductor cables, assuming uniform distribution of current throughout the conductor section and neglecting the effect of sheath currents is

$$x = j4.657f10^{-3} \log_{10} \frac{S}{r_{eq}} \text{ ohms per mile per phase (67)}$$

where r_{eq} = G.M.R. of conductor.

The effect of sheath currents in single-conductor cables is to produce an apparent reduction in reactance. The last term in the following expression covering this case represents the reduction due to sheath currents:

$$x = j4.657f10^{-3} \log_{10} \frac{S}{r_{eq}} - \frac{X_m^3}{X_m^2 + R_s^2} \text{ (68)}$$

In three-conductor cables the reduction in reactance due to induced sheath currents may generally be neglected.

Many three-conductor cables are made using sector-type conductors because the space factor is better. The sector-

type conductor does not have a definite mathematical shape, and many different forms have been placed on the market. Accurate data are not available on the sector conductors, but Simmons has proposed as an approximation that the distance between the centers of the small diameters of the sectors be taken in determining the value of S .

The type H shielded three-conductor cables are in general use. The shield consists of a thin metallic tape wound around the conductor insulation to provide control of the electrostatic stress. For the purpose of short-circuit calculations, the effect of the circulating induced currents in the shields has a negligible effect upon the positive- and negative-sequence resistance and reactance.

ZERO-SEQUENCE IMPEDANCE

The problem of determining the zero-sequence impedance of cable circuits is essentially the same as for overhead lines. The zero-sequence resistance is three times the resistance of three conductors in parallel. The zero-sequence currents are in phase in the three phases, and the proximity effect is greater but is still negligible for practical calculations. The effect of sheath currents is present with either single- or three-conductor cable with zero-sequence currents flowing.

Cable circuits are generally provided with one or more ground connections, and usually considerable leakage exists due to the medium surrounding the sheath. If damp earth or water is in contact with the sheath, then the earth may carry a considerable portion of the return current. In case the cable is installed in a dry fiber conduit, practically all the return current will flow in the cable sheath. In general it is necessary to consider the earth and sheath paths acting in parallel.

It can be shown¹ that the self-impedance of a conductor in a non-conducting conduit in the earth and with earth return is given by the same expression as that for an aerial conductor, namely,

$$Z_{aa} = R_c + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_c}{G.M.R.} \quad \text{ohms per mile (69)}$$

¹ WAGNER, C. F., and R. D. EVANS, *op. cit.*, *Elec. Jour.*, November, 1931, pp. 626-628.

The mutual impedance between two conductors in the earth with earth return is

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{d_{ax}} \quad \text{ohms per mile} \quad (70)$$

Consider the application of these formulas to the case of a three-conductor cable (Fig. 9). The procedure is the same and the equivalent circuits similar to the case of aerial conductors. It is evident that an equivalent circuit similar to Fig. 5c can

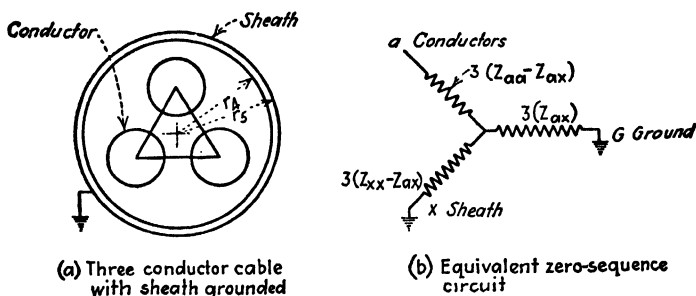


FIG. 9.

be set up. In Fig. 9b is shown such an equivalent circuit with constants having the following values:

Z_{aa} = Eq. (69) in which G.M.R. is that of the three conductors
and $R = \frac{R_c}{3}$

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{2D_e}{r_4 + r_5}$$

The self-impedance of the sheath Z_{xx} is equal to the mutual impedance Z_{ax} between conductors and sheath since the G.M.R. of the sheath is the same as the G.M.D. between conductors and sheath. The impedance of the sheath branch of the equivalent circuit, Fig. 9b, is therefore simply the sheath resistance since the self and mutual reactance cancel out. If the cable sheath is grounded at both ends, the overall self-impedance is branch a in series with g and x in parallel. If the cable is isolated from earth the total self-impedance is branch a and x in series.

The equivalent impedance of several parallel three-conductor cable circuits can be determined by using proper values of G.M.R. and G.M.D. of the various conductors. For example,

consider three similar three-conductor cables arranged in a horizontal plane as shown in Fig. 10. Let

$(\text{G.M.R.})_a$ = G.M.R. of three conductors in one cable

$(\text{G.M.R.})_{abc}$ = G.M.R. of conductors in three circuits

$$= \sqrt[3]{(\text{G.M.R.})_a^3 d_{ab}^2 d_{bc}^2 d_{ac}^2}$$

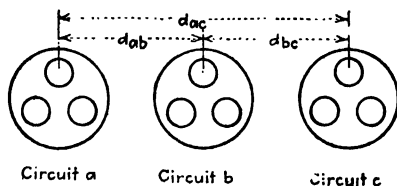


FIG. 10.—Three three-conductor cable circuits.

$(\text{G.M.D.})_a$ = G.M.D. from three conductors in one cable to

$$\text{sheath} = \frac{r_4 + r_5}{2}$$

$(\text{G.M.D.})_{abc}$ = G.M.D. spacing of conductors and sheaths in

$$\text{three circuits} = \sqrt[3]{(\text{G.M.D.})_a^3 d_{ab}^2 d_{bc}^2 d_{ac}^2}$$

The self and mutual impedance of the equivalent conductor replacing the three circuits is

$$Z_{aa} = \frac{R_c}{9} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{(\text{G.M.R.})_{abc}}$$

$$Z_{ax} = 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{(\text{G.M.D.})_{abc}}$$

$$Z_{xx} = \frac{R_s}{3} + 1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{(\text{G.M.D.})_{abc}}$$

Since the three circuits have been replaced by an equivalent circuit, the equivalent zero-sequence circuit of Fig. 9b applies in which

$$\text{Impedance of conductor branch} = 3(Z_{aa} - Z_{ax}) =$$

$$3 \left[\frac{R_c}{9} + j4.657 \log_{10} \frac{(\text{G.M.D.})_{abc}}{(\text{G.M.R.})_{abc}} \right]$$

$$\text{Impedance of sheath branch} = 3(Z_{xx} + Z_{ax}) = 3 \left(\frac{R_s}{3} \right)$$

$$\text{Ground branch} = 3(1.588f10^{-3} + j4.657f10^{-3} \log_{10} \frac{D_e}{(\text{G.M.D.})_{abc}})$$

When it is necessary to find the current distribution for a fault on one cable of a group between bussing points or for a

fault on one cable after the breaker on the faulted cable at one end has opened, all but the faulted cable can be replaced by an equivalent conductor. The circuit can be set up and handled

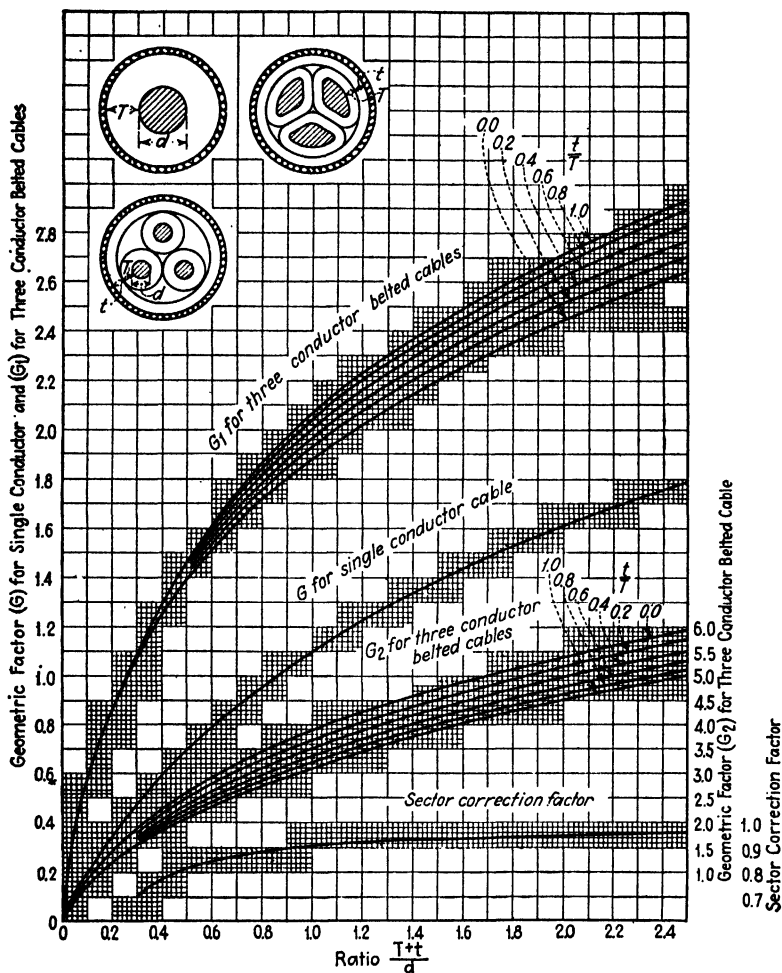


FIG. 11.—Geometric factors for single-conductor and three-conductor cables.

in the same manner as described for multiple overhead circuits with ground wires.

The zero-sequence impedance of single-conductor cables can be determined in the same manner as for the multiple three-

conductor cables, with approximate values of G.M.R. and G.M.D. for the various conductors.

POSITIVE-, NEGATIVE- AND ZERO-SEQUENCE CAPACITANCE

The capacitive reactance must in general be included when making calculations involving cable circuits because of the physically close spacing between conductors and between conductors and sheath. In three-conductor cables the positive- and negative-sequence reactance is different from the zero sequence, since the latter involves only the capacity between the three conductors and sheath. The positive-, negative-, and zero-sequence reactance of single-conductor cables are identical.

Simmons¹ has published data for the calculations of cable constants based on a semigraphical and analytical determination of the shape modulus or geometric factors of single- and multi-conductor cables. The curves in Fig. 11 are reproduced from this work. The values of geometric factors for a particular cable can be obtained by calculating the ratios $\frac{T+t}{d}$ and t/T and reading the required geometric factor from the curves corresponding to those ratios. The significance of T , t , and d is shown on the inserts in Fig. 11. The curves are for round-conductor cables. The geometric factor for sector-type three-conductor cables is smaller than the factor for a round-conductor area. The geometric factor for a sector-type three-conductor belted cable can be found by determining the factor for a round-conductor cable having the same conductor area and insulation thickness and multiplying this factor by the ordinate obtained from the sector-correction-factor curve at the bottom of Fig. 11.

The positive-, negative-, and zero-sequence capacitive reactance can be calculated from the following formulas, using the proper geometric factors.

Three-conductor belted cables:

$$x_1 = x_2 = -j \left(\frac{(G_2)10^6}{1.681887fK} \right) \text{ ohms to neutral per mile per phase}$$

$$x_0 = -j3 \left(\frac{(G_1)10^6}{1.681887fK} \right) \text{ ohms to neutral per mile per phase}$$

¹ SIMMONS, D. M., *op. cit.*, *Elec. Jour.*, July, 1932, p. 339.

Single-conductor cables:

$$x_1 = x_2 = x_0 = -j3 \left(\frac{G10^6}{1.681887fK} \right) \text{ ohms to neutral per mile per phase}$$

The value of the specific inductive capacity K depends upon the insulation. For impregnated paper a value of $K = 3.5$ is a good average figure to use. The value of K varies for impregnated paper from 3 to 4, varnished cambric 4 to 6, and rubber 4 to 9.

In type H shielded cables, the shield and sheath are connected together at frequent intervals; therefore, the shields around the individual conductors have the same effect as the sheath on a single-conductor cable. The capacitive reactance can be calculated exactly as for single-conductor cables in which the shields are the sheaths. The positive-, negative-, and zero-sequence reactances are the same.

CALCULATIONS INVOLVING CAPACITIVE REACTANCE

The constants of circuits and apparatus that have been developed are on a line-to-neutral base. The series-impedance constants can be set up in the three sequence networks and the capacitive reactances can be connected in shunt between the series impedances and neutral in the three networks. For short lines the shunt capacitive reactance can be placed at the electrical mid-point of the line or one-half can be placed at each end. If the circuits are long, it may be necessary to resort to an exact solution involving distributed constants. Such calculations can most readily be accomplished by the use of hyperbolic functions or general circuit constants.¹

INTERMEDIATE FAULTS

The general method of handling parallel circuits with mutual coupling has been indicated. The magnitude and distribution of currents and voltages for faults at points between bussing stations can be determined by similar procedure. It is frequently necessary to make calculations for the condition of a fault on one line of a group of parallel lines before and after a breaker as

¹ NESBIT, WILLIAM, "Electrical Characteristics of Transmission Circuits," Westinghouse Technical Night School Press, East Pittsburgh, Pa.

one end of the faulted circuit opens. If the circuits are closely spaced, it is necessary to consider the mutual coupling in the zero-sequence network but generally the coupling can be neglected in the positive- and negative-sequence networks if the circuits are adequately transposed.

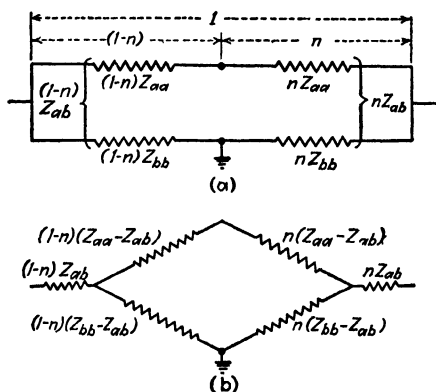


FIG. 12.—Zero-sequence network for intermediate fault on double circuit; breakers closed.

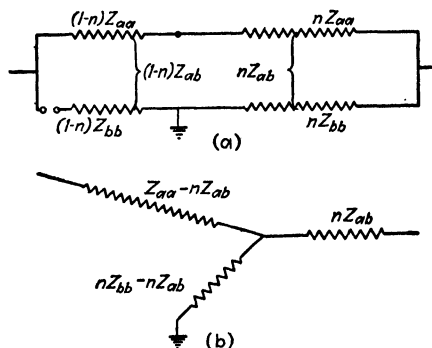


FIG. 13.—Zero-sequence network for intermediate fault on double circuit; one breaker open.

Consider the case of an intermediate fault on one of the two parallel circuits of Fig. 12a. For simplicity the circuits are each of unit length. It is evident that the circuit with mutual can be replaced by two circuits without mutuals, of exactly the same type as given in Fig. 5c. The equivalent circuit without mutual (Fig. 12b) can be determined by inspection. In Fig. 13a is shown the circuit for a fault on one line with the

breaker at one end open and in Fig. 13b the equivalent network without mutual. The condition for an intermediate fault on one circuit of a three-circuit line is shown in Fig. 14a. There is mutual coupling between each circuit and the two other circuits.

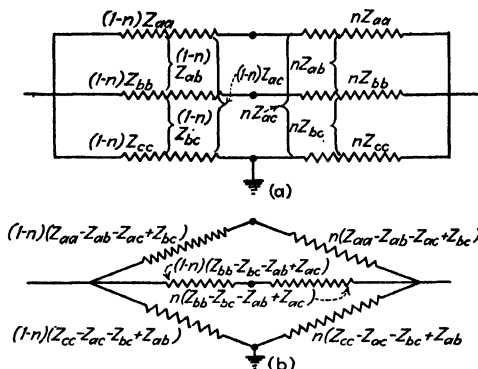


FIG. 14.—Zero-sequence network for intermediate fault on triple circuit; breakers closed.

The circuit consists of two sets of star impedances with mutuals between branches and can be replaced by two equivalent stars without mutuals. Any number of parallel circuits having mutual coupling can be handled in this manner when calculating intermediate faults.

CHAPTER VIII

VECTORS FOR RELAY SYSTEMS¹

In applying relays it is not necessary to consider involved vector systems. The vector system should be simple and free from complication. It is essential to have a clear understanding of transformer polarity before a vector system can be thoroughly understood and made useful as a means of checking relay and system connections.

A standard practice in regard to transformer polarity is now followed by transformer manufacturers so that the confusion

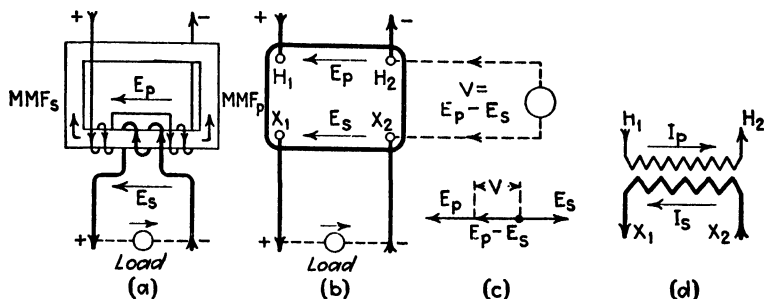


FIG. 1.—Elementary transformer diagram illustrating subtractive polarity.

which has existed in the past in regard to additive and subtractive polarity has been eliminated.

These standards are as follows:

1. All instrument transformers have subtractive polarity.
2. All power transformers have subtractive polarity.
3. All distribution transformers, 7500 volts or below regardless of capacity and 200 kva. or below, regardless of voltage, are of additive polarity. Transformers of higher voltage or larger capacity are of subtractive polarity. A distribution transformer is a transformer whose rating is 500 kva. or less.

An elementary diagram of a loaded transformer is shown in Fig. 1a and b.² In this diagram, E_p represents the direction

¹ A part of the data has been obtained from published data of the Westinghouse Electric and Manufacturing Company.

² AUCHINCLOSS, JOHN, Notes on Transformer Polarity and Connections, *Gen. Elec. Rev.*, November, 1926, p. 783.

of voltage induced in the primary winding opposing the impressed primary voltage at this particular instant in the voltage cycle. The current flow is from positive terminal to negative terminal and is opposed by the induced primary voltage E_p .

The flux produced by the load component of this primary current is equal and opposite to the flux produced by the secondary load current, and hence the only flux present in the core is that due to the magnetizing component of the primary current.

The flow of current in the secondary winding will be from left to right. The left-hand terminal is therefore positive and the right-hand terminal negative. The flow of current through the external load is therefore in the same direction as in the primary winding, and the polarity of the primary and secondary is the same as though no transformer existed. The polarity of such a transformer is defined as subtractive.

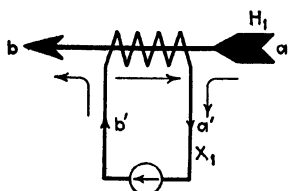


FIG. 2.—Subtractive polarity current transformer.

To test the polarity of such a transformer it is merely necessary to connect together two adjacent primary and secondary terminals and apply a voltage across the primary winding. The voltage measured by a voltmeter connected across the other adjacent primary and secondary terminals will be the difference between the primary and secondary voltages. This is shown in Fig. 1c. The conventional diagrammatic representation of such a transformer is shown in Fig. 1d. (Refer to Chap. XV for further information on polarity tests of transformers and instrument transformers.)

It is important to note that in a subtractive transformer the primary current is in the same direction as the secondary with regard to the external circuit but in opposite directions when considering the transformer windings. This is likewise true in a current transformer as shown in Fig. 2. The primary current is assumed to flow from a to b . At this same instant the secondary current is flowing from a' to b' , and the current may be traced through the instrument from a to a' back through b' to b just as though the transformer were not present.

Since all instrument transformers and power transformers have subtractive polarity, it may be considered that in the great

majority of relay problems subtractive polarity will prevail. Hence no detailed analysis will be made of additive polarity conditions.

The vector system here presented is one which has been found particularly convenient for relay-application problems. The position of the vectors provides a natural arrangement for the open vector system as well as the closed vector system. All

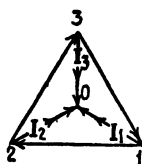


FIG. 3.

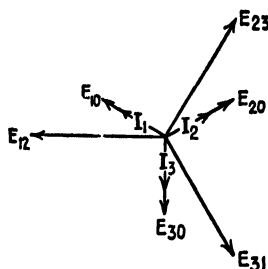


FIG. 4.

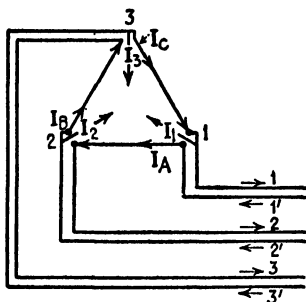


FIG. 5.

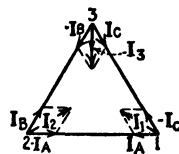


FIG. 6.

FIGS. 3, 4, 5, 6.—Fundamental vector system.

vectors are on basis of phase rotation 1, 2, 3. (Refer to Chap. XV for detailed analysis of phase rotation.)

Figure 3 gives the basic closed vector system which will be used throughout this discussion. Figure 4 illustrates the open vector system. By referring to Fig. 3, it will be noted that vectors E_{1-2} , E_{2-3} , and E_{3-1} form the closed delta. Voltage vector E_{1-2} extends from left to right, and E_{2-3} and E_{3-1} follow in a clockwise direction. Line-to-neutral voltages E_{1-0} , E_{2-0} , and E_{3-0} extend from their respective corners of the delta to the neutral point zero. This then gives a basic counterclockwise vector system. If the delta be rotated in a counterclockwise

direction around point 1, it will be seen that voltage E_{1-0} lags voltage E_{1-2} . Current I_1 is in phase with E_{1-0} for 100 per cent power-factor conditions. Hence, I_1 lags E_{1-2} by 30° . The same conditions hold for the other phases. If the delta be rotated around point 2, E_{2-0} and I_2 lag E_{2-3} by 30° . Likewise rotation around point 3 gives E_{3-0} and I_3 lagging E_{3-1} by 30° . Similarly, if the whole system of vectors in Fig. 4 be rotated around the neutral point, the vectors will all show up in the same positions as explained for Fig. 3. This system of vectors makes it an easy matter to go from the closed system to the open system in case one or the other should prove to be the better for a particular problem.

In order that there should be no question whatever concerning Figs. 3 and 4, a fundamental illustration of the three-phase circuit is given in Figs. 5 and 6. Figure 5 shows the two leads from each phase voltage brought out. On a single-phase basis, current I_A flows from conductor 1' in phase with voltage E_{1-2} back through conductor 2. Likewise current I_B flows from conductor 2' to 3 and current I_C from conductor 3' to 1. If conductors 1 and 1', 2 and 2', and 3 and 3' are, respectively, connected together, the three-phase system is formed, as shown in Fig. 6. Voltages E_{1-2} , E_{2-3} , and E_{3-1} are shown similar to Fig. 3. Current I_A is shown in phase with E_{1-2} , I_B in phase with E_{2-3} and I_C in phase with E_{3-1} . ($-I_A$), ($-I_B$) and ($-I_C$) are also indicated. These values of I_A , I_B , and I_C are the single-phase currents illustrated in Fig. 5. These currents combine to form the three-phase system. Referring to Fig. 5, the resultant current in conductor 1 is composed of I_A and ($-I_C$). In Fig. 6, I_A and ($-I_C$) combined vectorially give the resultant I_1 . Similarly I_2 and I_3 currents are formed. These values of I_1 , I_2 , and I_3 will be represented through this book as shown in Fig. 3.

Voltage-transformer Connections.—Before getting into the details relating to relay-terminal connections, a series of major potential transformer connections and vector diagrams will be given. The V primary to V secondary connection in Fig. 7 produces the same results as the delta-delta connection in Fig. 8. The delta-delta connection is, therefore, seldom used. The per-cent figures given show the voltage between the points indicated in percentage of the voltage between the lines, corrected for ratio of transformation.

Figures 9, 10, 11, and 12 show connections which are used in various relay schemes and the vector diagrams are included for

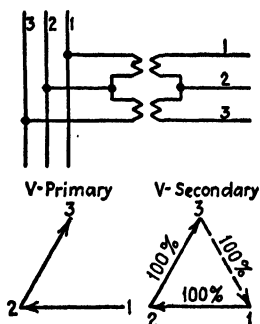


FIG. 7.

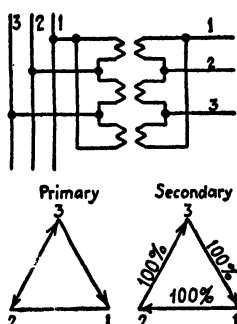


FIG. 8.

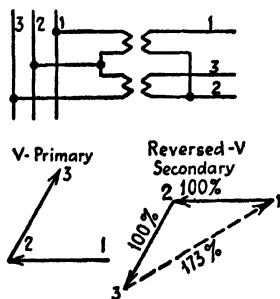


FIG. 9.

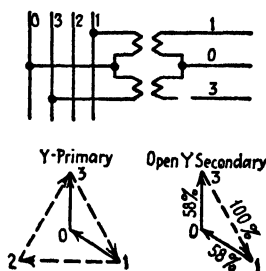


FIG. 10.

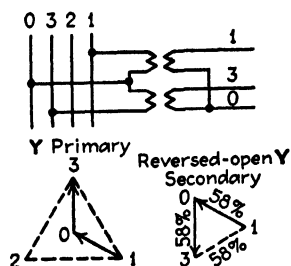


FIG. 11.

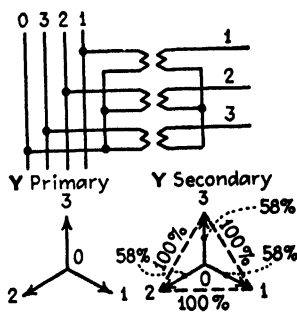


FIG. 12.

FIGS. 7, 8, 9, 10 11, 12.—Fundamental potential transformer connections.

each connection. These connections will be discussed in more detail when particular applications are described.

Relay-terminal Connections.—As far as the theory of directional protection is concerned, it is necessary to consider only the four terminals of an overcurrent directional relay as shown in Fig. 13. The current coil of the wattmeter element is connected in series with the overcurrent element.

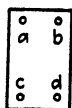


FIG. 13.—
Arrangement
of relay ter-
minals.

The two lower terminals *c* and *d* of Fig. 13 are the current-coil terminals. The two upper terminals *a* and *b* are the voltage-coil terminals of the wattmeter element. The potential coil is practically all reactance, so that the actual current in the potential coil itself lags the impressed voltage by approximately 90° . This fulfills the requirement of an induction element in order to produce rotation. However, in order to simplify the analysis of directional relays, voltage values impressed across the potential coil are always used and reference made to the relation between line current and potential transformer voltages.

The polarity and arrangement of the coils in a directional relay are such that, if *like* instantaneous voltage and current (within 90° phase relation) are impressed on terminals *a* and *c*, the wattmeter contact will open. If instantaneous opposite polarities of current and voltage are impressed on *a* and *c*, the wattmeter contact will close. Arrows used to indicate direction of polarity for the terminals of relays are always drawn so as to represent vectors within 90° phase angle of each other.

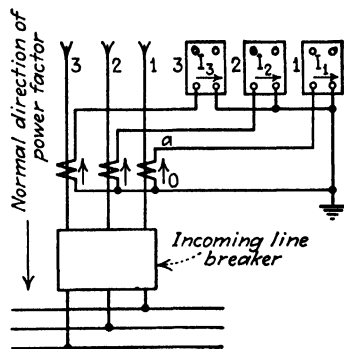


FIG. 14.—Relay current connections.

An incoming line is shown in Fig. 14 with the positive direction of power flow toward the bus. The interposition of current transformers effects no change in the direction of current. Current in line 1 may be assumed to flow in the main line to terminal *a* of the current transformer, from there passing through relay 1 and returning to zero, to continue toward the bus.

For directional-relay elements, it is not satisfactory to use current I_1 with voltage E_{1-2} , I_2 with E_{2-3} , and I_3 with E_{3-1} .

The reason for this is that in each case for 100 per cent power factor these currents already lag their respective voltages by 30° . For lagging power factors, as will exist under short-circuit conditions, it is possible to have a power factor which makes this angle greater than 90° . In such a case, the true directional wattmeter characteristic would be upset and incorrect operation result.

The first attempt to overcome this condition used the connection shown in Fig. 15a, where the current and voltage impressed

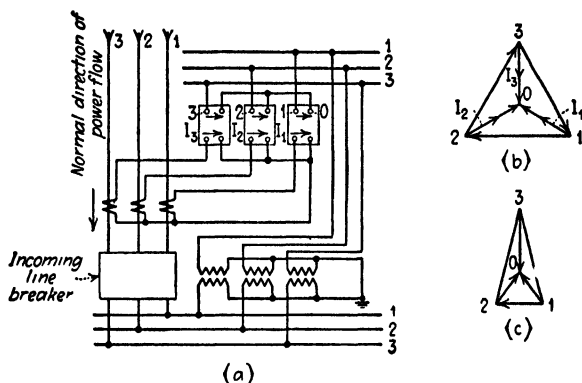


FIG. 15.—Three 4- to 15-amp. relays with potential coils Y connected.

across each relay are in phase. The relay-potential coils are connected in star. Current I_1 is used with E_{1-0} , I_2 with E_{2-0} and I_3 with E_{3-0} . These vector relations are shown in Fig. 15b. This connection does not give sufficient leeway for lagging power factor under short-circuit conditions. Another disadvantage of this relay connection is that the voltage is low at the time of short circuit, and, if current I_1 is used with E_{1-0} and I_2 with E_{2-0} , a single-phase fault across phases 1 and 2 will reduce voltages E_{1-0} and E_{2-0} , giving but slight torque to the directional element. This is illustrated in Figure 15c. The third relay would have its voltage E_{3-0} of high value, but line 3 will not have any short-circuit current with which to operate the relay. Referring to Fig. 15a, the connections are such as to fulfill the condition of like instantaneous polarity on terminals *a* and *c* (refer to Fig. 13) so that the directional contacts will be open with power flow into the bus.

The most satisfactory method of overcoming these difficulties is to connect the potential coil of the relay to a voltage which is 30° behind the current at 100 per cent power factor. Figure 16a gives the standard connection. The connections are shown for an incoming breaker, with V-V connected potential transformers, and Fig. 16b shows the vector relations. Current I_1 is used with voltage E_{1-3} , I_2 with E_{2-1} , and I_3 with E_{3-2} . To understand clearly this fundamental basic principle of directional-relay connections, refer to Fig. 17a, b, and c. The normal three-phase balanced connections will first be described in detail

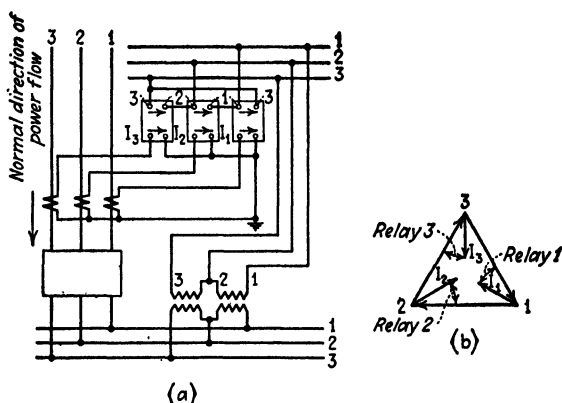


FIG. 16.—Three 4- to 15-amp. relays showing standard thirty-degree connection.

for an incoming and an outgoing breaker. This will be followed by an explanation of unbalanced short-circuit conditions.

The connections for the relays of breaker A (Fig. 17a) are identical with those of Fig. 16a. Figure 17a gives a simplified method of representing the relay connections. Currents I_1 , I_2 , and I_3 flow from left to right (terminals c to d). Voltage vectors E_{1-3} , E_{2-1} , and E_{3-2} are shown with left-to-right (terminals a to b) positive polarity. It has been already stated that like instantaneous polarities of two vectors impressed on terminals a and c will open the directional contacts.

The vectors for relays of breaker B show currents I_1 , I_2 , and I_3 reversed. The normal flow is away from the bus in this case. An easy way to establish a definite vector system is to represent currents I_1 , I_2 , and I_3 pointing into the delta for power flow into a bus and have I_1 , I_2 , and I_3 pointing out from the delta

for power flow *away* from the bus. This change in the direction of flow of current within the relays is effected by reversing the current-transformer connections as shown in Fig. 17 for the relays of breaker *B*. The actual direction of power is the same at breakers *A* and *B* as far as the main lines are concerned, but a change in reversal of current-transformer connections gives a method of representing power flow *into* and *away* from a bus.

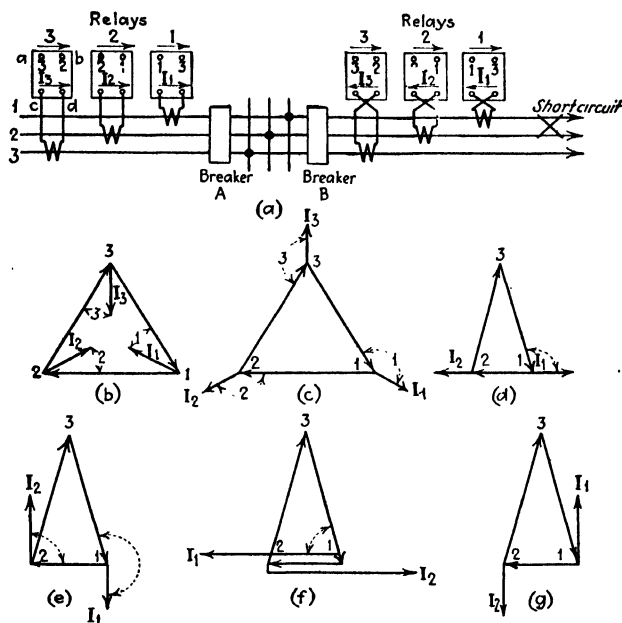


FIG. 17.—Standard thirty-degree connection for incoming and outgoing power.

Summarizing the above and referring to Fig. 17*a*, *b*, and *c*, left-to-right polarity of relays at breaker *A* shows the current and potential 30° out of phase (Fig. 17*b*) and hence the directional contacts are open. Left-to-right polarity of relays at breaker *B* show current and potential 150° out of phase (Fig. 17*c*), hence the directional contacts are closed.

Figures 17*d*, *e*, *f*, and *g* represent the vector relations for both sets of relays of Fig. 17*a* when a single-phase short circuit occurs between phases 1 and 2 to the right of breaker *B* as indicated.

Figure 17*d* and *e* covers the relays at breaker *B*. In Fig. 17*d*, I_1 shifts counterclockwise from its position in Fig. 17*c* to a horizontal position. This is on the assumption of a 100 per cent power-factor fault. Current I_2 of Fig. 17*c* shifts to a new horizontal position as shown in Fig. 17*d*. Voltage E_{1-3} and current I_1 still have an angle between them greater than 90° . Hence, relay 1 is still in the tripping direction, although the torque is small, due to a practically 90° phase relation. Current I_2 is in direct phase opposition with E_{2-1} , hence relay 2 trips positively.

However, the fault is not likely to be one of 100 per cent power factor. Taking the other extreme, Fig. 17*e* represents a 90° lag or zero per cent power-factor condition. Currents I_1 and I_2 have a 90° relation with E_{1-2} . E_{1-3} and I_1 are now almost directly in phase opposition, and relay 1 will close with positive torque. Current I_2 and E_{1-2} have theoretically a 90° phase relation, and no torque will result. However, zero degree power factor is an extreme condition which will not be met in practice. As the power factor increases, I_2 shifts to an out-of-phase position with E_{1-2} and closing torque will result.

The conditions for relays at breaker *A* (Fig. 17*f*) show currents I_1 and I_2 reversed in position from that shown in Fig. 17*d*. This is as it should be, for the direction of current is opposite in the two sets of relays. Currents I_2 and voltage E_{2-1} are in phase, which gives positive opening. This covers the 100 per cent power-factor condition. For the zero power-factor condition, refer to Fig. 17*g*. In this case, I_1 and E_{1-3} are nearly in phase with a resultant positive opening. I_2 and E_{1-2} are 90° out of phase, but any slight increase in power factor brings them into an in-phase position. Therefore, a fault to the right of breaker *B* will open breaker *B* and leave *A* closed. The reverse condition will obtain for a fault to the left of breaker *A*, namely, *A* will trip and *B* stay closed.

In addition to the connection shown in Fig. 16*a*, there are two major connections which are commonly used. The first of these is shown in Fig. 18*a*. The main step-up power bank of transformers is connected delta on the low side and star on the high side. The potential transformers are connected V-V to the delta bus. The breaker to be controlled and hence the current transformers are on the high-tension or star side of the transformer bank. Since the main bank of power transformers is

connected delta-star, the line-to-line voltage on the delta side is in phase with the line current on the star side. Figure 18b gives the vector diagram. The same respective currents and voltages are used on the relays in Fig. 18a as in Fig. 16a, but in Fig. 18a the phase angle is 60° , while it is only 30° in Fig. 16a.

With the system connections shown in Fig. 18a, it is possible to get a 30° relation of voltage and current on the relays in two ways; first, by connecting 70-volt relays in star in conjunction with two potential transformers connected V-V, and, second, by using three potential transformers connected star-star with

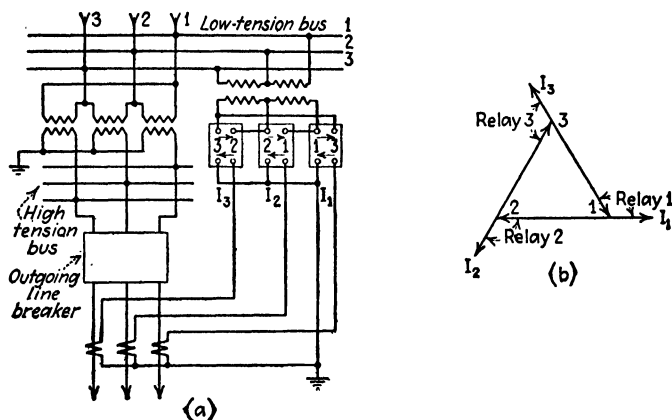


FIG. 18.—Sixty-degree connection for three 4- to 15-amp. relays with current transformers on Y side and potential transformers on delta side of transformer bank.

125-volt relays connected in star. The star connection of directional-relay potential coils should be avoided wherever possible for the reasons illustrated by Fig. 15c.

A second system connection sometimes encountered is that where the transformer bank is connected delta-star and where potential is obtained from the star side and current from the delta side. Figure 19a is a wiring diagram of this connection. The delta power-transformer primary side vectors are covered by Fig. 19b. The power-transformer secondary vectors are shown in Fig. 19c. The secondary is shifted 30° from the primary. The primaries of two potential transformers are connected across phase E_{3-1} as shown in Fig. 19a, so that E_{3-0} and E_{0-1} are added vectorially. This is further illustrated in Fig. 19d. On the secondary side of the potential-trans-

former winding, E_{1-0} is reversed, so that E_{2-0} and E_{0-1} have the vector arrangement shown in Fig. 19e. This gives a normal

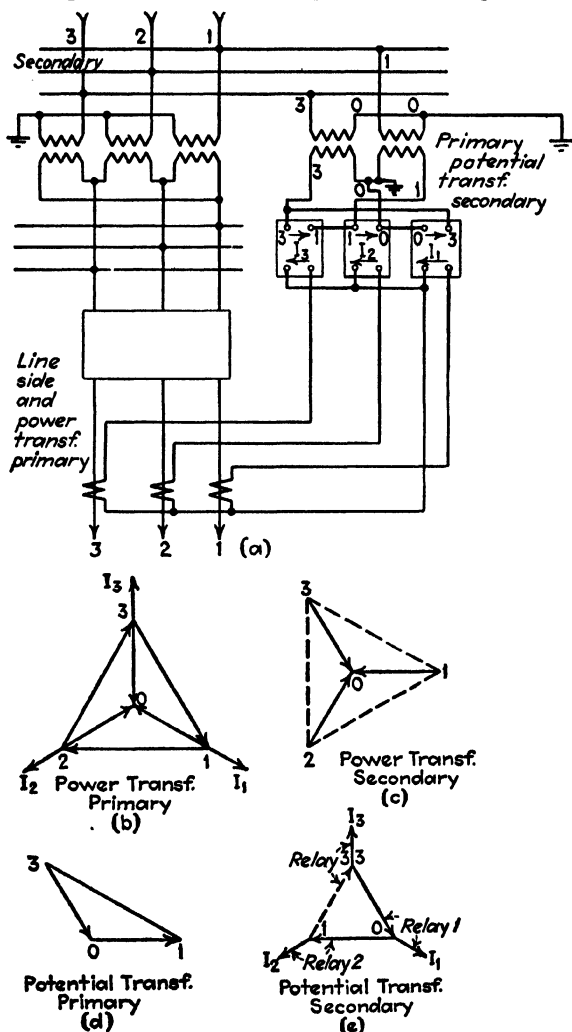


FIG. 19.—Thirty-degree connection for three 4- to 15-amp. relays with current transformers on delta side and potential transformers on Y side of transformer bank.

balanced delta, and connections are made to the directional relays to give the standard 30° relation.

The 90° Connection.—The 30° connection is used as the standard connection for directional relays and works out very satisfactorily. There is, however, a connection available which in some few cases may prove to have certain advantages. This connection is referred to as the 90° connection, but it should not be construed to mean that there is an actual 90° phase relation

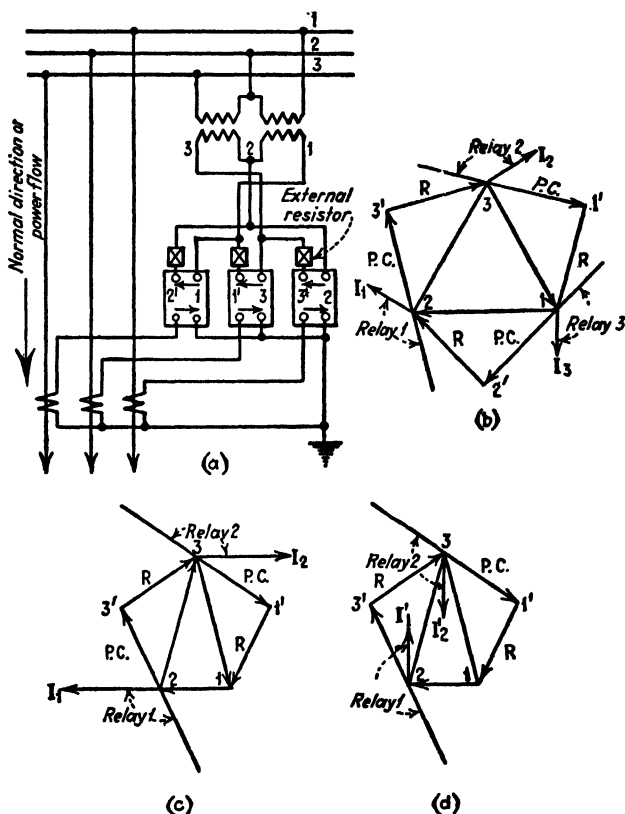


FIG. 20.—Ninety-degree connection for three 4- to 15-amp. relays.

between voltage and current in the relay. A diagram of this connection is shown in Fig. 20a. In series with each potential coil is a resistor which, for the 60-cycle relay, is approximately 600 ohms. When connected in series with the potential coil, this gives a value of resistance to the circuit equal to the reactance of the potential coil. Hence, if an external connection is made across the resistor and coil so that it lags a current by 90°,

then the voltage across the relay coil lags the current by approximately 45° . This is illustrated in Fig. 20b, where E_{2-3} lags I_1 by 90° . E_{2-3} is impressed across the potential coil plus the resistor. The drop across the potential coil is E_{2-3}' and across the resistor it is E_{2-3} . These two voltages add vectorially to give E_{2-3} . The potential across the relay E_{2-3} lags I_1 by approximately 45° .

Consider next the case of a single-phase fault on phases 1 and 2 as given in Fig. 20c. Fault currents I_1 and I_2 are in phase with E_{1-2} for a 100 per cent power-factor fault. Current I_1 and E_{2-3} still retain an out-of-phase relation greater than 90° , so relay 1 will operate. I_2 and E_{1-3} are also more than 90° out of phase. For a zero per cent power-factor fault, the vector condition is analyzed in Fig. 20d. Both currents I_1 and I_2 retain a phase angle with their respective voltages considerably greater than 90° .

Each connection has certain advantages depending on the power factor of the fault encountered. This power factor refers to the power factor at the particular set of relays being considered. The power factor will vary depending upon the location of the fault and type of fault. At the two extreme ends of the power-factor range, namely, 100 per cent and zero per cent, weak torque is produced on one of the two relays involved in a line-to-line fault when the 30° connection is used. However, the 90° power factor is a theoretical condition only and will in the great percentage of cases be somewhere between 15° and 60° lagging. The 30° connection has been found to give excellent results.

In special applications the polyphase directional relay can be used to advantage. This relay is similar in operation to three wattmeter elements mounted on a common shaft. In other words, three single-phase directional elements are combined in one case and operate on a common shaft. The overcurrent directional feature is obtained by using one polyphase relay in conjunction with three overcurrent relays. It is designed for the 90° connection, with the resistors mounted inside the relay case. The 45° connection tends to equalize the torque on the elements involved in a short circuit and hence gives more positive operation on a three-element relay working on a common shaft.

The polyphase-relay relations are the same as shown for the 90° connection of three single-phase relays, with the exception that the torque, instead of being applied to three separate relays, is applied to one common shaft relay.

Special Vector Conditions Involved in Impedance-relay Applications.—While discussing the vector analysis of line-type directional relaying, it will be well to consider the impedance-type relay.

The impedance relay is of the induction-disk type, similar to the ordinary overcurrent-type relay but with the addition of

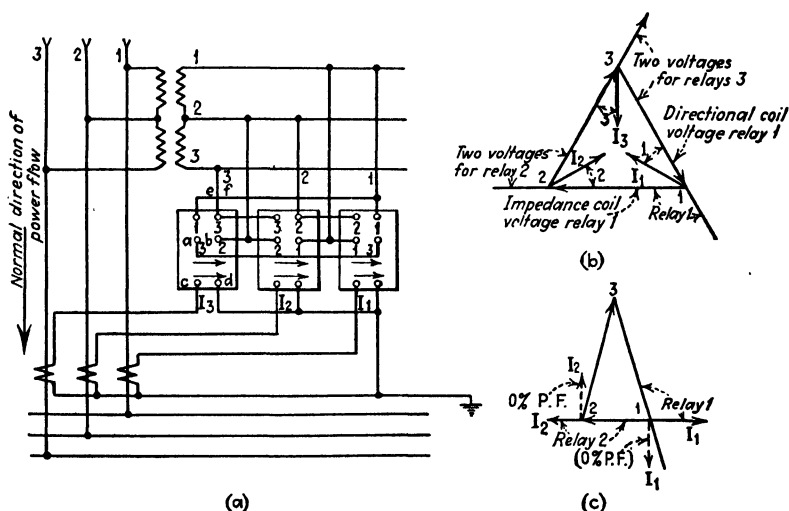


FIG. 21.—Standard thirty-degree connection for induction impedance relays. Impedance-coil voltage lags directional coil voltage by 120° .

a voltage-restraining coil which is connected mechanically to the disk and to the contact mechanism. Instead of operating the contacts directly, the movement of the disk winds up a spring, one end of which is fastened to a shaft that is geared to the disk shaft. The other end of the spring is connected by means of a lever to a rocker arm pivoted at its center and mounted directly above the disk. This rocker arm carries the contact on one end, and the core of the restraining coil is suspended from the other end. The pull of the voltage coil, which opposes the closing of the contacts, is directly proportional to the voltage. The time of operation, since it is directly proportional to the

voltage and inversely proportional to the current, is proportional to the impedance or the distance from the fault.

The current and potential connections for three line impedance relays are shown in Fig. 21a. In addition to studs a , b , c , and d , as in the overcurrent directional relay, there are two additional studs e and f . These are the terminals for the impedance-voltage coil. The vector relations and connections for the directional-element potential coil and relay-current coil are identical with those already fully discussed for the overcurrent directional relay. If the impedance-voltage coil were connected in parallel with the directional potential coil (*i.e.*, the same

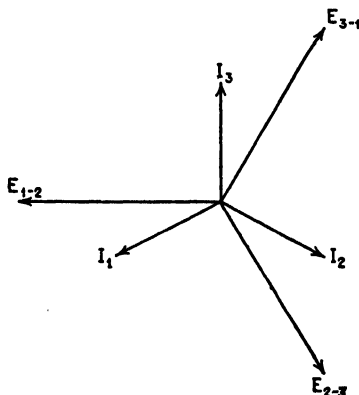


FIG. 22.

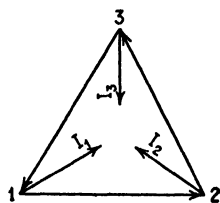


FIG. 23.

FIGS. 22 and 23.—Fundamental vector system for reversed phase rotation.

voltage used on impedance coil as on directional voltage coil), it can be seen by referring to Fig. 21c that, while both relays receive the same fault current, the phase 2 relay would operate, since it is the one with the low restraining voltage. At the same time, the directional voltage of this relay would be equally low. If, however, the connection shown in Fig. 21b is used, connecting E_{1-3} to the directional potential coil and voltage E_{2-1} to the impedance-voltage coil of phase 1 relay, the conditions are changed. When a fault occurs between lines 1 and 2, the phase 1 relay, since it has low restraining voltage E_{2-1} on its impedance-voltage coil, will operate. Instead, however, of having the same lowered voltage on its directional potential coil, the directional voltage E_{1-3} will be nearly normal.

Reversed Phase Rotation.—The basis for the vector relations illustrated in Figs. 5 to 21 is the standard counterclockwise phase rotation 1, 2, 3. If the phase rotation is opposite, or 1, 3, 2, it is necessary to reconstruct the vector analysis and use terminal-relay connections to correspond.

Figure 22 illustrates the open vector system for 1, 3, 2, counterclockwise rotation and in Fig. 23 is shown the closed delta system of vectors.

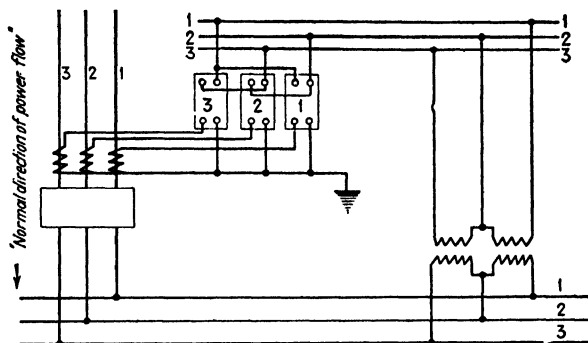


FIG. 24.—Thirty-degree connection with reversed phase rotation.

The 30° relay connection for overcurrent directional relays is shown in Fig. 24 on the basis of phase rotation 1, 3, 2. Current I_1 and voltage E_{1-2} are connected to relay 1, current I_2 and voltage E_{2-3} to relay 2, and current I_3 and voltage E_{3-1} to relay 3.

The relay connections for Figs. 5 to 21 can all be reconstructed on the basis of 1, 3, 2 phase rotation. However, since phase rotation 1, 2, 3 is standard for manufacturers' wiring diagrams, only one example is given for reversed phase rotation.

CHAPTER IX

APPLICATION OF INSTRUMENT TRANSFORMERS TO RELAY SYSTEMS^{1,2}

The failure of a relay system to give complete protection or the possibility of faulty operation under certain conditions of short circuit is in many instances due to the incorrect application of instrument transformers. Current transformers and potential transformers must be considered as a part of the relay system, and as careful thought and analysis should be given the characteristics of the instrument transformers as is given to the relays.

The requirements imposed upon current transformers to give correct operation as part of a relay protective scheme are entirely different from the characteristics necessary for accurate metering. Current transformers are in most cases designed with the meter problem primarily in view, and this factor makes it all the more difficult to apply them so as to give satisfactory relay protection under the many varied fault conditions for which a protective system must function quickly and accurately. It might be assumed that a special set of current transformers should be used for those relay schemes which require current-transformer characteristics different from the standard line available, but here again the question of economics, available space, and wiring duplication or complication must be considered. In many cases meters and relays are initiated from the same set of transformers, and perfectly satisfactory operation will result. In other instances separate sets of current transformers are used, but these are often of the bushing type in order to save initial cost and space. Too much emphasis cannot be given to this important part of a relay system.

¹ Many of the data in this chapter have been obtained from published data of the Westinghouse Electric and Manufacturing Company.

² N.E.L.A. Handbook and Supplement.

GENERAL CURRENT-TRANSFORMER THEORY

In an ideal current transformer the primary ampere turns are always exactly equal to the secondary ampere turns, and the secondary current is, therefore, always directly proportional to the primary current. In an actual current transformer, however, this is never the case. Every core material thus far discovered requires a certain number of ampere turns to induce in it the magnetic flux which, in turn, causes the secondary voltage. The secondary ampere turns, therefore, are equal to the primary ampere turns minus the exciting ampere turns. This subtraction is, of course, vectorial, as is illustrated in Fig. 1. E_p and E_s are the voltages in the primary and secondary windings, respec-

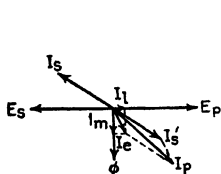


FIG. 1.—Current-transformer vector relations.

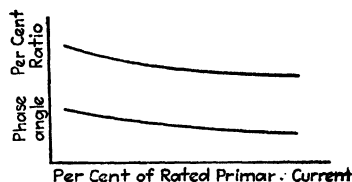


FIG. 2.—Ratio and phase-angle curves of current transformer.

tively. In order to maintain the flux ϕ , the primary circuit must supply a current I_i , in phase with the voltage, to overcome the iron loss; and a current I_m , at the right angles to the voltage, to magnetize the core. These two currents combine to form the exciting current I_e . If a burden connected to the secondary draws a current I_s lagging behind the voltage as shown, a corresponding current, I'_s must flow in the primary. The total primary line current, therefore, is the sum of I_e and I'_s or I_p . Figure 1 applies to a transformer which has an equal number of primary and secondary turns.

If the primary current is reduced, the secondary current will also be reduced, and, since the secondary impedance is fixed, the secondary voltage and the flux in the core will be reduced in the same proportion. But, on account of the shape of the magnetization curve of iron, the exciting current I_e goes down in a different ratio. The result is that the ratio and phase-angle curves of current transformers generally are not straight lines but tend to tip upward at the left end, as shown in Fig. 2. That

is, with a given impedance in the secondary circuit, the exciting ampere turns form a larger proportion of the total at small primary currents than at large, and it is the exciting current which causes the errors.

In the metering problem all the errors of a current transformer are caused by the exciting current since the ratio is chosen so that the maximum normal current in a particular circuit is within the capacity of the transformer and hence no saturation takes place. In fact, the ratio of current transformers should be selected so that normal full load imposes about 65 per cent rating. However, in the relay problem heavy overloads must be considered and the resultant saturation obviously causes a large ratio error. Before getting farther into the relay problem, however, let us continue with the general methods of improving the characteristics of the current transformer.

The most accurate current transformer is one in which the exciting ampere turns are least in proportion to the secondary ampere turns. Exciting ampere turns may be reduced in three principal ways:

1. By improving the quality of the magnetic material.
2. By decreasing the amount of core material (this naturally produces more saturation and ratio error at heavy overloads).
3. By reducing the magnetic-flux density in the core.

Much has been accomplished in the last few years in the way of improved materials. Silicon steel is obtainable with losses which would have been thought impossible a few years ago, and several high-permeability alloys have been developed with losses far below those of the best silicon steel. Some of these alloys can be used in current transformers to advantage, although their high cost bars them, at present, from use in other transformers.

The second and third methods of reducing the exciting ampere turns are closely connected. If the amount of the core material can be decreased by decreasing the mean turn of the core, the gain is considerable; but, if the core material is reduced by cutting down the cross section, it tends to increase the flux density and thus more or less neutralizes the good effect.

The flux density is determined by the usual transformer formula

$$B = \frac{E \times k}{f \times A \times N}$$

where B = flux density in the iron.

E = voltage induced in N turns.

k = a constant.

f = frequency.

A = area of the core.

N = number of turns.

The flux density, accordingly, can be reduced by reducing E , the secondary voltage, or by increasing f , A , or N . The quantity E depends on the burden connected to the secondary of the transformer, and f is fixed by the circuit in which the transformer is used. The designer, therefore, has control of only the area of the core and of the number of turns. The area cannot be made too large without using a large amount of core material, so most high-accuracy current transformers are designed with a large number of ampere turns.

After the transformer is designed, built, and installed, however, the flux density still is not fixed. The secondary voltage, within the capacity of the transformer, will be whatever is necessary to circulate the secondary current through the impedance of the secondary circuit; and, even after the impedance of the secondary circuit is fixed, the secondary current, and consequently the voltage, varies with the primary current.

From all this it may be seen that the best conditions for high accuracy in a current transformer are: (1) high-grade core material of suitable shape and cross section, (2) a large number of ampere turns, and (3) a small burden. It is difficult to write a specification which takes proper account of all these factors.

CURRENT-TRANSFORMER CHARACTERISTICS

Since a current transformer is in series with the line, it is obvious that on heavy overcurrents its core may become saturated. Under the conditions approaching saturation, the ratio of the transformer will naturally depart from its nominal ratio and the secondary current will no longer be a fixed percentage of the primary current. The change in ratio on overcurrents for four types of transformers having similar burdens connected to their secondaries is illustrated in Fig. 3. These curves are typical of transformers on the market today, and an understanding of their significance is important.

The application of current transformers to a relay system involving a single set of overcurrent relays on a radial feeder need not be given any special study in most instances. In such cases relays are set for 200 or 300 per cent normal load, and satisfactory operation will result provided this amount of current is developed, so that saturation beyond the 300 per cent point does not upset the relay operation, even though the ratio may increase rapidly at higher overcurrent values.

In cases where several sections are relayed in series by progressive time and current settings, it is important to know the

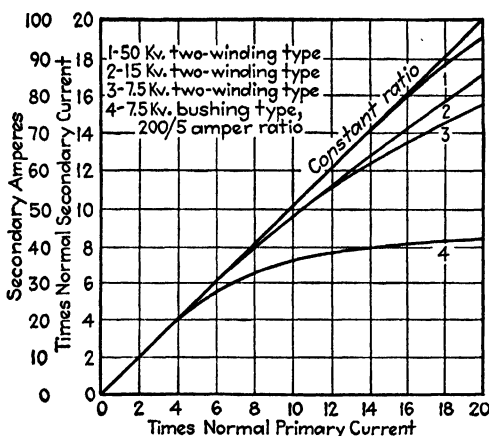


FIG. 3.—Typical current-transformer ratio curves with non-saturating burden of 25 volt-amp., 50 per cent power factor, 60 cycles, 5 amp.

characteristics of all the transformers at high overcurrent values, so that saturation at different points and in varying degree in the current transformers does not upset the selective action of the several sections. To take care of this it is desirable to use the same general type of current transformer at the switching points which are to be relayed selectively.

Many relay systems are based on the differential-current principle. A typical example is that of the standard differential protection shown in Fig. 4. The connections are made so that the secondary current circulates between the two sets of current transformers which, under ideally balanced conditions, should produce zero current through the relay. For such a protective system, the relay should trip only on faults occurring within the

apparatus so protected, and must be immune from "through"-fault operation.

Referring to Fig. 4, assume that current transformer 1 has a ratio characteristic represented by curve 1 and that current transformer 2 has a ratio characteristic represented by curve 3 in Fig. 3. Further, assume that an ordinary overcurrent relay is used, and set to trip at a current differential of 8 amp. Should a short circuit occur on the low-voltage side of the power transformer and external to it, the short-circuit current in both windings would be the same multiple of normal. If it is assumed that this is 15 times normal, reference 'o Fig. 3 will indicate that transformer 1 produces 75 amp. secondary current and transformer 2 produces 64 amp. with a *difference* current through the relay of 11 amp., which is above the pick-up point of the relay.

It is not always possible to have both sets of current transformers in a differential-current scheme of like ratio characteristics. This was recognized several years ago and a special differential-current relay was developed which operates on a differential percentage basis rather than on a fixed ampere basis, but even such a relay will operate incorrectly in case transformers have such varied characteristics as shown by curves 1 and 4. A study of the current-transformer curves must be made for each relay system if proper assurance of correct relaying is to result.

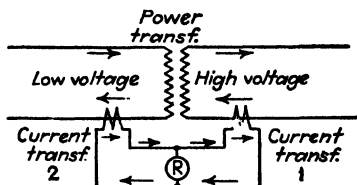


FIG. 4.—A differential-protection scheme using a standard overload relay. Arrows show instantaneous direction of currents.

GENERAL TYPES OF CURRENT TRANSFORMERS

Current transformers as associated with relay systems are of three principal types: the wound type, the through type, and the bushing type. Transformers having more than one primary turn are called wound-type current transformers. Sufficient turns are used in the wound type to keep the exciting current relatively small to obtain high accuracy.

Current transformers, generally for 1000 amp. and higher, which are made with a cable or straight copper bar, forming a single-turn primary winding, are known as through-type trans-

formers. They are particularly adapted to heavy-overcurrent conditions because the tendency for the primary winding to assume a circular shape at heavy overcurrent does not exist. The accuracy of a current transformer with a given secondary burden depends upon the number of ampere turns in the transformer. Since the through-type transformer has no primary winding but rather makes use of the current carried by the cable or bus to energize the core, it is necessary to have a relatively large primary current to make up the required ampere turns.

The bushing-type transformer consists of an annular core and secondary winding placed on the outside of the bushing of a high-tension oil circuit breaker. Bushing current transformers necessarily have a very long magnetic path, as compared with the ordinary current transformer, because of the diameter of the bushing. The long magnetic path together with only the one turn in the primary requires a high magnetizing current, and accordingly the transformers are rather inaccurate for the lower ratios. A bushing-type current transformer of ratio 150/5 amp. will give good results under ordinary conditions of relay loading. As the ratio increases, the errors decrease, and, when a ratio of 300/5 is reached, the bushing transformer gives practically as good results for relay purposes as that of the wound type. Ratios of less than 150/5 amp. are not recommended for the various balanced relay schemes, and even at this value it might be preferable to connect the secondaries of two transformers in series. Ratios of 200/5 amp. usually give satisfactory results with a single transformer, if the burden is kept to a minimum.

The Wound-type Current Transformer.—There are several classifications of wound-type current transformers, the proper selection depending upon the burden to which the current transformer is to be subjected. A relatively cheap transformer is available for voltages of 5000 volts and under, having a rating of 25 volt-amp. and being compensated for a burden of 2.5 volt-amp. Under normal operating conditions the best accuracy is obtained when operating with a burden at or near the compensated value. In the relay problem this is not important because it is the short-circuit characteristics which determine the application. Obviously, where the same transformers are used for metering and relaying, it is important to keep both factors in mind.

The relatively cheap, wound-type current transformer is necessarily smaller, having less iron and hence saturating at lower values of short-circuit current for a given burden. The volt-ampere rating is correspondingly reduced and the application limited. Typical ratio curves are shown in Fig. 5. This transformer is designed primarily for a watt-hour-meter burden.

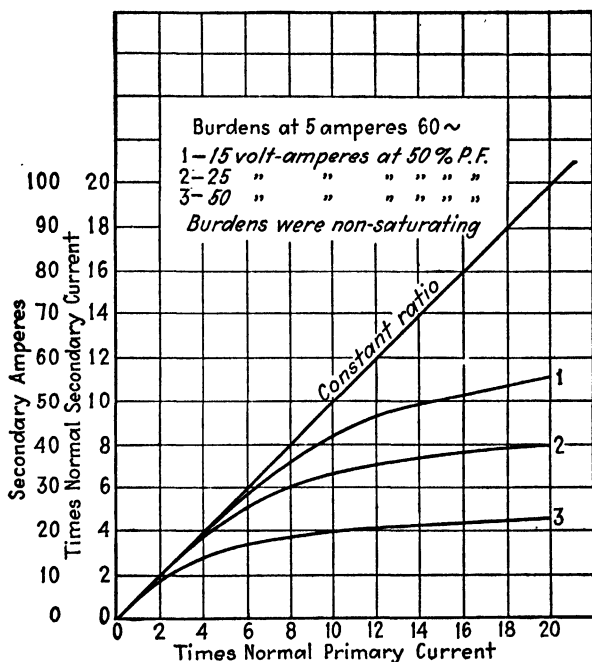


FIG. 5.—Ratio curves of typical 25 volt-amp., 5000-volt current transformer designed for watt-hour-meter load only.

Figure 6 gives ratio curves for a larger and more expensive current transformer designed for burden including relays as well as meters. In Fig. 7, are shown typical curves on 7500 and 15,000 volt, 50-volt-ampere current transformers.

In Fig. 8 are shown typical ratio curves for high-voltage oil-filled wound-type current transformers.

The Through-type Current Transformer.—Since the performance of a current transformer is dependent, to a considerable extent, on the number of primary ampere turns, it is possible to make higher current ratings in a through type with an opening

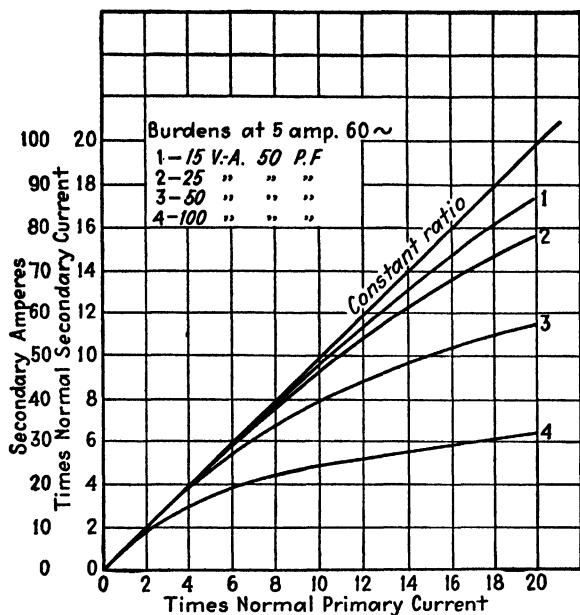


FIG. 6.—Typical ratio curves of 5000-volt current transformer designed for meter and relay burden.

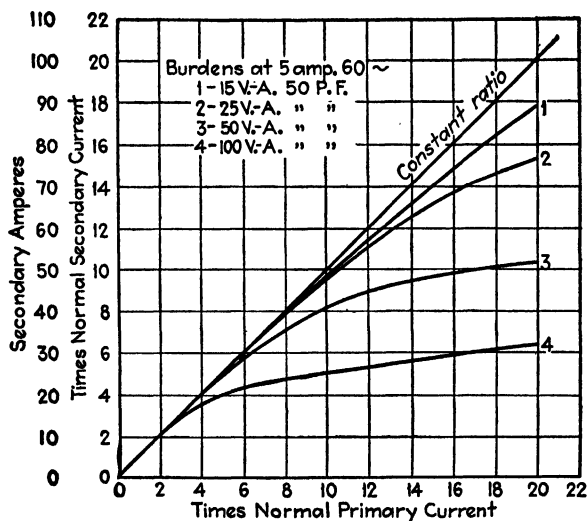


FIG. 7.—Typical ratio curves of 7500 to 15,000-volt current transformer designed for meter and relay burden.

through which a cable or bus bar can be inserted as a primary winding. This general type of transformer is also designed with

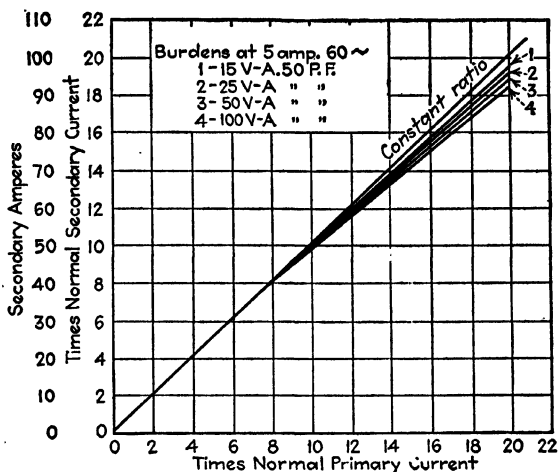


FIG. 8.—Typical ratio curves of high voltage oil filled current transformer.

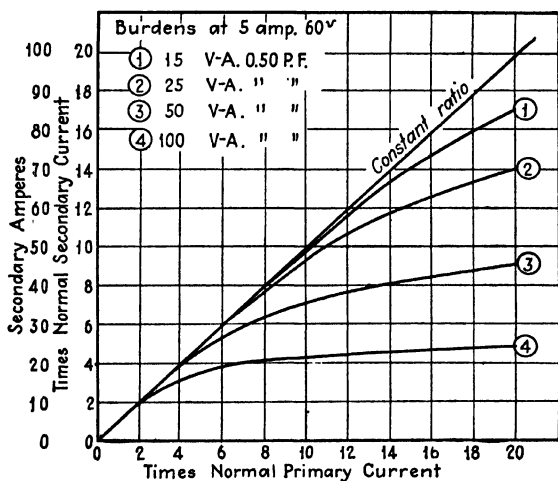
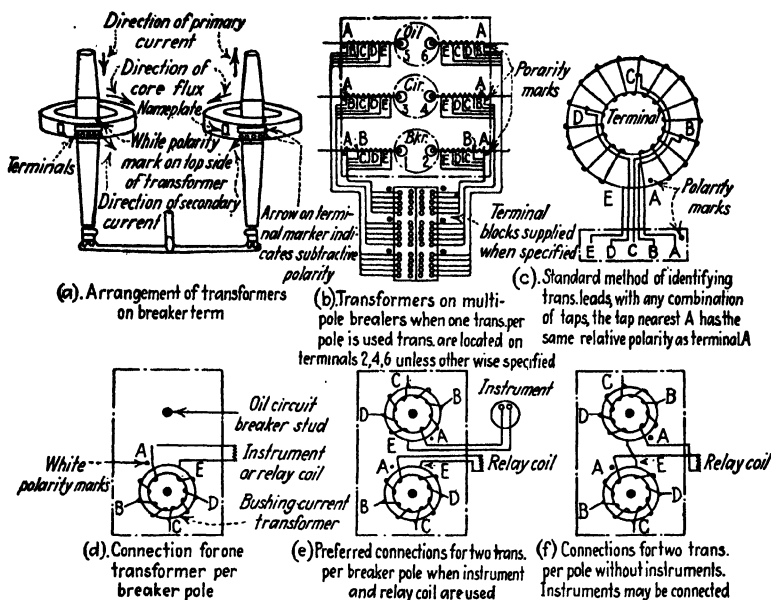


FIG. 9.—Typical ratio curves of 2500- to 15,000-volt through-type current transformer.

primary mounting and provided with suitable terminals for connecting the cable or bus. Indoor and outdoor units are available. Typical ratio curves are given in Fig. 9.



400-AMP. MULTI-RATIO TABLE

Approximate ratio*	Turn ratio	Secondary taps
†	5 to 1	A—B
75 to 5	15 to 1	B—C
100 to 5	20 to 1	A—C
125 to 5	25 to 1	C—D
175 to 5	35 to 1	D—E
200 to 5	40 to 1	B—D
225 to 5	45 to 1	A—D
300 to 5	60 to 1	C—E
375 to 5	75 to 1	B—E
400 to 5	80 to 1	A—E

1200-AMP. MULTI-RATIO TABLE

Approximate ratio*	Turn ratio	Secondary taps
100 to 5	20 to 1	B—C
200 to 5	40 to 1	A—B
300 to 5	60 to 1	A—C
400 to 5	80 to 1	D—E
500 to 5	100 to 1	C—D
600 to 5	120 to 1	B—D
800 to 5	160 to 1	A—D
900 to 5	180 to 1	C—E
1000 to 5	200 to 1	B—E
1200 to 5	240 to 1	A—E

2000-AMP. MULTI-RATIO TABLE

Approximate ratio*	Turn ratio	Secondary taps
800 to 5	160 to 1	B—C
1200 to 5	240 to 1	A—C
1400 to 5	280 to 1	B—D
1600 to 5	320 to 1	B—E
1800 to 5	360 to 1	A—D
2000 to 5	400 to 1	A—E

* Not compensated for secondary loading. See ratio curves for various loadings.

† Requires special calibration.

FIG. 10.—Connection diagrams for bushing-type current transformers for use in oil circuit breakers.

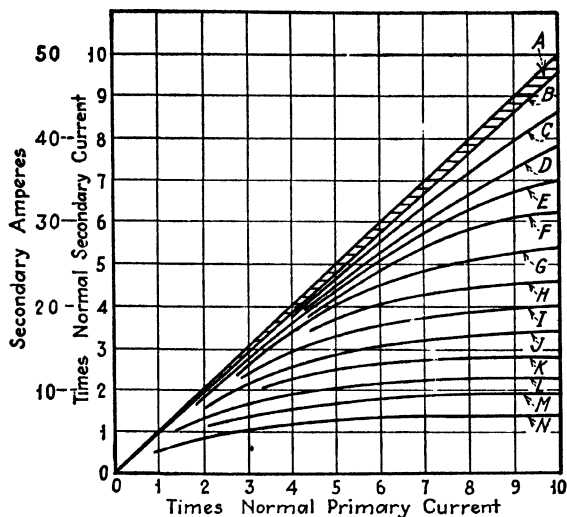


FIG. 11.—Composite curves of bushing-type current transformers showing ratio characteristics at 60 cycles.

Nominal ratio	15 V.A. 0.5 P.F.	25 V.A. 0.5 P.F.	50 V.A. 0.5 P.F.	100 V.A. 0.5 P.F.
*75/5	<i>G</i>	<i>I</i>	<i>M</i>	<i>O</i>
†100/5	<i>F</i>	<i>I</i>	<i>M</i>	<i>L</i>
*100/5	<i>E</i>	<i>H</i>	<i>K</i>	<i>N</i>
*125/5	<i>C</i>	<i>F</i>	<i>J</i>	<i>M</i>
*175/5	<i>A</i>	<i>D</i>	<i>H</i>	<i>L</i>
†200/5	<i>C</i>	<i>E</i>	<i>I</i>	<i>M</i>
*200/5	<i>A</i>	<i>C</i>	<i>G</i>	<i>K</i>
*225/5	<i>A</i>	<i>B</i>	<i>F</i>	<i>J</i>
†300/5	<i>A</i>	<i>A</i>	<i>F</i>	<i>J</i>
*300/5	<i>A</i>	<i>A</i>	<i>D</i>	<i>H</i>
*375/5	<i>A</i>	<i>A</i>	<i>B</i>	<i>G</i>
†400/5	<i>A</i>	<i>A</i>	<i>D</i>	<i>H</i>
*400/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>F</i>
†500/5	<i>A</i>	<i>A</i>	<i>D</i>	<i>F</i>
†600/5	<i>A</i>	<i>A</i>	<i>B</i>	<i>C</i>
†700/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>C</i>
†800/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>B</i>
†900/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>A</i>
†1000/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>A</i>
†1200/5	<i>A</i>	<i>A</i>	<i>A</i>	<i>A</i>

* From tap on 400/5 multi-ratio unit. † From tap on 1200/5 multi-ratio unit.

FIG. 12.—Characteristics of typical 73-kv. bushing-type current transformer. Refer to composite curves of Fig. 11.

The Bushing-type Current Transformer.—In Fig. 10 is shown a typical arrangement of connections for bushing-type current transformers. This indicates the use of a single transformer per pole of a circuit breaker, two current transformers per pole, connected to separate burdens, and two current transformers in series.

A typical set of composite curves for bushing-type current transformers is given in Fig. 11. Reference to Fig. 12 will give

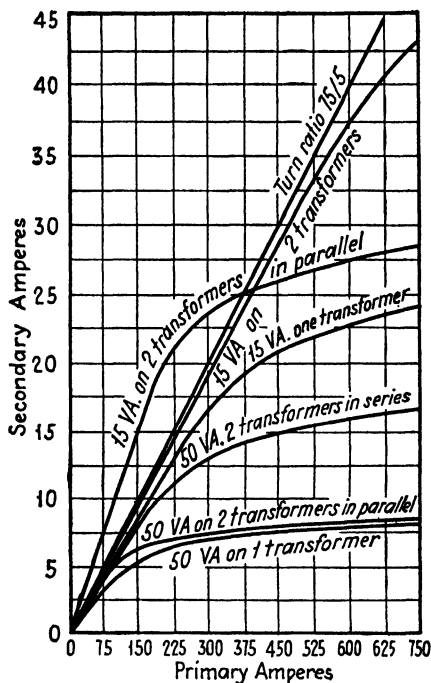


FIG. 13.—Typical ratio curves for 75/5 bushing-type current transformer.

ratio characteristics of a typical 73-kv. bushing-type current transformer.

SERIES OR PARALLEL CONNECTION OF SECONDARIES

A typical set of characteristic curves for a 75/5-turn-ratio transformer is given in Fig. 13. This low ratio is not recommended for burdens involving relays, but these curves are given in order to illustrate how the ratios of a single transformer and two in parallel break down at the higher values of burden. The

poor performance of the parallel secondary connection can be explained as follows.

With two current transformers connected in series, as shown in Fig. 14, supplying a 10-volt-amp. load at 5 amp., each transformer supplies 5 volt-amp. With the transformers connected in parallel, as in Fig. 15, supplying 10 amp. to the same load impedance, the burden increases to 40 volt-amp. Each current transformer is called upon to supply 20 volt-amp., which is four

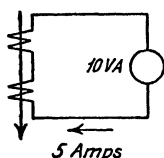


FIG. 14.

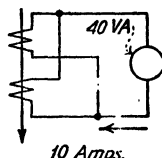


FIG. 15.

FIGS. 14 and 15.—Series and parallel connections of current transformers.

times greater than each transformer must supply when connected in series. This greatly increased burden destroys the ratio completely on low-ratio bushing-type current transformers.

APPLICATION OF CURRENT TRANSFORMERS TO TOTALIZING AND DIFFERENTIAL SCHEMES

Circulating current in the secondaries of current transformers with dead primaries when used in totalizing or differentially connected schemes may under certain conditions give incorrect relay operation.

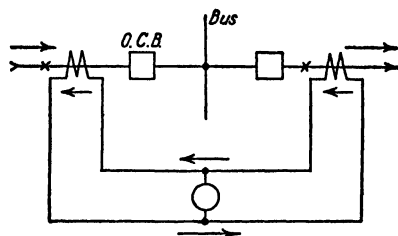


FIG. 16.—Connections for differential-current protection applied to a bus.

In the differential scheme for apparatus protection, the current flowing into a bus or piece of equipment is balanced against the current flowing out, as shown in Fig. 16. For normal conditions the currents are the same, and since the current transformers are connected in series, the current merely circulates through the

secondaries. No current will then flow through the relays. When a fault occurs inside the differential loop, the currents in the secondaries of the current transformers oppose each other and the current must pass through the relay in order to complete the circuit of the current flow.

Little difficulty is encountered in obtaining proper relay operations for faults within the differential loop. However, the characteristics of the current transformers must be such that with a

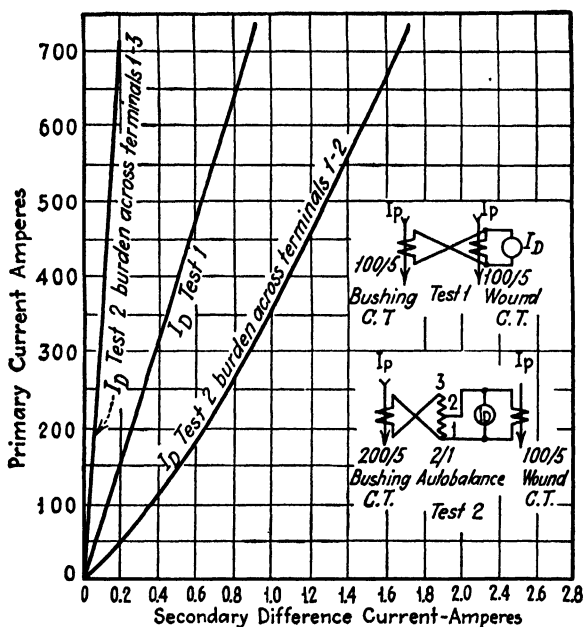


FIG. 17.—Test curves showing difference current when bushing-type current transformer is balanced against wound type.

fault near the station, but not within the differential loop, the unbalance will not be great enough to operate the differential relays. Trouble of this nature is frequently encountered where wound- and bushing-type transformers are balanced against each other or where current-balance autotransformers are used to correct for differences in the ratio of the current transformers used in the differential scheme.

In Fig. 17, test 1 shows the results of balancing a 100/5-ratio wound-type against a 100/5-ratio bushing-type current transformer representing through-fault conditions. For the range

of currents shown the ratios hold pretty well. With higher values of current, the ratio of the bushing transformer will break down, thus operating the relay. In test 2, Fig. 17, a 200/5-ratio bushing transformer is balanced against a 100/5-ratio wound-type transformer and the ratio difference corrected with a 2/1 autobalance transformer. The right-hand curve shows the *difference* current with the relay connected across terminals 1 to 2 of the autobalance transformer and the left-hand curve with the relay connected across terminals 1 to 3. The unbalance is greater for

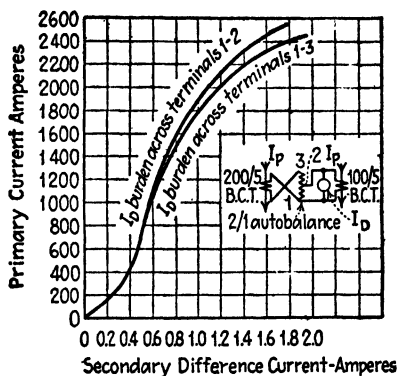


FIG. 18.—Test curves showing *difference* current when balancing bushing-type current transformers.

the former. The wound-type current transformer is compensated for zero-ratio error at 60 per cent primary current with a 30-volt-amp. burden. The bushing type is uncompensated. The effect of the burden is indicated by the difference in the two curves.

In Fig. 18 are shown curves similar to those in Fig. 17 except that in this test a 100/5-ratio bushing-type current transformer was substituted for the wound type. For this case the unbalance is least with the relay connected across terminals 1 to 2 of the autobalance transformers.

In selecting current transformers of different voltage and current ratings for use in differential schemes, it is always advisable to obtain those having similar characteristics and the proper ratios. Autobalance transformers should not be used except in cases where there is no alternative. The application should be determined on the basis of short-circuit currents. The opera-

tion of the scheme for the maximum through-fault condition should be checked, using the characteristic curves of the current transformers for the burden with which they will be operated. The differential relays must be set higher than the maximum unbalance current that exists for the worst through-fault condition.

Another factor that should be given consideration is the magnetizing current taken by the secondary when the primary is open. This condition exists when differential protection is applied to a group of circuit breakers and the operation of the station is such that one or more of the breakers may be open.

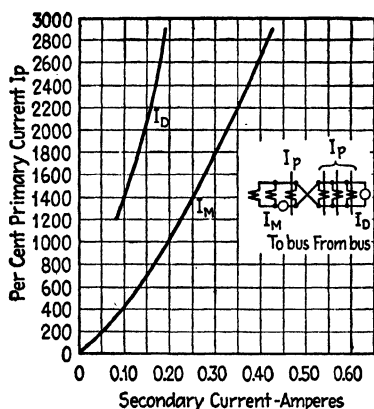


FIG. 19.—Test curves showing effect of open primaries in current transformer circuits.

The magnetizing current for the secondaries of the current transformers on the open breakers must be supplied by the other transformers. An ideal condition to exemplify this is shown in Fig. 19. The current transformers are all the same type and ratio. The set-up is representative of the condition when two breakers are open, the current flowing in on one circuit and out on three. It will be noted that a through fault of 3000 amp. produces a differential current of less than 0.2 amp.

CURRENT-TRANSFORMER BURDEN¹

In order to determine whether a combination of meters and relays and other devices connected in the secondary circuit of one or more current transformers is permissible, tables of volt-ampere burdens of all the equipment involved should be carefully checked. The volt-ampere or impedance values for relays or trip coils should always be based upon the minimum tripping point possible with the device, irrespective of the actual tripping point at which the relay is to be set in an individual installation. This is necessary for the reason that, after apparatus has been

¹ From published data of the General Electric Company.

installed, the setting may be changed by the operator to the lower tripping values.

For the purpose of comparison, volt-ampere data are given on the basis of 5 amp. The volt-ampere burden of any other current I_2 at which the relays operate may be obtained with sufficient accuracy from the formula

$$VA \text{ (volt-amperes) at } I_2 = \frac{(I_2)^2}{5^2} \times (VA \text{ at 5 amp.})$$

The burden imposed by each individual device in each circuit of the combination should be ascertained in terms of either

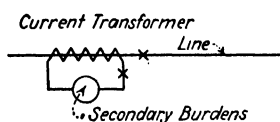


FIG. 20.—Fundamental current-transformer circuit.

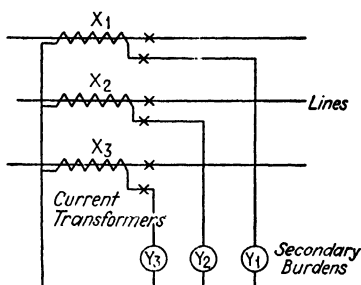


FIG. 21.—Y-connected current transformers.

impedance or volt-amperes, and these all added arithmetically. To the burden of each circuit, thus obtained, must be added the burden imposed by the conducting leads, this being equal to 25 times the resistance (I^2R).

Figures 20, 21, and 22 cover the normal cases of current-transformer connections. In Fig. 20, a single-phase connection is illustrated. The total burden on the current-transformer secondary (Fig. 20) equals the arithmetical sum of the impedance or volt-amperes of all devices connected in series, plus the burden of the connecting leads.

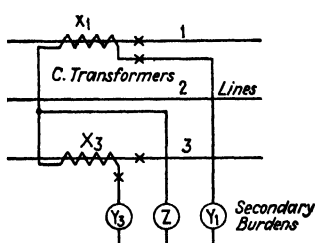


FIG. 22.—Two current transformers on three-phase circuit.

In a three-phase, three-wire, or a three-phase, four-wire connection (Fig. 21), the circuit should be treated as three single-phase combinations with a common return through a fourth conductor. Each transformer carries its own burden only.

The fourth metallic conductor is required to give magnetic balance for the secondary leads, and with it each transformer circuit is made independent. The fourth wire is connected between the *star* point of the current transformers and the *star* point of the secondary connected burden. This is done in order to provide a low-reactive path for single-phase current arising from ground faults, such as two simultaneous grounds on different lines of an ungrounded system, or one ground on a grounded system. Without this connection, the only other path is across the high reactance of the remaining unaffected secondaries. Accordingly the burdens on the transformers connected as in Fig. 21, are as follows:

Burden on $X_1 = Y_1$

Burden on $X_2 = Y_2$

Burden on $X_3 = Y_3$

For Fig. 22, the burden calculations can be obtained by the following:

a. If the ratio of Z/Y_1 is greater than 3.2, then $X_1 = Y_1 + Z$;
if the ratio of Z/Y_3 is greater than 3.2, then $X_3 = Y_3 + Z$.

b. If the ratio of Z/Y_1 is less than 3.2 but greater than 0.4, then $X_1 = Y_1 + 0.75Z$; if the ratio of Z/Y_3 is less than 3.2 but greater than 0.4, then $X_3 = Y_3 + 0.75Z$.

c. If the ratio of Z/Y_1 is less than 0.4, then $X_1 = Y_1 + 0.5Z$; if the ratio of Z/Y_3 is less than 0.4, then $X_3 = Y_3 + 0.5Z$.

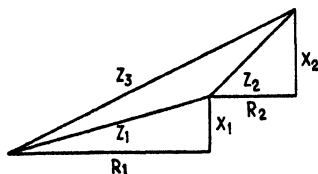


FIG. 23.—Vector addition of current-transformer burdens.

Accurate Calculation of Burdens.—The true total volt-amperes required for a burden is the vector sum of the various impedances multiplied by the square of the secondary current (I^2Z). This vector sum is the square root of the sum of the squares of the total resistance and reactance of the secondary burden. The total reactance is the product of the inductance of the burden in henrys and $2\pi f$, where f is the frequency.

The result so obtained is usually less than that obtained by the arithmetical addition of the burdens, particularly if a device of low power factor forms a considerable part of the total burden. For example, if the volt-ampere burden of a device of high power

factor be added to a burden of a device of low power factor, the results obtained by calculation in accordance with each of the above mentioned methods is illustrated in Fig. 23, where R_1 and X_1 are the resistance and reactance, respectively, of the high-power-factor device, and R_2 and X_2 are the resistance and reactance of the low-power-factor device. The total volt-amperes for the two devices is as follows for the arithmetical method at 5 amp:

$$(5)^2 Z_1 + (5)^2 Z_2 = 25Z_1 + 25Z_2$$

By the vectorial method the total burden is $25Z_3$, which is considerably less.

SELECTION OF CURRENT TRANSFORMERS FOR USE ON CIRCUITS HAVING A GROUNDED NEUTRAL¹

Current transformers for use on the line side of a power transformer, as illustrated by diagrams A and B, Fig. 24, should be

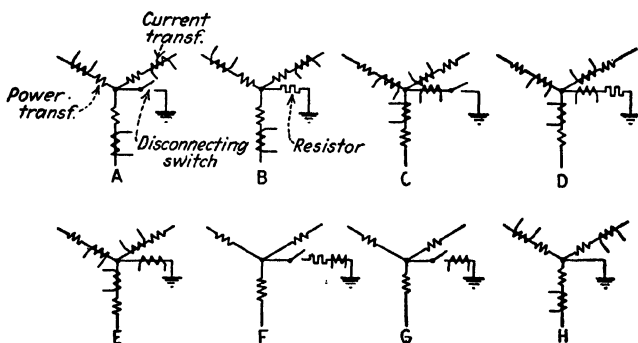


FIG. 24.—Typical applications of current transformers to power-transformer circuits.

selected on the basis of the phase-to-phase voltage, regardless of whether the neutral is grounded directly or through a switch or resistance, or is not grounded. A possible exception is with permanently grounded power transformers having 2.73 insulation (test voltage of 2.73 times the Y voltage), as shown in diagram H, Fig. 23. In this case it may be possible to use current transformers having insulation equal to that of the power transformers plus $12\frac{1}{2}$ per cent.

¹ From published data of the General Electric Company.

Current transformers for use at the neutral, either in the phases or between the neutral point and the ground, as shown by diagrams *C* and *D*, Fig. 24, should be of the same insulation as those used on the line side (*i.e.*, being of voltage rating on the basis of phase-to-phase voltage), if there is a disconnecting switch or a resistance on the ground side of the current transformer.

Where the neutral of the power-transformer bank is "dead-grounded," without resistance or disconnecting switch, either with full or reduced transformer insulation, as shown by diagram *E*, Fig. 24, or where a resistance or a disconnecting switch is on the side of the current transformer away from the ground (so that the current transformer itself is permanently dead-grounded), as illustrated by diagrams *F* and *G*, Fig. 24, current transformers of 15,000 volts may be used, except in the cases where the phase-to-phase voltage is less than 15,000 volts, when the neutral current transformers should correspond to the phase-to-phase voltage. On systems of voltages exceeding 15,000 volts, where 15,000-volt current transformers are used, the secondary leads should be grounded at the current transformer and the primary equipped with by-pass protectors unless a one-turn primary current transformer is used. Such transformers are safe only when the neutral ground is perfectly reliable and of low resistance. They are subject to high voltages and become a source of danger in their primary as well as their secondary circuits if the neutral ground becomes unreliable or of high resistance.

MECHANICAL AND THERMAL CHARACTERISTICS OF CURRENT TRANSFORMERS

Current transformers may fail under short-circuit conditions for either of two reasons. The heat generated in the windings may be sufficient to destroy the insulation, or, if the transformer is of the wound type, the mechanical force produced by the short-circuit current may be large enough to damage the coils. Standard wound-type transformers will carry 50 to 75 times normal current (depending on the type used) for 1 sec. without damage. Special bracing can be put on wound-type transformers making them mechanically good for 150 to 300 times their rated current. Transformers can be built for even higher ratings, but they are large in size and necessarily expensive. It is important

to specify the thermal time limit on transformers carrying more than 100 times normal current.

POTENTIAL DEVICES

In directional relay schemes it is necessary in most instances to use potential connections. These connections can be obtained from standard potential transformers, bushing potential devices, or capacitor potential devices.

Potential Transformers.—Standard voltage transformers are available in the dry type for the lower voltages (15 kv. and below) and the oil-immersed type for the higher voltages. These transformers have a capacity of 200 volt-amp. and are compensated to give correct ratio at 40 volt-amp. This compensation value need not be considered as far as relay applications are concerned, because the accuracy up to full rating of the transformers is well within relay requirements.

Voltage transformers are compensated for their iron loss at rated voltage. When used on either a higher or a lower voltage an error is introduced. In general this error will not be more than 0.15 per cent when the applied voltage is from 50 to 110 per cent of rated voltage. A voltage transformer should never be used on a circuit whose voltage is more than 10 per cent above the rated voltage of the transformer. Reference should be made to the chapter on Vectors for typical potential-transformer connections.

Potential transformers may also be used as small power transformers to supply loads in excess of 200 volt-amp. For such applications the capacity rating is based on heating and not accuracy. These ratings are generally from two to five times the standard rating of 200 volt-amp.

Principle of Operation.—The potential transformer is, in principle, an ordinary constant-potential transformer (as a distribution transformer), especially designed for close regulation, so that the secondary voltage under any condition will bear as nearly as possible a constant ratio to the primary voltage and differ from it in phase by as near 180° as possible.

The secondary voltage cannot be exactly proportional to the primary voltage or exactly opposite to it in phase for all conditions, on account of the losses in the transformer and the magnetic leakage between coils. This may be explained by reference

to the vector diagram, Fig. 25, for a 1:1 ratio transformer. The symbols on the diagram have the following significance:

OE_p = voltage impressed on the primary winding.

OE_s = voltage delivered to the instruments by the secondary winding, or the transformer-secondary terminal voltage.

OI_0 = exciting current required to establish flux in the core. It flows only in the primary winding and is composed of the components I_m , required to magnetize the core, and I_i , required to supply the core losses.

OI_1 = load current, or the current delivered to the instruments.

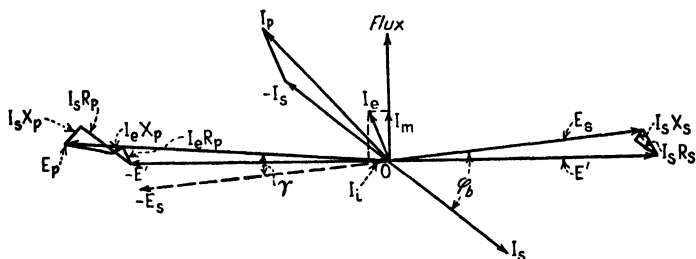


FIG. 25.—Vector analysis of potential transformer.

OI_p = vector sum of I_e and $-I_s$, or the resultant current that flows in the primary winding.

$I_p R_p$ and $I_p X_p$ = resistance and reactance drops, respectively, in the primary winding due to its resistance and reactance only and the exciting current. These drops are practically constant as long as the primary impressed voltage is constant.

$I_s R_p$ and $I_s X_p$ = resistance and reactance drops, respectively, in the primary winding due to its resistance and reactance only and the load current. These drops are entirely dependent on the magnitude of the burden connected to the secondary.

OE' = voltage induced in the secondary winding by the flux.

$-OE'$ = reversed secondary induced voltage, or the counterpart of E' in the primary winding.

$I_s R_s$ and $I_s X_s$ = resistance and reactance drops, respectively, in the secondary winding due to its resistance and reactance only and the load current. These drops are also dependent on the magnitude of the burden connected to the secondary.

ϕ_b = angle between the secondary terminal voltage and the load current. This angle is fixed by the power factor of the burden.

γ = angle between the reversed secondary terminal volts $-E_s$ and the primary impressed volts E_p . This angle is designated as the phase angle of the transformer. In a perfect transformer it would be zero.

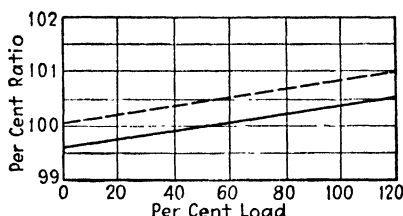


FIG. 26.—Ratio error curves of potential transformer.

The vector diagram as shown has been greatly distorted in order to simplify explanation. In reality the resistance and reactance drops are very small.

Errors.—From the foregoing it is seen that the presence of the voltage drops within the transformer causes the secondary voltage to be lower than the ratio of secondary to primary turns would indicate. This error is called the “ratio error.” Also, these same drops cause the reversed-secondary-voltage vector to differ in phase from the impressed-voltage vector. This error is called the “phase-angle error.” Potential transformers are so designed that these drops are extremely small and cause only very small errors.

The ratio error, in any transformer, can be neutralized for one particular condition of load, frequency, and voltage by *compensating* the transformer. This means that the actual ratio of turns differs from the marked ratio of the transformer by an amount just sufficient to make up for the voltage drops present under this condition. This is illustrated in Fig. 26. The dotted

curve shows the percentage of marked ratio for an uncompensated transformer, and the full line shows the same transformer compensated to deliver correct voltage at half load.

The *phase-angle errors* cannot be compensated for by changing the turn ratio, as is done for the ratio errors. They need not be considered when using voltmeters, frequency meters, synchroscopes, or protective and regulating relays. They are important only when the transformer is used with watt-meters, watthour meters, and similar instruments where indication depends not only on the voltage but also on its phase relation to the line current. The analysis of phase-angle errors will therefore not

be covered in that it does not enter into the application of relays.

Bushing Potential Device.—

The potential device makes use of the inherent potentiometer characteristics of the condenser bushings on high-tension circuit breakers and transformers. Unlike the insulator potentiometer scheme, it depends upon a relatively large charging current through the bushing and not upon a leakage current over the weather casing.

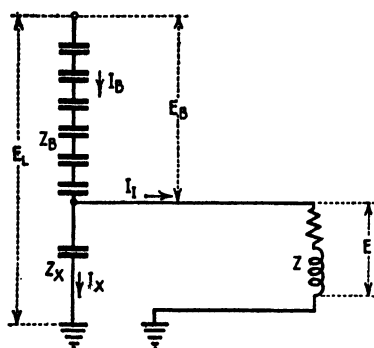


FIG. 27.—Schematic diagram of simple potentiometer circuit.

Theory of Potential Device.—Owing to the fact that standard condenser bushings are designed for a voltage drop in excess of the usual instrument voltage, some means must be provided for lowering the voltage across the last step to the usual 110 volts for meters and relays. This may be done by paralleling with the last layer a condenser of suitable size, or by the use of a transformer or resistor. The transformer method is best as it allows a greater current to be drawn from the potentiometer. However, with the transformer alone only instruments which have been specially calibrated can operate with accuracy. Any instrument having resistance in its circuit will cause the instrument voltage to be out of phase with the line voltage.

The schematic diagram for this elemental potentiometer is shown in Fig. 27. Neglecting the transformer or referring all

constants to one side of the transformer for simplicity, then from Fig. 27 the potentiometer ratio is

$$\frac{E_L}{E} = \left[Z_B \left(\frac{1}{Z} + \frac{1}{Z_X} \right) + 1 \right] \quad (1)$$

At no load the value of Z is infinite, consequently if we substitute ∞ for Z in the above equation we obtain the no-load ratio as follows:

$$\frac{E_L}{E} = \left[Z_B \left(\frac{1}{\infty} + \frac{1}{Z_X} \right) + 1 \right] = \left[\frac{Z_B}{Z_X} + 1 \right] = \frac{Z_B + Z_X}{Z_X} \quad (2)$$

This gives the potentiometer ratio on open circuit as being proportional to the total impedance of the bushing divided by the impedance of the last step.

The effect of load may be seen as follows, substituting the following terms in Eq. (1):

$$Z_B = -jX_B \text{ (capacity reactance only)}$$

$$Z_X = -jX_X \text{ (capacity reactance only)}$$

$$Z = R + jX \text{ (resistive with either inductive or capacitive reactance)}$$

Then the ratio

$$\frac{E_L}{E} = \left[-jX_B \left(\frac{1}{R + jX} + \frac{1}{-jX_X} \right) + 1 \right] = \left[\frac{-jX_B}{R + jX} + \frac{X_B}{X_X} + 1 \right] \quad (3)$$

In order that E_L/E be a numerical quantity, *i.e.*, that there be no phase-angle displacement between E_L and E , all j quantities in the above equation must cancel. By inspection it may be seen for any value of load resistance R except zero that the j terms will not cancel and there will be an angular difference in phase between the line voltage E_L and the instrument voltage E . Since practically all commercial forms of meters and instruments have some resistance in their windings, it can be seen that this elemental potentiometer or potential device is inherently subject to large errors unless the instruments have been carefully calibrated to account for this phase-angle displacement.

If an instrument having negligible resistance such as an electrostatic voltmeter were connected to the terminals of the above device, the value $R_1 = 0$ and Eq. (3) becomes

$$\frac{E_L}{E} = \left[X_B \left(\frac{1}{\pm X} + \frac{1}{\pm X_X} \right) + 1 \right] \quad (4)$$

Here all the j terms have canceled out, E_L/E is a numerical value and hence the phase angle between E_L and E is zero.

It is interesting to note that if $+X = -X_x$, or if the load is highly inductive with negligible resistance and is resonant with the last step of the bushing X_x , the ratio $E_L/E = 1$ and full voltage would be impressed across the transformer primary thus causing its failure. This case is practically never completely attained in practice, but it can be approached closely enough to be dangerous unless care is taken in designing the potentiometer.

If we place a variable reactor in series with one of the transformer secondary leads as shown by Fig. 28, we obtain an

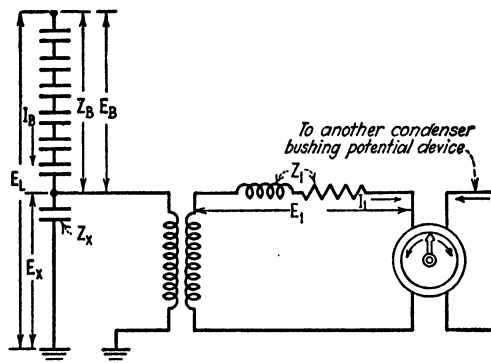


FIG. 28.—Schematic diagram of potentiometer circuit which gives improved characteristics.

improved potentiometer or potential device that is more nearly independent of the instrument or burden impedance, in so far as voltage and phase-angle variation is concerned. This reactor, when properly tuned, compensates for phase-angle errors caused by the instrument resistance, and the potential device gives a voltage proportional to and in phase with the line-to-ground voltage on the bushing. This is shown by the vector diagram, Fig. 29, which has been drawn from actual test data. At no load (open circuit) the reactor is ineffective since no current is flowing through it. Consequently the no-load voltage depends only upon the potentiometer ratio of the condenser bushing and the ratio of the transformer as shown above for the elemental device. That the relation between E_L and E is now approximately independent of the load impedance is evident from the following equation based on Fig. 28. In this case, as before, all constants

have been referred to one side of the transformer circuit. Let Z_B be the bushing impedance from the central conductor to the tap, and Z_X be the last layer impedance. Then, under all load conditions within the rating of the device, the bushing charging current may be considered as being $I_B = \frac{E_L}{(Z_B + Z_X)}$. For convenience in analysis assume $Z_B = -jX_B$ and $Z_X = -jX_0$. Let the reactor impedance Z_1 be pure inductance and equal to jX_0 . Then

$$\frac{E_L}{E} = \frac{-jI_B(X_B + X_0)}{-jI_B X_0} = \frac{X_B + X_0}{X_0} \quad (5)$$

Hence the voltage ratio is approximately independent of instru-

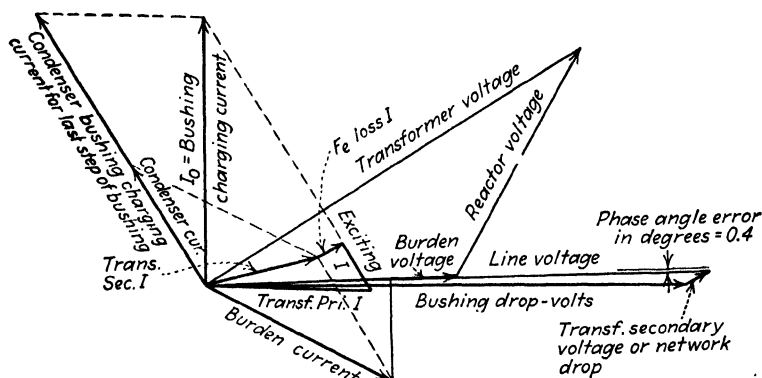


FIG. 29.—Vector analysis of potential device.

ment impedance, and the phase angle between E_L and E is approximately zero. The greater the number of steps in the bushing the better will be the approximation.

At times, the no-load voltage may be slightly higher than is desirable if good regulation is to be obtained. The addition of a small condenser across the transformer secondary causes the no-load voltage across the last layer, and consequently the no-load voltage of the device, to be lower than normal. This can be readily understood if we remember that in a condenser bushing the voltage across any one layer is inversely proportional to the capacity of that layer. In other words, if we place a small condenser across one layer of a condenser bushing, we cause the voltage across that layer to decrease in proportion to the amount of capacity added or in proportion to the total capacity resulting.

As shown in Figs. 30 and 31, the small condenser has been paralleled with the last layer through the transformer. If the transformer ratio is 10/1 and the capacity added is 1 microfarad, the effect on the bushing is the same as if 0.01 microfarad has

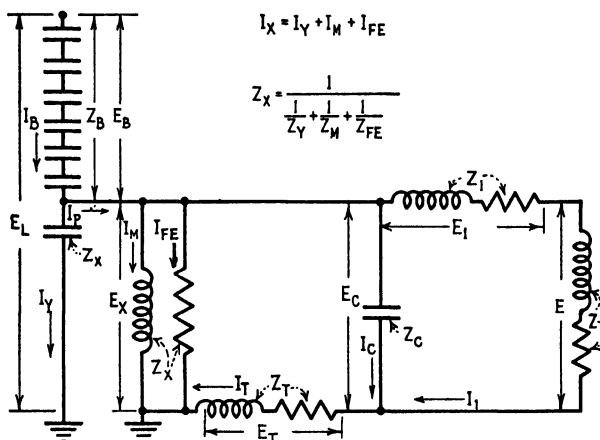


FIG. 30.—Complete schematic diagram of potential device.

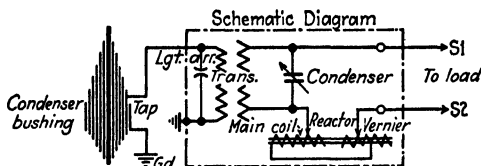


FIG. 31.—Circuit arrangement of potential device.

been paralleled directly across the last layer or through a 1/1 transformer.

The relation between E_L and E for the completed potential device taking into account the transformer impedance, as shown schematically by Fig. 30, is given by the following equation:

$$E_L = \frac{E}{Z} \left[\frac{1}{Z_C} \left(Z_B + Z_T + \frac{Z_T Z_B}{Z_X} \right) (Z_C + Z + Z_1) + \left(\frac{Z_B}{Z_X} + 1 \right) (Z + Z_1) \right] \quad (6)$$

All constants are referred to one side of the transformer. The exciting current is represented by the inductive component, the

iron loss by the resistive component, and the last-step capacity by the capacity component of Z_x . The impedance of the transformer is represented by Z_T placed back of the exciting impedance. The position of this impedance is somewhat in error, but its effect on the performance is very small.

Under normal conditions and within reasonable limits, the voltage across the burden remains fairly constant, regardless of burden impedance. As the burden is increased, the voltage drop across the reactor and across the last layer is increased. Hence the bushing operates with a variable voltage on the last step when used with the potential device. It is desirable to keep this voltage as near the normal operating voltage as possible, particularly under load conditions. This is done by means of the small condenser, since, as pointed out above, it causes the no-load voltage of the last step (and consequently of the potential device) to be lower than normal, thereby providing a liberal margin for the rise of voltage caused by the instrument load. By this means, it is possible to operate the potential device without in any way interfering with the normal operation of the bushing.

As has been stated, the voltage across the last layer increases with an increase in instrument burden on the potential device. One explanation of how this increase takes place is given by Eq. (4) when the total reactive impedance is approximately equal to the total capacitive impedance and a resonant condition is approached. As indicated by the above mentioned equation, the potentiometer ratio approaches *one* when resonance takes place and the voltage across the last layer, *i.e.*, the transformer-primary voltage, tends to approach the line-to-ground voltage across the entire bushing. A second explanation is as follows: The circuit ordinarily is predominantly capacitive, and at no load or light load the reactive impedance is insufficient to cause resonance. As the load is increased across the terminals of the device, the current through the reactor increases thereby increasing the effective reactive impedance in the circuit, and resonance is approached. When the device terminals are short-circuited, maximum current flows through the reactor and consequently there is maximum reactive impedance in the circuit which may be just sufficient to cause resonance.

When two impedances Z_1 and Z_c are connected in parallel, the total impedance Z_T is given by the following equation:

$$Z_T = \frac{Z_1 Z_c}{Z_1 + Z_c} \quad (7)$$

If Z_c is capacitive ($-Z_c$) and is approached in value by the reactive impedance Z_1 , it can be seen from Eq. (7) that the denominator approaches zero in value and the total impedance Z_T approaches infinity. Since the condenser bushing consists of several capacity impedances in series, the last one of which is paralleled with a reactive impedance, it is evident that the last layer impedance will tend to approach an infinite value when the reactive impedance is sufficient to cause resonance. In other words, when the load on the device approaches short-circuit conditions, the last layer impedance tends to approach an infinite value.

When a voltage is applied to any number of impedances in series, one of which approaches an infinite value as in the case just explained, the entire voltage tends to pile up across that impedance whose value approaches infinity. Theoretically, then, it is possible to obtain full line-to-ground voltage across the last layer of the bushing, if sufficient reactance is available in the potential device to cause resonance. The bushing potential device is so designed that, along with the inherent resistances and losses in the circuit, the above set forth conditions cannot be fully obtained. The theoretical discussion given will, however, help to explain why an increase in voltage across the transformer primary always takes place when load is applied to the device.

To prevent damage to the bushing caused by overloading the device, or short-circuiting its terminals, a small protective spark gap has been permanently connected across the transformer primary, as shown in Fig. 31. This gap functions well below the dangerous voltage, so that there is no possibility of the bushing being damaged. It furthermore acts as a protective device for the instruments connected to the secondary of the device protecting the instruments against abnormal line conditions such as would be caused by lightning, switching, etc. The operation of the gap due to abnormal line conditions or to an overload on the device is in no way detrimental to the long life of the bushing.

Potential-device Ratings.—The condenser-bushing potential device has a limitation in the maximum allowable secondary burden due to insulation limitation of the condenser bushing. Two types of device are available, each of which has the following limitations of burden depending upon line voltage.

Voltage class, kilovolts	Operating voltage, kilovolts	Burden, 60 cycle, 110 volt, volt- amperes
92	88	12 at 100 % P.F.
115	110	15 at 100 % P.F.
138	132	20 at 100 % P.F.
161	154	27 at 100 % P.F.
196	187	50 at 100 % P.F.
230	220	60 at 100 % P.F.
297	287	120 at 100 % P.F.

Quite often the power factor of the secondary burden is relatively low such as in the average induction-type relay. In such cases, the effective volt-ampere burden on the potential device may be reduced by improving the burden power factor with an additional condenser installed in the device.

As an illustration of the advantage gained by using condenser compensation, consider the application of 90° connected directional overcurrent relays, where the relay burden plus external resistor is 15.2 volt-amp. at 78 per cent power factor.

Relay burden in volt-amperes (110

volts)..... $11.8 + j9.45$

Minimum effective burden of relay. $11.8 + j0.$

Reactive component of relay burden $0 + j9.45$

Compensating condenser capacity $\cdot \frac{9.45 \times 10^6}{110^2 \times 2 \times 60} = 2.08 \text{ mf.}$

Thus a condenser of 2.08 mf. connected in parallel with the relay reduces its effective burden to 11.8 volt-amp. at 100 per cent power factor. The compensating condenser connected in parallel with the relay burden and tuned for 60 cycles will have no detrimental effect on the operation of the relays.

Generally, the performance of the standard silicon-steel potential device is satisfactory for most transmission-line relay

applications. At low line voltages, like those experienced when a fault is close to the bus, the phase-angle error is quite large, especially at the higher burdens, and may affect the operation of relays using the reactance principle for their operation.

A new potential device has recently been developed which has improved ratio and phase-angle characteristics. The device is similar to the one described above except that Hypernik is used in the core of the transformer and reactor. Owing to the high-permeability and low-loss characteristics of this material, the phase-angle errors are considerably less at the lower line voltages. The Hipernik device is applicable for energizing such burdens as reactance relays where greater phase-angle accuracy is required.

Capacitor Potential Devices.—Capacitor potential devices have been made which use the same potential-device network as already described except that a stack or string of carrier-current coupling capacitors are used in place of the condenser bushing. The capacitor units are arranged for suspension or pedestal mounting and connected phase to ground. Potential devices when used with capacitors are available at a rating of 100 volt-amp. for line voltages as low as 44 kv.

The capacitor potential device is well adapted for energizing impedance and reactance relays, in that the phase-angle errors over a range of 5 to 120 per cent of line voltage do not exceed 5°. In addition, it is applicable to various schemes of high-tension relay protection up to full rating, where relay accuracy is essential.

CHAPTER X

DIFFERENTIAL PROTECTION¹

Differential protection is a general system of protection based upon balancing the secondary current in current transformers so that an operating winding of a relay when bridged across this balanced circuit will have zero current during the normal power-flow conditions upon which the connections are based. Any change in the power-flow conditions which upsets the balance will necessarily cause current to flow in the operating winding of the relay. This general system of protection is applied to transformers, rotating apparatus, and to station and substation buses.

TRANSFORMER PROTECTION

The transformer represents such a large capital investment and is of such importance to the operation of the present-day power system that every means should be taken to supply it with protection, provided this protection does not interfere with the transmission-line relaying so as to cause undue outages. This means that such protection should be sensitive to transformer bushing or internal-winding faults and yet be inoperative for through faults.

There are three general classifications of abnormal conditions which should be considered:

1. Sustained overloads
2. Excessive temperatures
3. Internal faults

The first two of these types of abnormal conditions are similar, the difference being only a matter of time. It is the duty of the switchboard operator to cut into service additional transformers so as to prevent damage to the apparatus due to sustained overloads. Under peak-load conditions of relatively short duration, such as half an hour or possibly one hour, it is usually permissible

¹ General description and characteristics of relays taken from published data of the Westinghouse Electric and Manufacturing Company and General Electric Company.

to overload the equipment, provided the temperature does not become excessive. Temperature-indicating devices permit the operator to keep a check on the condition of the equipment. It is usually not desirable, however, to apply relays to disconnect the equipment automatically, as the overload may be but a temporary condition not justifying dropping the entire load.

Bushing and internal-winding faults, however, should cause the equipment to be relayed out of service as promptly as possible. Winding faults may be classified as phase-to-phase short circuits, short-circuited turns, open circuits, and grounds. Most winding faults develop as a ground on one phase winding. Although relays used for the protection of apparatus, as contrasted with transmission lines, are not frequently called upon to function, the importance of the apparatus which they protect warrants their use. Experience has proved the efficacy of ratio differential relays for the protection of transformers and generators, because of their speed of operation and the fact that, while remaining inoperative to relatively large through-fault currents, they are sensitive to small internal-fault currents.

Overcurrent Relays for Differential Protection.—A few years ago, it was common practice to use ordinary overload relays connected differentially between two sets of current transformers, as shown in Fig. 1, the current-transformer ratios being chosen so that, on through power flow, the secondary currents of the current transformers were as nearly equal as possible. No current would enter the relay circuit because of its relatively high impedance and because the secondary currents are balanced. Hence the relays are unaffected under normal load and under through-short-circuit conditions. An internal fault, however, causes an unbalance in the currents of the differentially connected current transformers, as shown in Fig. 2, and operates the overload relay. A hypothetical case has been assumed where the short-circuit currents from the 66-kv. and 13.2-kv. sides are such that the current-transformer secondary currents are equal. Ordinarily, of course, these currents are not equal, but even if no short-circuit power is available from one side, there will still be sufficient current from the other side to cause relay operation.

If the current transformers of exact inverse ratio to the power-transformer voltages are not available, then current-balance

autotransformers must be inserted in the secondary of the current-transformer circuit in order to balance the currents. This case is illustrated in Fig. 3. Current-balance autotransformers are also required if a delta connection is used on one set of current transformers and a star connection on the other set.

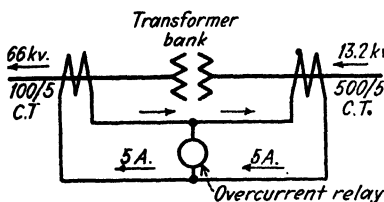


FIG. 1.

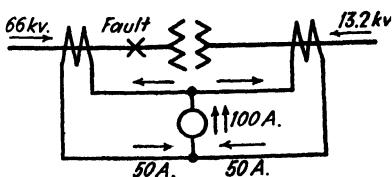


FIG. 2.

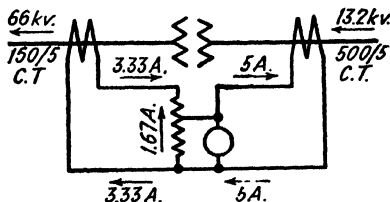


FIG. 3.

FIGS. 1, 2, and 3.—Schematic diagrams of transformer differential protection.

The equal-balance conditions just discussed obtain only under such ideal conditions as follows:

1. Fixed power-transformer ratio
2. Identical current-transformer ratio curves
3. Equal lengths of leads from each side of current transformers to the relays
4. Accurate selection of the proper taps on current-balance autotransformers, where such transformers are used

It is evident that such ideal conditions do not exist for installations of transformer-differential protection. The high-side and low-side current transformers are usually of different types having, of course, different ratio curves. It is usual to have bushing-type current transformers on the high-tension side of the transformer bank and wound-type on the low-tension side. Furthermore, the lengths of current-transformer leads are usually not the same, and there is a magnetizing component of current on one side which does not exist on the other. A given set-up may be fairly well balanced for normal load conditions, but a through

short circuit may easily unbalance the circuit so that, unless a high-ampere tap setting is used, sufficient current will be produced to trip the relay. The use of a high-ampere tap setting cuts down the sensitivity of a relay scheme, which means longer time to clear an internal fault.

Operating Principles and Characteristics of Percentage-differential Relays.—The percentage-differential relay is designed for the differential protection of alternating-current apparatus including motors, generators, and transformers. It is a current-operated induction relay so designed that its operating current varies in proportion to the load. The current required to operate the relay, assuming the instantaneous direction of currents in two sets of current transformers to be the same, increases in proportion to the line current instead of remaining constant for all load conditions. This feature of the relay tends to prevent faulty tripping on through faults, due to unbalanced current-transformer characteristics or loading, and allows the relay to be set for close protection at normal loads.

Other methods of differential protection generally operate on a constant difference between two currents, so that a slight difference in current-transformer ratios, or a slight error in balancing when magnified by a through fault, may permit circulating current to operate the relay incorrectly. The overcurrent relays used in such schemes must, therefore, be given either high-current settings or high-time settings to prevent faulty tripping. Either one of these corrections reduces considerably the effectiveness of this type of protection.

The Transformer-type Ratio-differential Relay.—The transformer-type ratio-differential relay is suitable for the protection of all two-winding transformers and some multiple-winding transformer applications. Since the current-transformer secondary currents of a power transformer are usually unequal, the transformer-type relay is provided with taps so that current-balancing autotransformers are unnecessary on two-winding transformer applications when the secondary currents are within a 2:1 ratio. The transformer-differential relay is less sensitive than the generator relay, being designed to operate on a fixed percentage differential of approximately 25 or 50 per cent. This prevents the relay from operating on unbalances caused by the transformer magnetizing-current rushes as well as a wide

difference in the current-transformer performance, as when wound-type transformers are balanced against bushing type. These effects are particularly pronounced in case of heavy through faults.

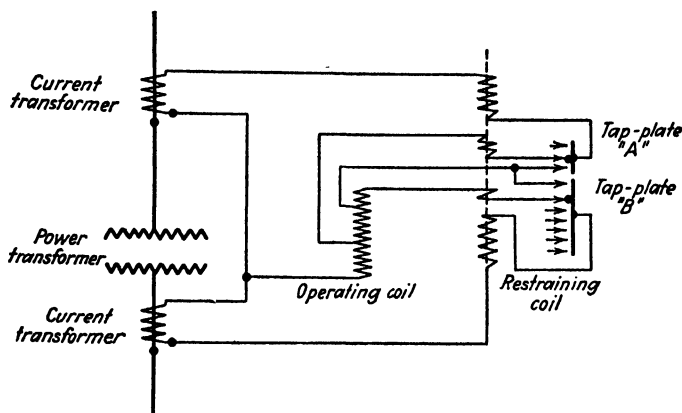


FIG. 4.—Schematic connections of one phase of a three-phase percentage-differential transformer relay.

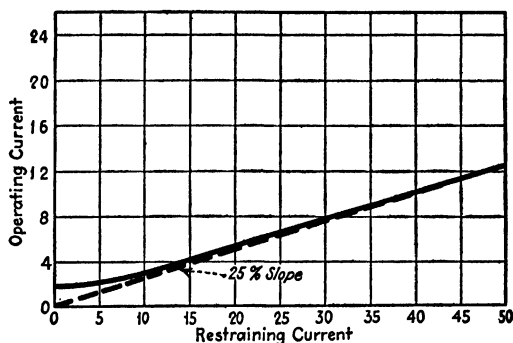


FIG. 5.—Operating characteristics of relay shown in Fig. 4.

In Fig. 4 are shown the connections for one phase of the three-phase percentage-differential transformer relay. Both the operating and restraining coils have taps so that any inequalities that exist, due to the impossibility of obtaining ideal ratios in standard or bushing transformers, may be balanced. Figure 5 gives the operating characteristics of the relay shown in Fig. 4.

A simplified diagram of a single-phase induction percentage-differential transformer relay is shown in Fig. 6, and operating characteristics of this relay are shown in Figs. 7, 8, 9, and 10.

A connection diagram for use with delta-delta connected power transformers is given in Fig. 11, the current transformers being

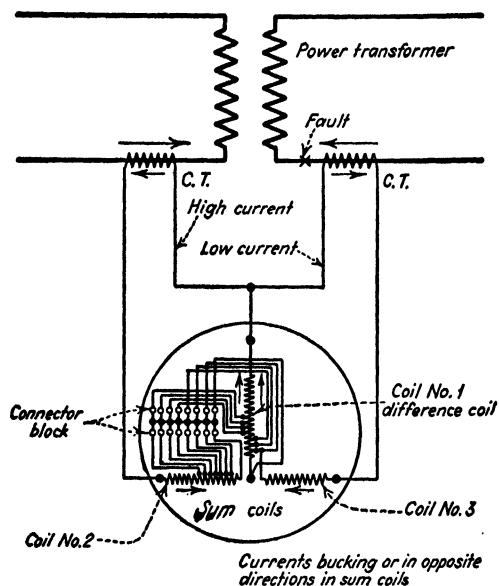


FIG. 6.—Simplified diagram of single-phase induction-type percentage-differential relay applied to single-phase transformer. Current flow indicates fault conditions.

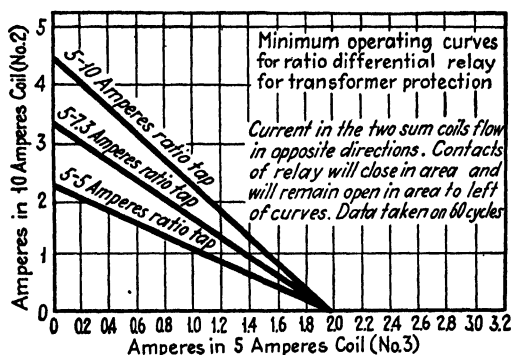


FIG. 7.—Sensitivity characteristics of transformer-differential relay when current flows into transformer fault from both sides of the bank, or when current from either side is zero.

connected in star. If the current-transformer ratios on the high and low sides are exactly the inverse ratio of the transformer voltages, then the 5-to-5 amp. tap on the relay can be used. If

the current-transformer ratios do not match the voltage ratio of the power transformers, then a balance can be effected within the relay by the use of taps which give a ratio range of 5-to-5 up to 10-to-5.

The final tap selected should be based on an analysis of current-transformer ratio curves which include values to at least 10 times normal load. In many cases values up to 15 and 20 times normal load will be required. The volt-ampere load of the complete circuit, including relay and conductors, should be determined to a fair degree of accuracy. The relay-characteristic curves should be checked to see that there is ample margin for

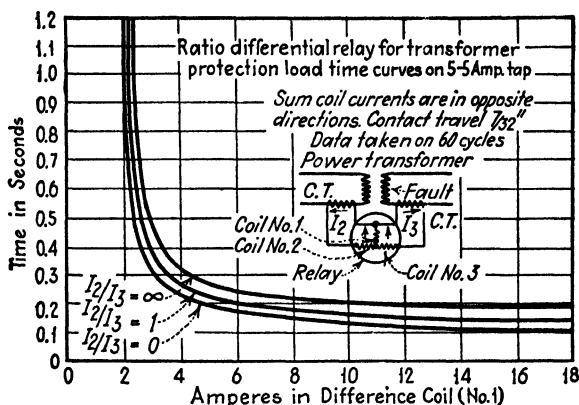


FIG. 8.—Operating time characteristics of transformer-differential relay.

maximum through short circuits. The operating margin for heavy through faults may be increased by taking advantage of the fact that the per cent unbalance required to operate the transformer-type ratio-differential relays at light loads is not exactly 50 per cent but is somewhat greater than this value. This margin over 50 per cent is present even up to normal full load, and it is therefore possible to set the relay at a tap somewhat off balance for normal conditions.

It is usually not necessary to make calculations as to the number of transformer turns required to be short-circuited in order to cause sufficient unbalance for relay operation. It will be found that on large transformers one or two short-circuited turns are all that is required. These calculations must be based on actual design data and no general limits can be determined as it depends

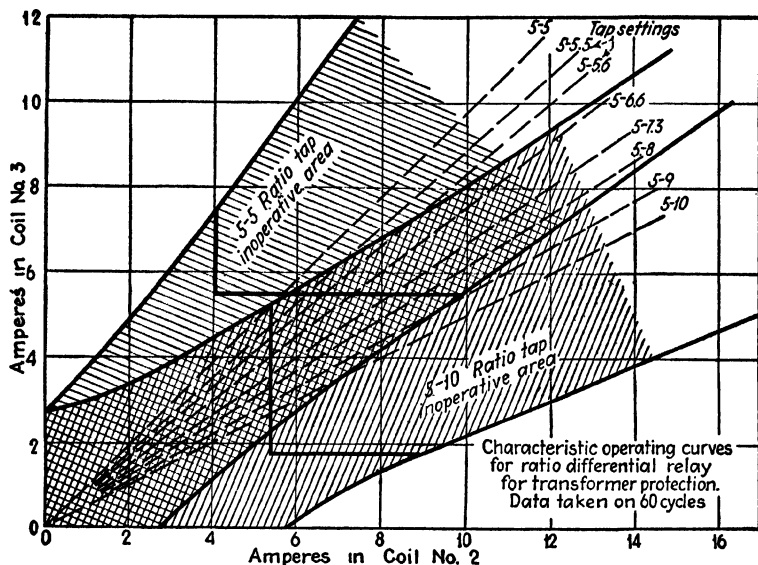


FIG. 9.

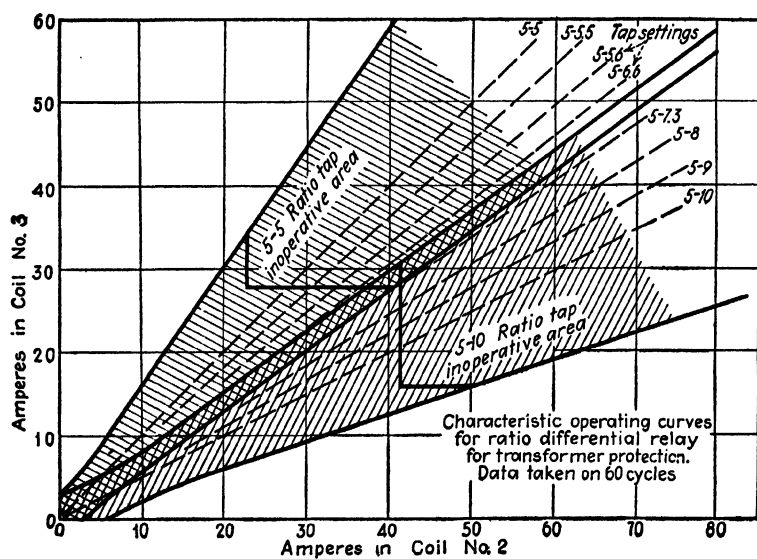


FIG. 10.

FIGS. 9 and 10.—Characteristic operating curves for transformer-differential relay. Current in coils 2 and 3 in same direction.

largely on the type and design of transformers. In tap-changing-under-load transformers, the exciting winding of the boost- and buck-regulating transformers has a kilovolt-ampere rating of only 10 to 20 per cent of the main windings, and hence special provision is often necessary in order to provide sensitive protection to the regulating transformer winding.

Delta-star Connected Transformer Banks.—A somewhat different problem presents itself for delta-star connected power-transformer banks. In making differential connections for such

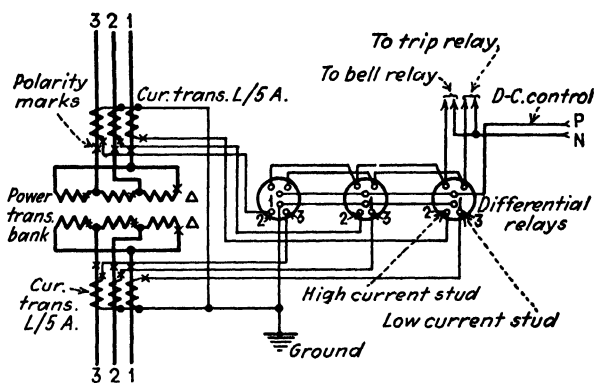


FIG. 11.—Diagram of connections for differential protection of delta-delta transformer bank.

installations, careful attention must be given to the phase relations of the current-transformer secondary currents. This is especially true where the current transformers are located outside of the delta connection of the power bank, as is usually the case. For these conditions, shown in Fig. 12, the current transformers on the star side of the power-transformer bank are connected in delta and balanced against the star-connected current transformers on the delta side of the power-transformer bank. This gives the proper phase relation, not only for balanced three-phase conditions, but also for line-to-line and line-to-ground through short circuits.

The question as to how to properly connect a delta-connected set of current transformers to a star-connected set so as to get the correct circulating currents between the two can be easily determined by the use of vectors, as follows.

In Fig. 12, polarity marks are indicated on the main transformer windings as well as on the current-transformer windings. The end of the windings on which the polarity mark is placed will be considered as plus and the opposite end of the winding will be considered as minus. The voltage and current vectors for the

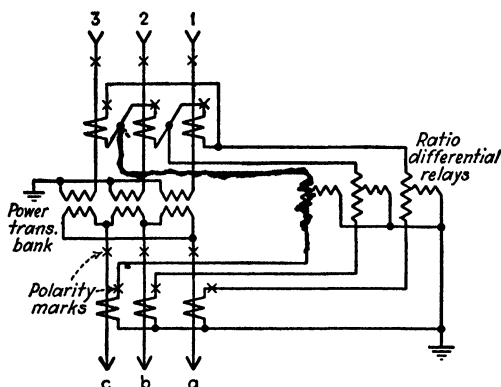


FIG. 12.—Schematic diagram of differential protection applied to Y-delta transformer bank.

star-connected main transformer winding are shown in Figs. 13 and 14. In Fig. 13 they are represented by the closed-delta system of vectors and in Figs. 14 by the open system of vectors.¹

For the delta-connected main transformer winding, the vectors are represented by the closed-delta system as in Fig. 15, and the

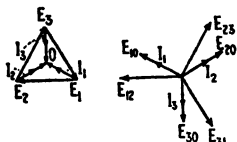


FIG. 13.

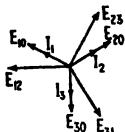


FIG. 14.

FIGS. 13 and 14.—Vector analysis of Y-connected main transformer winding, Fig. 12.

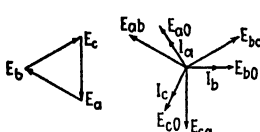


FIG. 15.

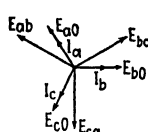


FIG. 16.

FIGS. 15 and 16.—Vector analysis of delta-connected main transformer winding, Fig. 12.

voltage and current vectors are represented by the open system of vectors in Fig. 16. It should be carefully noted that the current vectors shown are the line-current values.

The problem is to connect the current transformers so that the proper circulation of current between the two sets of current transformers will result, *i.e.*, that these currents be in phase.

¹ The fundamental vector notations are explained in Chap. VIII.

The vector relations of the current-transformer secondary currents are shown in Figs. 17, 18, and 19.

For example, the current I_a in the secondary of the current transformer in lead a on the delta-connected side of the bank is shown in Fig. 17, and it is desired to find a current from the secondaries of the current transformers on the star-connected side of the bank which will be in phase with I_a . In Figs. 14 and 17, it is shown that the vector sum of $-I_1$ and $+I_3$ gives a value I_{3-1} which is 180° out of phase with I_a , and this is the desired phase relation. This resultant vector I_{3-1} is, of course, obtained by connecting the positive end of the current transformer in leg 3 to

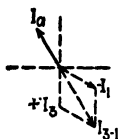


FIG. 17.

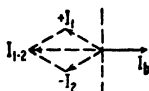


FIG. 18.

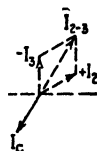


FIG. 19.

FIGS. 17, 18, and 19.—Vector analysis of current-transformer connections, Fig. 12.

the negative end of the current transformer in leg 1. The resultant current I_{3-1} will be greater than I_a by the ratio of 1.73:1, provided the current transformers are the exact inverse ratio of the power-transformer ratios, but this difference can be taken care of by selecting the proper taps on the relay.

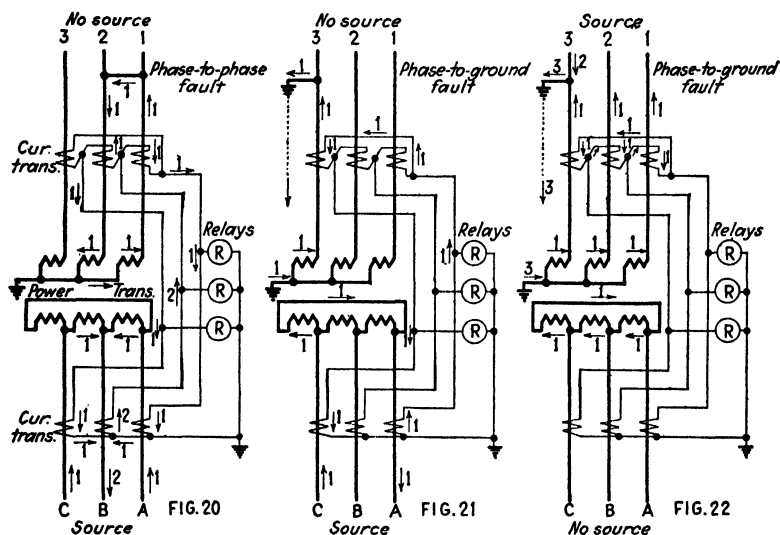
In the same way I_b is balanced by the resultant vector I_{1-2} as shown in Fig. 18, and I_c is balanced by the vector I_{2-3} shown in Fig. 19. The delta connection of the current transformers in the star-connected side of the main transformer bank is then made to conform to these results, shown in Fig. 12, and the connections are completed through the ratio-differential relays to the star-connected current transformers on the delta side of the bank.

This vector analysis is based on subtractive polarity of both the main power transformers and the current transformers. If the polarities of the power transformers and the current transformers should be different, the delta must be formed accordingly.

The objection to the use of star-connected current transformers on the star side of a star-delta transformer bank arises if the power-transformer neutral is grounded. Under this condition the bank supplies ground or zero-phase sequence current to through ground faults on the star side. With star-connected

current transformers the only path for this secondary current is through the differential relays causing them to trip off the transformer bank. On the other hand, with delta-connected current transformers on the star side the secondary zero-sequence current circulates through the current-transformer secondaries instead of through the relays.

Figures 20 to 25¹ illustrate the current flow for various through faults. Figures 20, 21 and 22 are for the standard connection

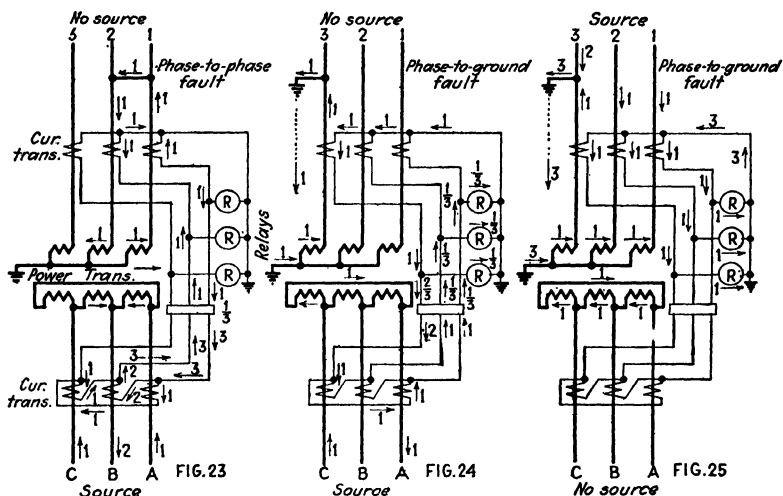


FIGS. 20, 21, and 22.—Analysis of through-short-circuit current flow of power-transformer bank with differential protection properly connected.

which does not cause relay action. Figures 23, 24, and 25 are for corresponding conditions of through faults if the current transformers are star-connected on the star side of the bank and delta-connected on the delta side of the bank. This connection gives rise to relay current in case of a through ground fault as illustrated by Figs. 24 and 25. On account of the double-delta transformation on one side of the transformer bank, this secondary current is three times the secondary current on the other side of the bank, and hence, assuming a 1:1 transformation for the power and current transformers, a set of auxiliary 3:1 current transformers are required to balance the secondary currents.

¹ Acknowledgment is given to E. H. Klemmer for this analysis.

With the standard differential connection as shown in Figs. 20, 21, and 22, the auxiliary current transformers are not required as there is a delta transformation on each side. For simplicity, phase-to-phase and phase-to-ground faults only were considered, since, being single-phase faults, they give rise to in-phase currents throughout, which avoids the complications of a vector analysis.



FIGS. 23, 24, and 25.—Same analysis as shown in Figs. 20, 21, and 22 showing incorrect relay operation in Figs. 24 and 25 due to incorrect current-transformer connections.

It may be of interest to note that the proper phasing of the connections involved in the differential protection can readily be checked by this method. A complete check would involve the consideration of two combinations of phase-to-phase faults, *i.e.*, 1-2 and 2-3, and also a phase-to-ground fault, *i.e.*, 1-G. If the connections satisfy these three conditions, they will also satisfy the three-phase fault condition or combinations of phase-to-phase and ground faults.

Of course, in an actual installation, connections should be checked by test to insure that the phase relations are correct. This can easily be done by the use of a portable phase meter. Connect one voltage coil to any convenient 110-volt circuit, as in the test connections (Fig. 26), and connect the current coil in

series with the secondary circuit of the current transformers, first on one side of the ratio-differential relay and then on the other. For each of these two series connections, note the reading of the pointer on the scale of the phase meter. If the current transformer and relay connections have been properly made, the two readings should be practically the same except for slight divergence of a few degrees due to the magnetizing component. This, of course, is unbalanced and flows through the relay. If the readings are out of phase, to any extent, the connections should be checked and changed so that the currents are in phase.

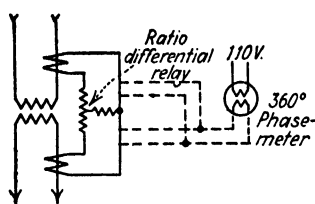


FIG. 26.—Schematic test connections for differential protection.

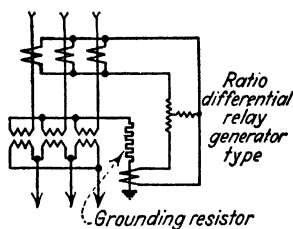


FIG. 27.—Schematic diagram of sensitive differential protection of ground faults.

This should be repeated on each of the three phases. It must be remembered that the portable phase meter is an approximate device and requires rated current and voltage to produce anything close to accurate results. If such full load current is not available, a correction curve of the meter should be used.¹

SPECIAL CASES OF DIFFERENTIAL PROTECTION

Ground Faults.—Protection against a fault to ground within a transformer can ordinarily be taken care of by the usual connection of ratio-differential relays. They are sufficiently sensitive to make special protection unnecessary in most cases where the neutral of a bank of transformers is grounded through a resistor. There are, however, certain exceptions where the neutral resistor has particularly high resistance, or where there is short-circuit power available only from the grounded side. A scheme of protection is shown in Fig. 27 which can be added to the regular phase-to-phase differential protection in order to get sensitive protection for a fault to ground within the transformer. The

¹ Refer to Chap. XV for further test information.

ratio-differential relay used should be of the generator type, as it is more sensitive than the transformer type.

Current Transformers within the Delta.—The use of transformer-bushing-type current transformers on the delta-connected side of a transformer bank places the current transformers within the delta, and a special connection is therefore necessary. These connections for a star-delta connected transformer bank are shown in Fig. 28. The necessity of using the delta connection of current transformers on the star-connected side of the bank is

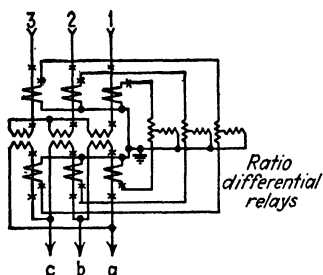


FIG. 28.—Schematic diagram of differential protection of Y-delta transformer bank with current transformers within delta.

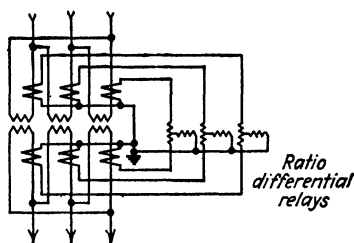


FIG. 29.—Schematic diagram of differential protection of delta-delta transformer bank with current transformers within delta.

eliminated. However, this scheme has a decided weakness, in that it protects only the windings and does not include the bushings. Wherever possible, current transformers outside the delta should be used so as to get overall protection of the transformer windings and bushings. If either side of the transformer bank is connected to a system having the neutral grounded through a resistance at the source, ground-current protection should be included in addition to the differential protection as shown in Fig. 27.

Differential-relay connections for a transformer with both windings connected in delta and with current transformers inside the delta are shown in Fig. 29. Ground-current protection should also be added in case either side of the bank is connected to a system having the neutral grounded through a resistance at the source.

Zigzag-grounding Transformers.—The complete protection required on a zigzag-grounding transformer is shown in Fig. 30. The ratio-differential relay used to protect against internal faults

should be of the generator type. The ratio curves of the neutral transformer should be approximately the same as those of the line transformers, all current transformers having the same ratio. In order to prevent a dangerous temperature within the transformer from being reached while delivering power to through short circuits, a thermal relay is used. It should be given a time setting the same as the time rating of the transformer, usually one or two minutes. A special-gear 40-sec. overcurrent relay may be used in place of the thermal relay, for some applications.

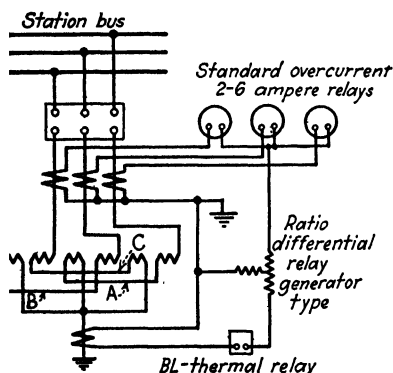


FIG. 30.—Differential protective scheme for zigzag-grounding transformer.

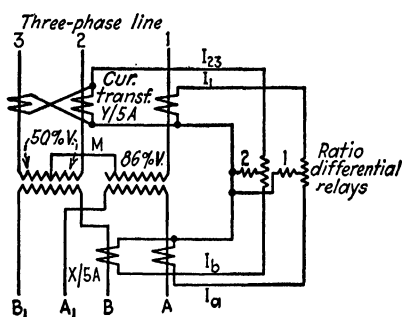


FIG. 31.—Scott-connected transformer differential protective scheme.

Scott-connected Transformers.—A diagram of connections for the application of differential protection to a Scott-connected transformer bank is shown in Fig. 31. The vector analysis of the voltages and currents is given in Fig. 32. In this latter diagram, *a* is drawn for the three-phase line, *b* for the three-phase side of the transformer bank, and *c* for the two-phase side of the transformer bank. The current vectors applied to relay 2 are shown in *d*, and those applied to relay 1 are given in *e*. These vectors were determined in a manner similar to that explained for Figs. 13 and 19.

Relay 1 can be connected directly to the current I_1 from the three-phase side and the current I_a from the two-phase side, provided, of course, the proper current taps in the relay are used. For relay 2, the current I_b from the two-phase side must be balanced by the cross-connected current transformers in phases 2 and 3 of the three-phase side. This is shown in Figs. 31 and 32.

The method of determining the proper current tap to be used in the relays is as follows.

Let X represent the ratio of the current transformers on the two-phase side and Y represent the ratio of the current transformers on the three-phase side. From the fact that the total volt-amperes in the three-phase line are approximately equal to

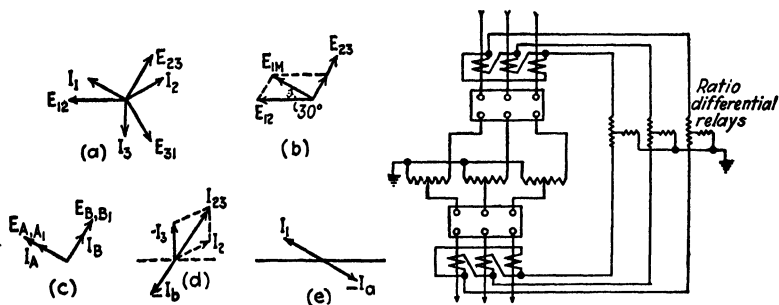


FIG. 32.—Vector analysis of differential protection for Scott-connected transformer bank.

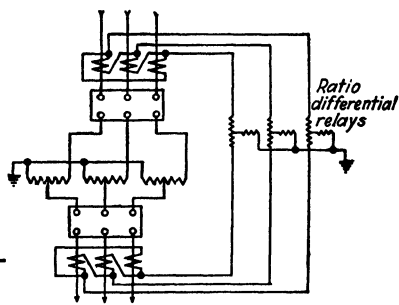


FIG. 33.—Autotransformer differential protection.

the total volt-amperes in the two-phase line, the following formula, applicable to relay 1, can be derived:

$$\frac{I_1}{I_a} = \frac{E(2\text{-phase}) \times X}{E(3\text{-phase}) \times Y \times 0.866} \quad (1)$$

Taps on the *sum* coils of relay 1 must then be chosen in this ratio. A similar formula for relay 2 is as follows:

$$\frac{I_{23}}{I_b} = \frac{E(2\text{-phase}) \times X \times 2}{E(3\text{-phase}) \times Y} \quad (2)$$

It will be noted that this balance ratio for relay 2 is equal to $\sqrt{3}$ times the balance ratio of relay 1.

Autotransformers.—Connections for differential protection of an autotransformer are shown in Fig. 33. Since these transformers usually have interlacing tertiary windings to form a path for the flow of zero-sequence current, it is essential that the current transformers be connected in delta. The tertiary windings are not shown in the diagram, as they do not ordinarily have any external connections. If the current transformers were connected in star, relay operation would be caused by the flow of zero-sequence current.

Percentage-differential Relays for Three-winding Transformer Protection.—Two types of four-element (three restraining coils and one operating coil) percentage-differential relays are available for the protection of three-winding transformers. Three single-phase relays are required for the protection of a three-phase three-winding bank of transformers. These relays differ from the percentage-differential relay applied to two-winding transformer banks in that the four-element relays have no internal-current-balance autotransformers. If the current transformers used with the relay will not give equal secondary currents in the correct phase relation, when properly connected,

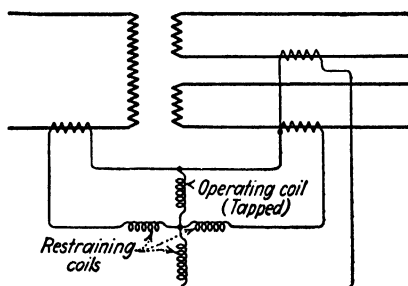


FIG. 34.—Schematic diagram of percentage-differential protection of three-winding transformer.

external auxiliary current-balancing autotransformers are required. Usually, two sets of autotransformers (three each) are used, although, in some instances, one set may be sufficient.

One type of percentage-differential relay for three-winding transformers is provided with percentage-ratio taps of 15, 25, and 40 per cent. The second type is designed for a 25 percentage ratio. The choice of a particular tap is based upon the performance of the current transformers under through-fault conditions and upon the severity of the magnetizing-current inrush of the power transformer to be protected.

In some cases, modern power transformers give very high values of magnetizing-current inrushes when the first breaker is closed on a three-winding transformer. The magnetizing inrush may be of too high a magnitude and too long a duration for the pick-up value of these relays. In such cases it is necessary to provide auxiliary time-delay features to restrain the action of the relay until the magnetizing-current inrush has diminished below the pick-up value of the relay.

In Fig. 34, is shown a schematic diagram of connections for the percentage-differential relay applied to a three-winding transformer. The terminal connections of the 25 per cent-slope relay are given in Fig. 35, and the characteristics of this relay are

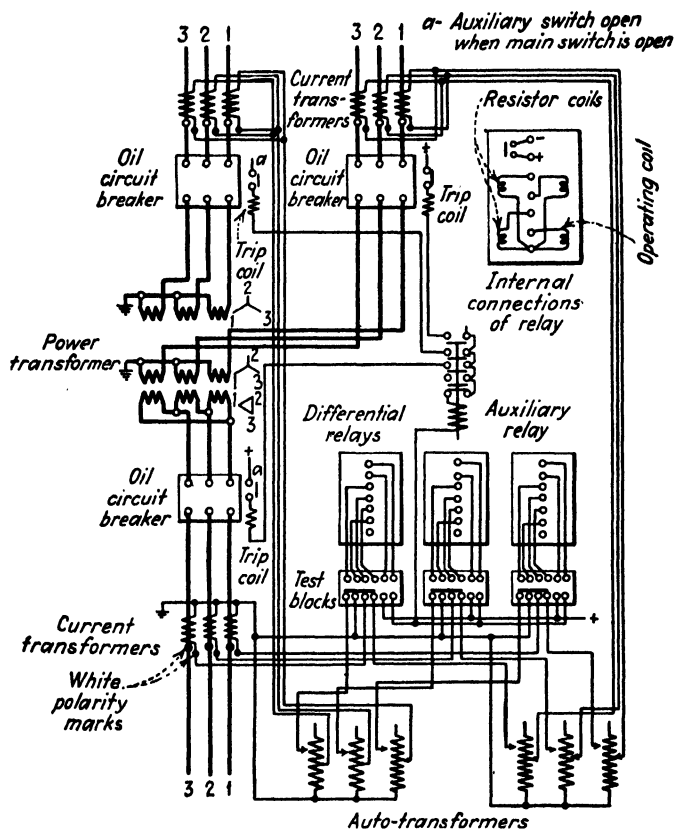


FIG. 35.—Terminal connections of 25 per cent-slope relay for differential protection of three-winding transformer.

shown in Fig. 36. Figures 37, 38, and 39 give the characteristics of the 15, 25, and 40 per cent-slope relay, and Fig. 40 gives the connection diagram for this relay as applied to a three-winding transformer.

A typical diagram for a transformer having two star-connected windings and a delta tertiary used only as an interlacing winding

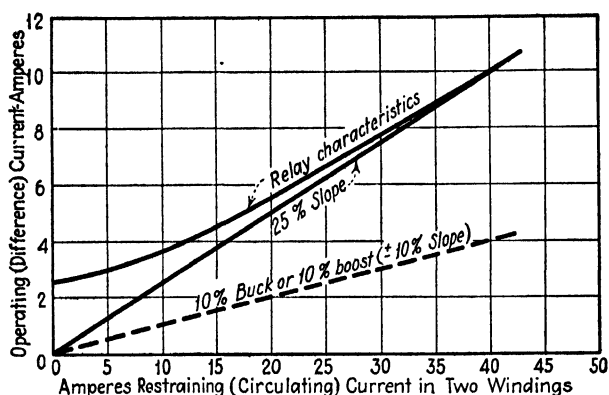


FIG. 36.—Operating characteristics of 25 per cent-slope differential relay.

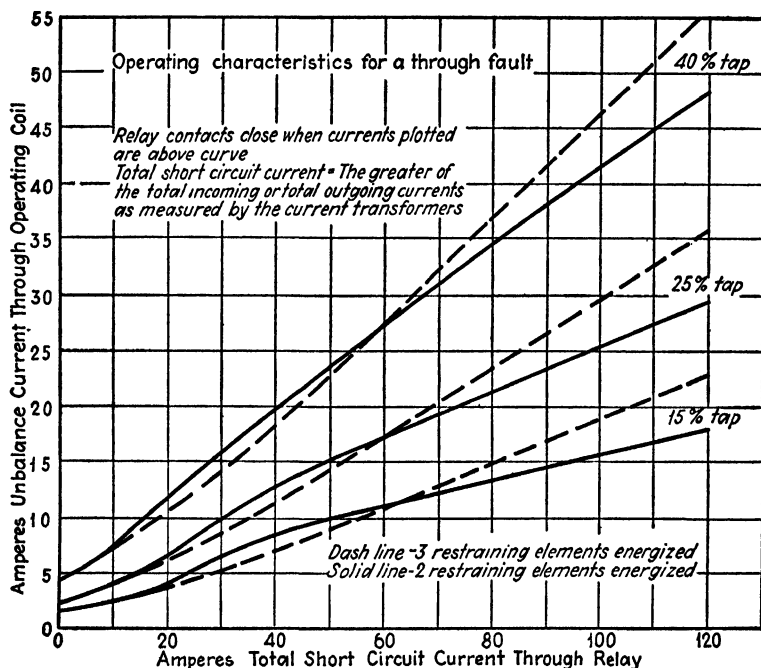


FIG. 37.—Operating characteristics for a through fault when 15-25-40 per cent relay is used for protecting three-winding transformer.

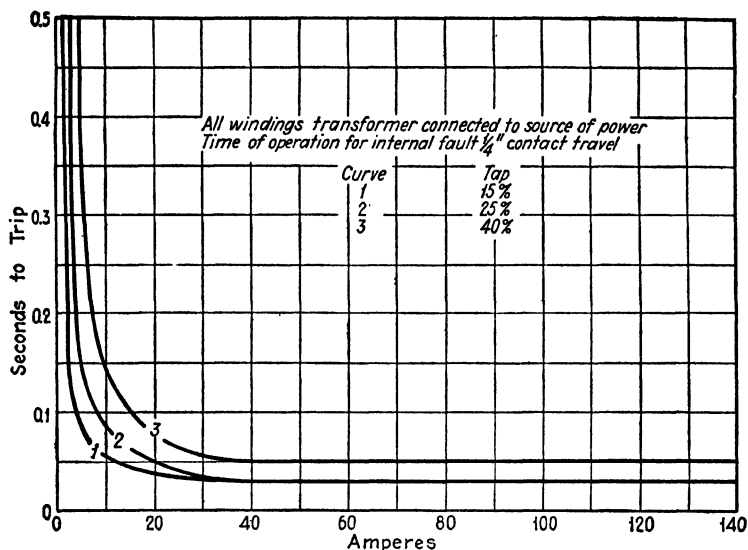


FIG. 38.—Time-ampere curves of 15-25-40 per cent relay for protecting a three-winding transformer, when all windings are connected to source of power.

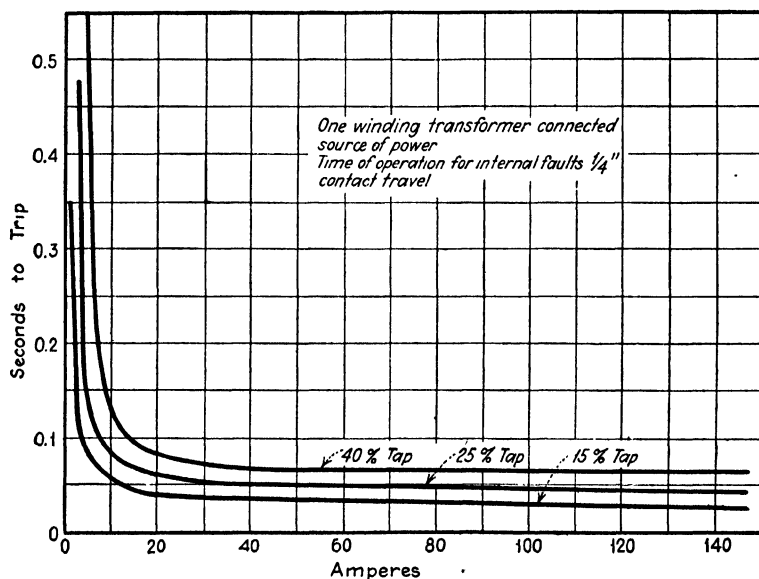


FIG. 39.—Time-ampere curves of 15-25-40 per cent relay protecting a three-winding transformer, when one winding is connected to source of power.

is shown in Fig. 41. As in the connections for an autotransformer, such a transformer connection requires that the current transformer be connected in delta on each side of the bank. If this is not done, an external ground fault will cause circulating

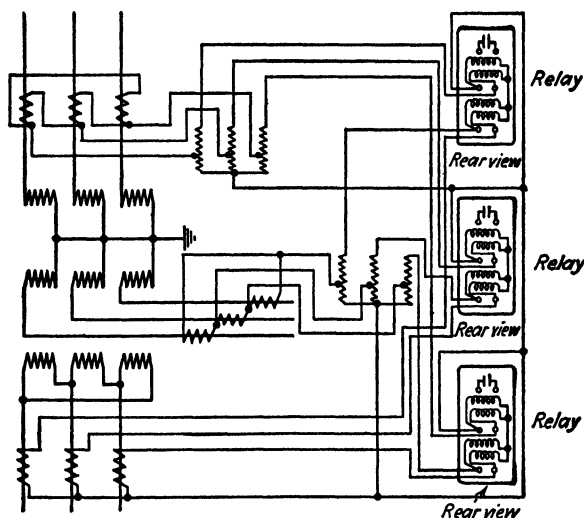


FIG. 40.—External connections for protecting three-winding transformer with 15-25-40 per cent relay.

current in the delta which, in turn, will produce zero-sequence current in the operating coil of the differential relays of sufficient magnitude to operate them. If delta-connected transformers are

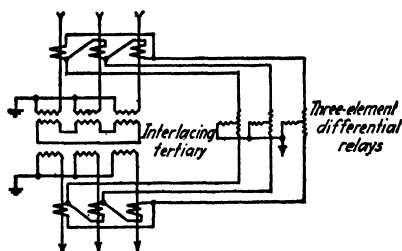


FIG. 41.—Schematic diagram of differential protection for Y-Y-connected transformer.

used, this zero-sequence component will circulate in the secondary delta of the current transformers and will not appear in the relays.

In case the delta-tertiary winding is used for power purposes, current transformers must be added to this winding and four-

element relays must be used. If the current transformers are placed outside the delta connection of the tertiary winding, the current transformers for the tertiary winding must be connected in star as in Fig. 42.

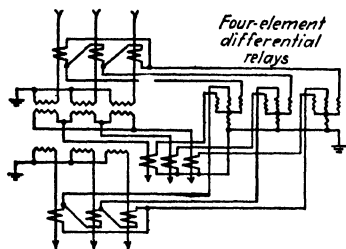


FIG. 42.

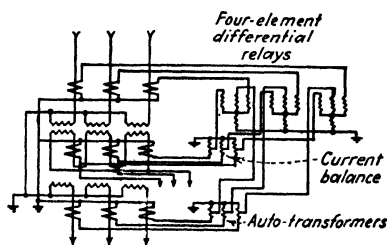


FIG. 43.

FIGS. 42 and 43.—Schematic diagrams of differential protection for three-winding transformers.

On the other hand, if these current transformers are placed within the delta of the tertiary winding, two alternatives are available. The first method is to connect the current trans-

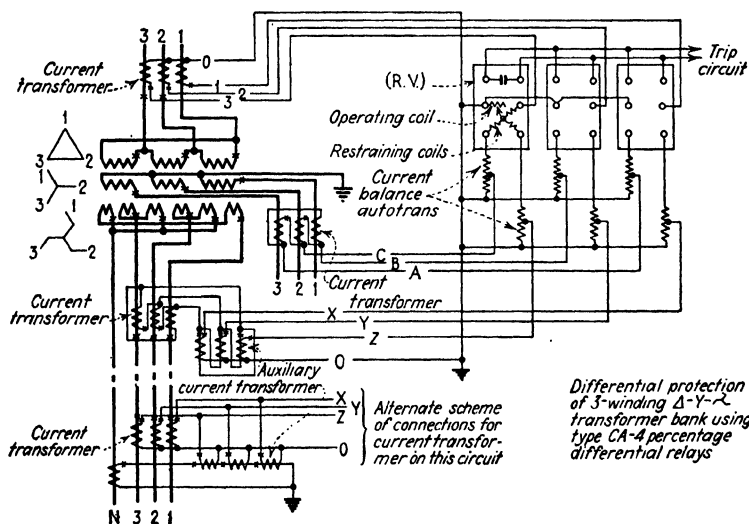


FIG. 44.—Differential protection of a delta-Y-zigzag transformer.

formers in delta the same as is done on the two star-connected transformer windings. A second method, more commonly used, is to connect all the current transformers in star, as shown in Fig. 43. For cases where the current transformers are not in

exact inverse ratio to the voltage ratio of the transformer windings, it is necessary to use current-balance autotransformers.

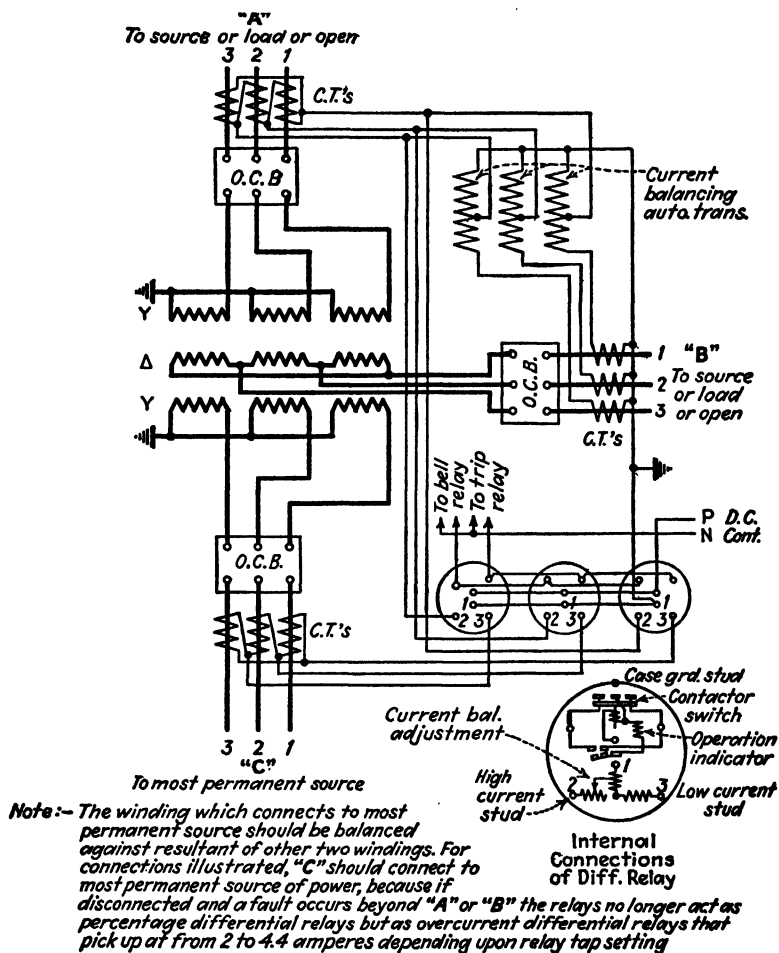


FIG. 45.—Differential protection of three-winding delta transformer (one Y winding connected to most permanent source) using three percentage-differential relays.

A special case of three-winding differential protection for a delta-Y-zigzag transformer is shown in Fig. 44.

It is not essential in all cases to use a four-element ratio-differential relay for three-winding transformers, provided a careful study of short-circuit current values is made. If the proper

connections are not made when three-element relays are used, it is possible to get a condition where the ratio-differential relay acts merely as an overcurrent relay. This is illustrated in Figs. 45 and 46.

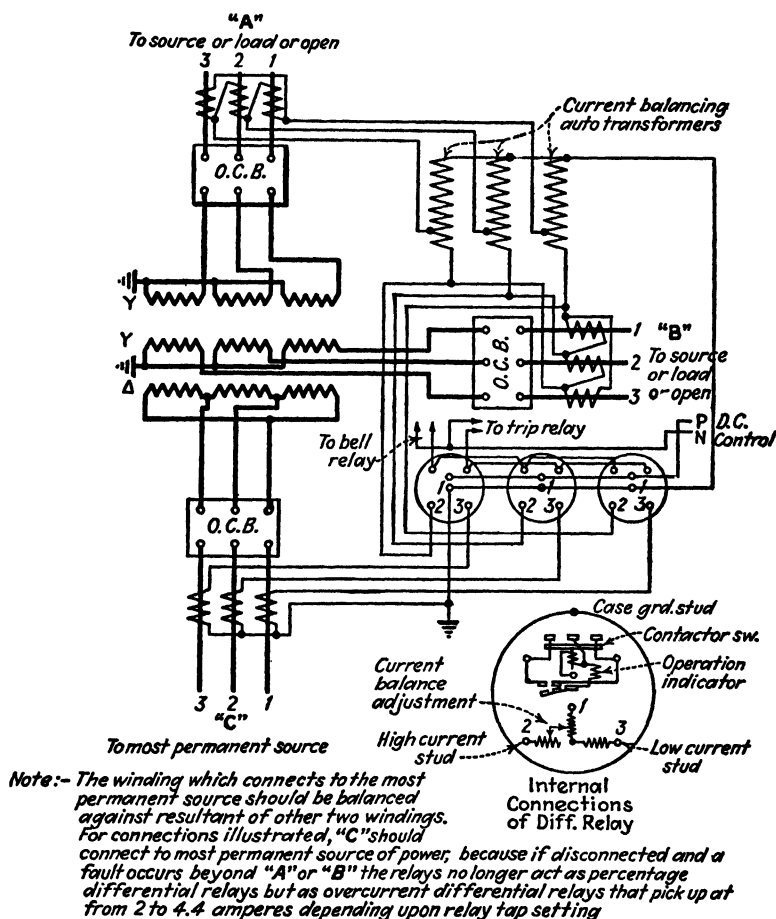


FIG. 46.—Differential protection of three-winding Y-Y-delta transformer (delta winding connected to most permanent source) using three percentage-differential relays.

Protection of Regulating Transformers.—When tap-changing equipment is provided to operate taps on the winding of a power transformer, this equipment may be included within the main-

transformer differential protection if a percentage-differential type of relay is used. There are, however, many installations of tap changing under load equipment using a separate regulating transformer having a kilovolt-ampere rating of 10 or 20 per cent of that of the main transformer bank. The overall differential protection will therefore not give sensitive protection against faults within the regulating transformer.

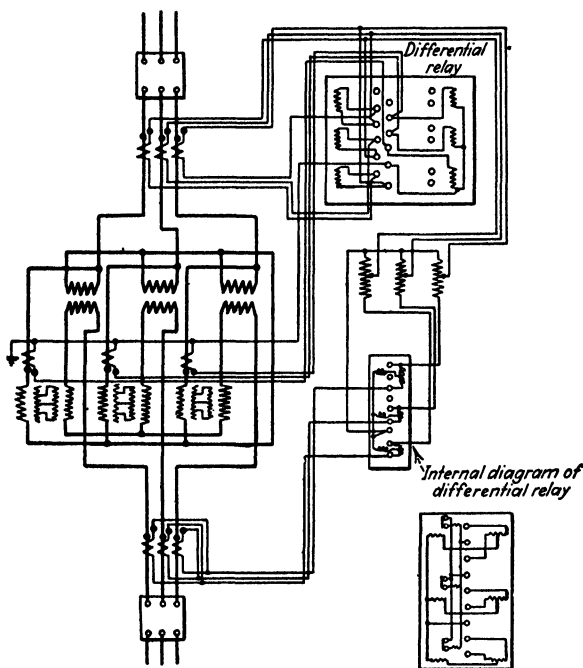


FIG. 47.—Percentage-differential relay connections for the protection of a regulating transformer.

In Fig. 47 is shown a scheme of connections wherein the current flowing into the exciting winding of the regulating transformer is balanced against the current flowing through the primary connections of the power transformer. This is accomplished by connecting the operating coil of a percentage-differential relay to the exciting winding and the restraining coil of this relay to the primary leads. The coils of the relay are so proportioned that a tripping action occurs when the current in the operating coil is

50 per cent greater than that in the restraining coil. The relay will receive no current from the exciting winding of the regulating transformer when this winding is in the zero buck or boost position, but any current up to full load of the exciting winding will flow when set for buck or boost position. The percentage-differential feature of the relay prevents operation of the relay for through faults while the regulating transformer is in the maximum buck or boost position. In order to take care of inrush currents at the time the transformer is placed on the line, it is common practice to adjust the relay to operate when one and one-half to two times full-load current of the exciting winding flows in the operating coil of the relay when there is no current present in the restraining element.

It is also necessary to use an additional current-differential relay for the overall transformer protection. These currents will differ by a percentage depending upon the position of the regulating unit. Any faults to ground in the series winding will be detected by this protection where no operation would be had from the relays in the exciting winding if the unit happened to be on a zero or very low percentage position.

Inrush Current When Transformer Bank Is Energized.—If a 50 per cent slope-ratio differential-induction type relay is used and the contacts set with larger than normal gap, it will usually be found that any difficulty due to the energizing of one winding of the transformer bank (as is the case when being cut into service) and its resultant large inrush current will be overcome by the inherent time lag in the relay. This time lag is not serious because if a fault should develop at the time the bank is energized, the short-circuit current will speed up the relay appreciably and minimize the damage due to the short circuit.

Later in this chapter it is shown that when high-speed differential protection is used (applied to those systems where stability demands high-speed relaying of lines and equipment), it is necessary to add desensitizing relays to prevent the differential relays from operating during the first inrush. In very special cases where the induction normal-speed type of differential relay is subject to tripping with completely open contact gap, desensitizing equipment can be added. This may be particularly the case with certain three-winding transformers protected with the four-element relay.

PROTECTION OF GENERATORS AND ROTATING APPARATUS

The protection of generators and other rotating apparatus is as important as that of transformers and is similar to the protection of transformers in a number of respects. Sustained overloads and excessive temperatures can usually best be taken care of by the switchboard operator. Internal-winding faults, however, should cause the apparatus to be disconnected from the line as quickly as possible. Winding faults in generators, as in transformers, may be classified as phase-to-phase short circuits, short-circuited turns, open circuits, and grounds. Armature failures usually start as a ground on one of the phase windings and quickly develop into one or more of the other types of faults, if the generator is not disconnected promptly. Serious fire and damage to the unit will result, if the machine is not disconnected during the very inception of the fault.

In order to obtain the most effective differential protection the neutral of the generator should be grounded either solidly or through a resistance. Reactance grounding is also sometimes used. Where a number of generators at one station, are operated in parallel, provision is usually made for the grounding of any one generator of the group.

In order to get as much ground current as possible for operating the relays, it is preferable to ground the neutral solidly. However, there are other considerations which determine whether the ground should be solid or through resistance. Increasing the amount of ground resistance will, of course, reduce the magnitude of the short-circuit current and will tend to keep the voltage from dropping to a low value and thus assist in preventing synchronous machines from dropping out of step. If feeders are connected directly to the generator bus without intervening transformers, a resistor ground should be used to eliminate the "bump" on the generators during feeder-line-to-ground faults.

On generators that are not grounded at all, differential protection is limited in its value, as an internal fault to ground must be sufficiently severe to develop into a phase-to-phase fault before the differential relays will operate. The possibility of damage to the machine is, therefore, increased. Also, at the time of a ground fault, an ungrounded generator may be subjected to considerable voltage stress due to the flow of unbalanced charging

current and this voltage stress may cause a phase-to-phase flashover.

In order to protect the generators satisfactorily, the differential protection should meet the following requirements:

1. It must be sensitive to failures in the generator armature winding and not affected by short circuits outside of the machine.
2. It must function rapidly so as to disconnect the generator before serious damage can result.
3. It must be arranged to trip out the main breaker, neutral breaker, field circuit breaker, and, in the majority of cases, it should trip the emergency steam valve.

Overcurrent Relays for Generator-differential Protection.—Overcurrent relays have been used for differential protection of generators for many years. A schematic diagram of this connection is given in Fig. 48, the protection being shown for only one phase, as that for the other phases is similar. This form of protection consists of balancing the current entering the apparatus against the current flowing from it and is accomplished by connecting a current transformer at each end of each phase winding. The secondaries of each pair of current transformers are connected in series, and the overcurrent relay is connected across these secondaries.

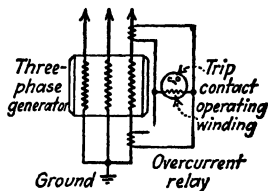


FIG. 48.—Schematic diagram of overcurrent-differential protection applied to one phase of a three-phase generator.

Even though identical wound-type current transformers are used on each end of the generator winding there is normally a small current flowing through the relay. This current may be due to minor variations in the current-transformer ratio curves and to differences in the loadings of the current-transformer secondaries. Unequal loading may be caused by unequal lengths of leads to the relay, or by additional relay or meter load in series with one of the current transformers and not the other. Though normally small, this current through the relay may become appreciable during a through short circuit. In order to prevent the relays from operating under through-fault conditions, it is necessary to raise the operating point of the relay. This obviously reduces the sensitivity of the scheme and requires more current and a longer time for the relays to respond.

The Generator-type Percentage-differential Relay.—The generator-type percentage-differential relay can be set to operate on internal-fault currents as low as 2.5 per cent of the smaller secondary current. It is usual practice to set the relay on the basis of 5 or 10 per cent slope rather than 2.5 per cent in order to preclude any possibility of the relay tripping on heavy overloads or system faults.

In Fig. 49 is shown a schematic diagram of connections of a single-phase percentage-differential relay applied, for simplicity's

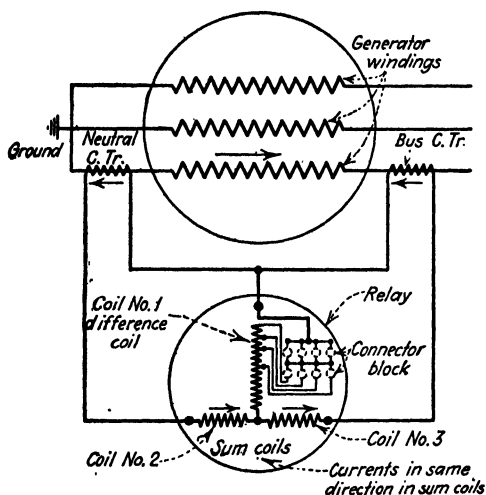


FIG. 49.—Schematic diagram of percentage-differential relay connections, for simplicity of illustration, applied to one phase of a three-phase generator.

sake, to but one phase of a generator winding. Under normal conditions current passes through the current transformers, through coils 2 and 3 in series, and back to the current transformers. The currents in the *sum* coils 2 and 3 are additive and produce an opening torque. When the secondary currents from the current transformers become unbalanced due to an internal fault in the protected apparatus, or to inherent differences in current transformers and differences in secondary-current-transformer loading, the differential current must pass through coil 1 to the common junction of the current-transformer secondaries. Coil 1 is the operating winding and produces a torque in the closing direction of the contacts.

In Figs. 50 and 51 are shown the characteristics of the generator-type differential relay shown in Fig. 49 for through faults (current in the same direction in the sum coils). Figure 51 gives

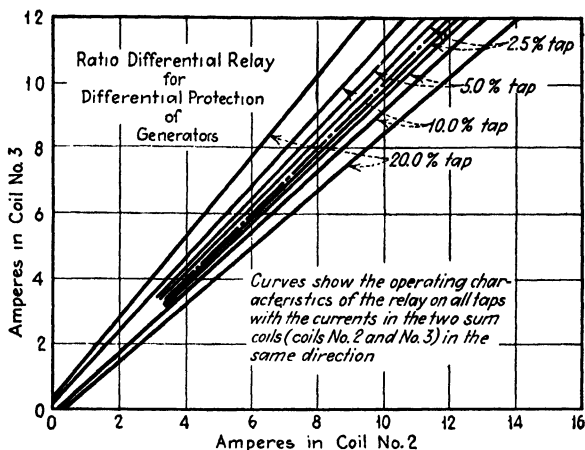


FIG. 50.

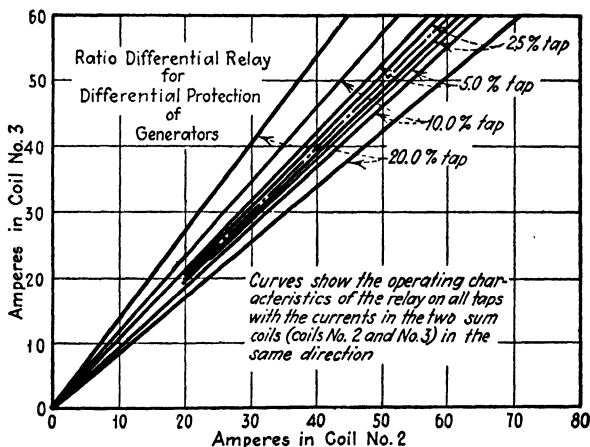


FIG. 51.

FIGS. 50 and 51.—Characteristic curves generator-differential relay with restraining coil currents in the same direction.

the through-fault characteristics drawn to a scale so that short-circuit conditions can be readily checked. The relays are inoperative for points which are included in the envelope for a given tap setting. A given percentage of unbalance is required to

operate the relay rather than a given current. As the current in the *sum* coil increases, the opening torque increases, requiring more current in the *difference* coil to operate the relay.

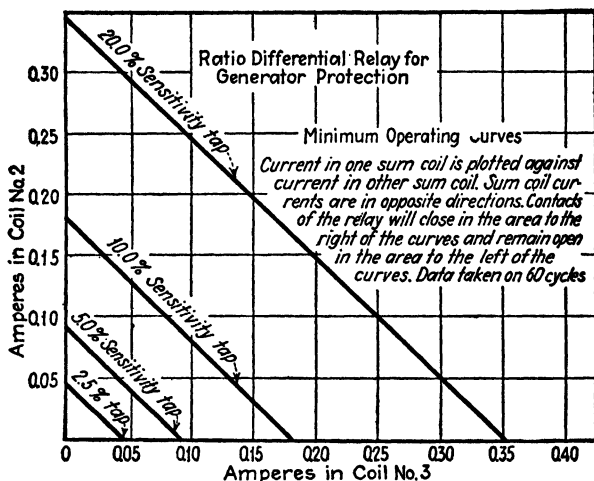


FIG. 52.

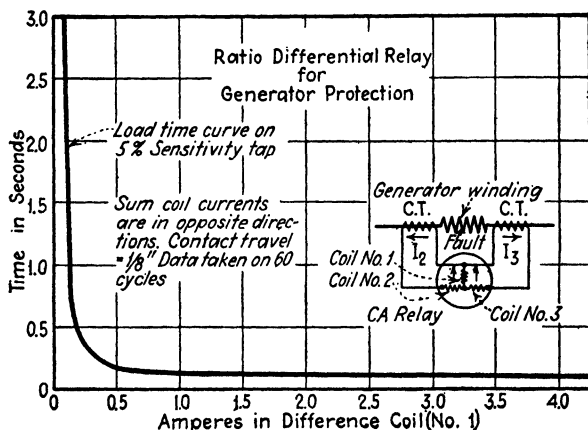


FIG. 53.

Figs. 52 and 53.—Operating curves of generator-differential relay with restraining coil currents in opposite directions.

In Fig. 52 the operating characteristics of the generator-type differential relay (Fig. 49) are shown with *sum* coil currents in opposite directions (as in the case of an internal fault in the

machine). The time-current characteristics of the generator-differential relay (Fig. 49) are given in Fig. 53.

A second type three-phase induction-type percentage-differential relay is available. This relay comprises three mechanically separate units in a common case. Each unit has two shaded-pole driving magnets acting on an induction disk. One of these, the operating magnet, drives the disk in the contact-closing direction, and the other, the restraining magnet, drives the disk in the contact-opening direction.

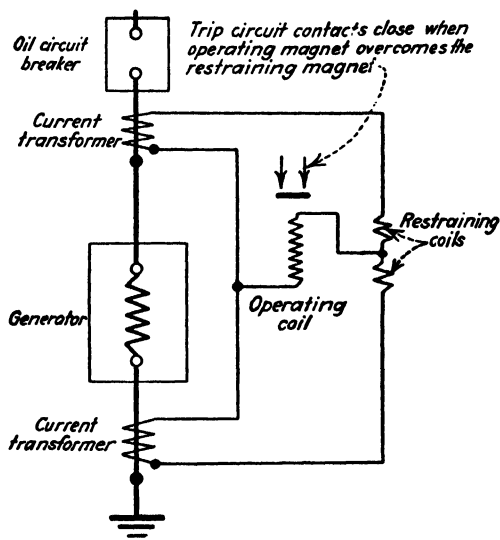


FIG. 54.—Schematic diagram of three-phase induction-type percentage-differential relay connections for generator protection.

The operating and restraining coils of each unit of the three-phase percentage-differential generator-type relay are connected, as shown in Fig. 54. These current transformers are selected so that equal secondary currents flow on an external fault, making the relay inoperative under such conditions. Figure 55 gives the operating characteristics of the three-phase relay.

The protective scheme is shown in detail in Fig. 56 for a star-connected generator and applies whether the neutral is grounded or ungrounded. Each winding is protected separately for phase-to-ground and phase-to-phase faults, the current in the neutral end of a winding being balanced against that at the line end.

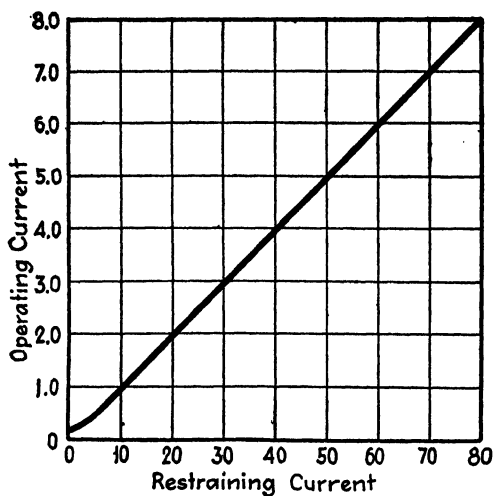


FIG. 55.—Operating characteristics of relay shown in Fig. 54.

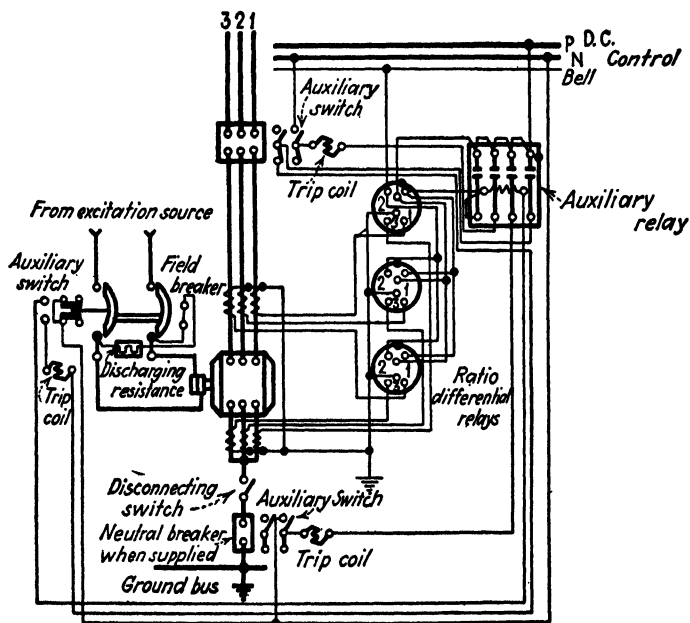


FIG. 56.—Complete wiring connections of generator differential protection.

If the machine neutral is ungrounded, it will still be protected for ground faults if the neutral is grounded elsewhere.

Similar protection for a delta-connected generator is shown in Fig. 57, except that the tripping circuits are not shown in detail.

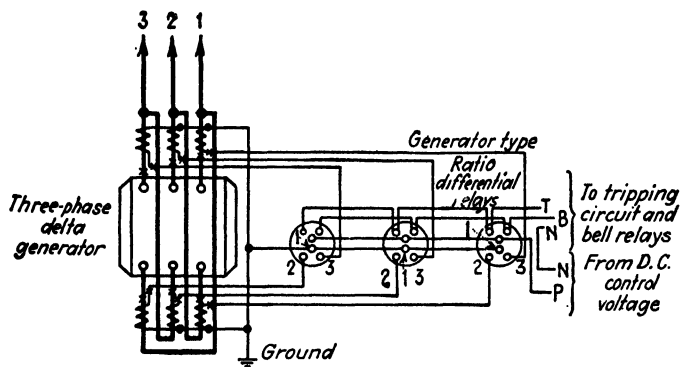


FIG. 57.

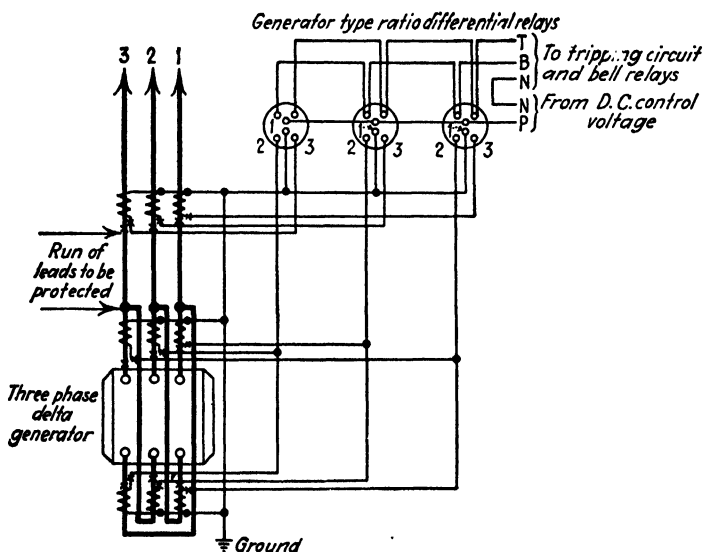


FIG. 58.

FIGS. 57 and 58.—Differential protection delta-connected generators.

Again each phase winding is independently protected by balancing, in a ratio-differential relay, the currents in the two ends of a winding. With this scheme of connection for delta-connected generators, protection is obtained for phase-to-phase

faults and also for ground faults between the winding and frame, if both the frame and system neutral are grounded.

Similar protection for a delta-connected generator, where it is also desired to protect the generator leads to the station bus with the same relays, is shown in Fig. 58. It should be noted that this scheme requires a total of nine current transformers for a three-phase generator. The current from one lead to the bus is balanced in the ratio-differential relay against the resultant

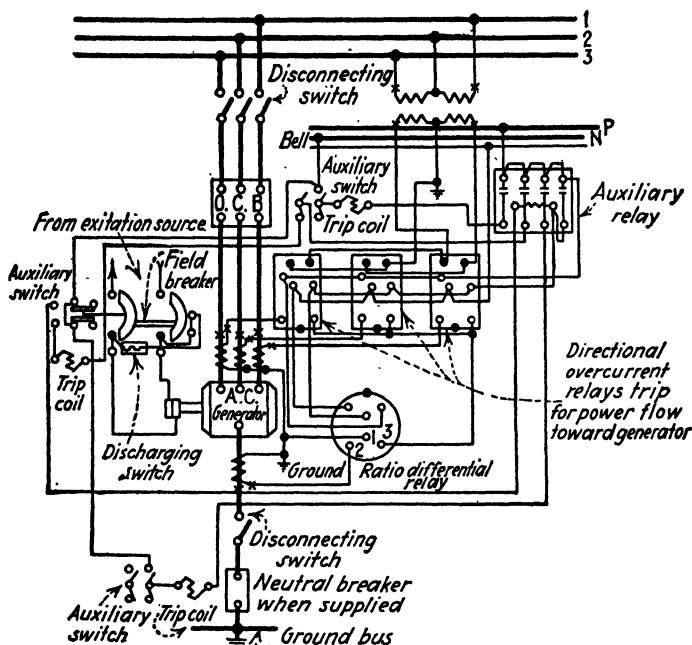


FIG. 59.—Differential protection of generator with only one neutral lead brought out.

current at the farther ends of the two windings which are connected to that particular lead to the bus.

In case only one neutral lead is brought outside the frame of a star-connected generator, the protective scheme shown in Fig. 59 may be used. The vector sum of the line phase currents is known as the residual line current, and it will be noted that this is balanced in the ratio-differential relay against the neutral current. Any internal fault to ground will cause a neutral current to flow, and, providing the system is grounded elsewhere,

a residual line current will flow in the opposite direction. Either or both of these currents may operate the differential relay. However, phase-to-phase faults which do not involve the frame will not set up any neutral or any residual line current and hence will not operate the differential relay. In case there are other machines in parallel which will feed power back into the

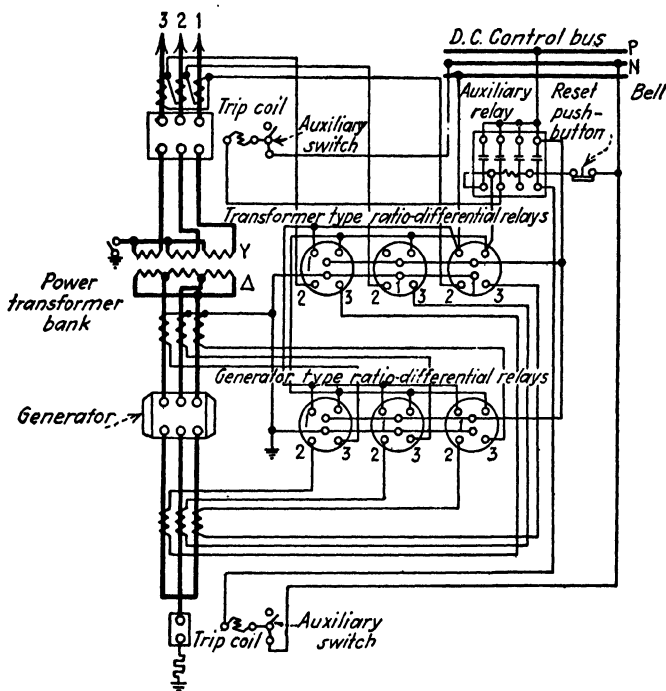


FIG. 60.—Combined generator and transformer differential protection.

fault, phase-to-phase faults may be detected by adding directional overcurrent relays, as shown.

A diagram of connections for differential protection of generators and transformers using one set of transformer-type ratio-differential relays, one set of generator ratio-differential relays and three sets of current transformers is shown in Fig. 60. By this means, differential protection is given to the generator windings, generator leads, transformer windings, transformer bushings, transformer leads, and circuit-breaker bushings. In case very sensitive protection of the generator is desired, this scheme can be modified by using double secondary current transformers

in the generator neutral, thus separating the connections of the two sets of relays.

Parallel-winding Generators.—In machines of recent design, it has sometimes been found advantageous to connect each phase winding in two or more parallel paths. This arrangement has made it possible to protect against all types of internal faults, including open circuits in the windings and short-circuited turns, by a combination of the so-called "split-conductor" protective scheme and the differential scheme.

A diagram of connections of this combination is shown in Fig. 61. One set of ratio-differential relays is arranged to operate on a predetermined unbalance between the opposite ends of each combined phase winding, just as described for the simple differential scheme. The split-conductor protective scheme consists of an additional set of ratio-differential relays connected to operate on an unbalance between any of the two parallel branches. Any fault such as an open circuit, or short-circuited turns in one winding which produces an unbalance between the current in the two parallel windings, will cause this second set of relays to close their contacts. Nine leads are required for the application of this method.

A simplified form of the parallel-winding method of protection, shown in Fig. 62, may be used where the extreme sensitivity of the more complicated scheme is not justified and where all twelve leads are brought out of the machine. The number of relays and current transformers is reduced by the use of the simplified form to the number required for the ordinary differential scheme, and, at the same time, protection against all types of internal faults is secured.

The disadvantage of this scheme lies in that it is difficult to build a machine without certain differences between the two parallel paths, thus causing a slight unbalance between the currents in the two windings. This unbalance, added to that caused by the slight discrepancies between the current transformers, particularly on heavy through faults, makes it impossible for the relays to be set to operate as sensitively as for the simple differential scheme. As a result, sensitivity is reduced for phase-to-ground and phase-to-phase faults.

In Fig. 63 is shown a scheme of protection which gives a single, overall, all-differential scheme and yet includes partial split-

conductor protection. In large water-wheel generators, there may be 8 to 10 parallel paths per phase. The complications of

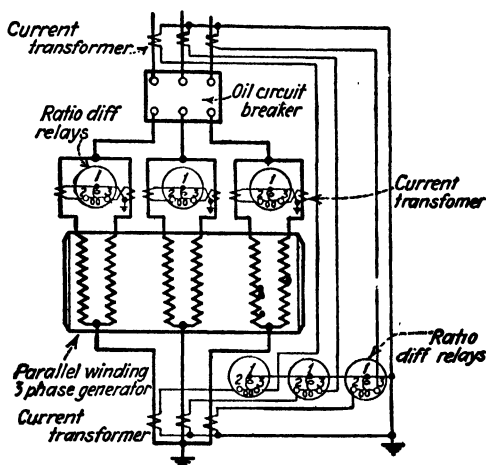


FIG. 61.

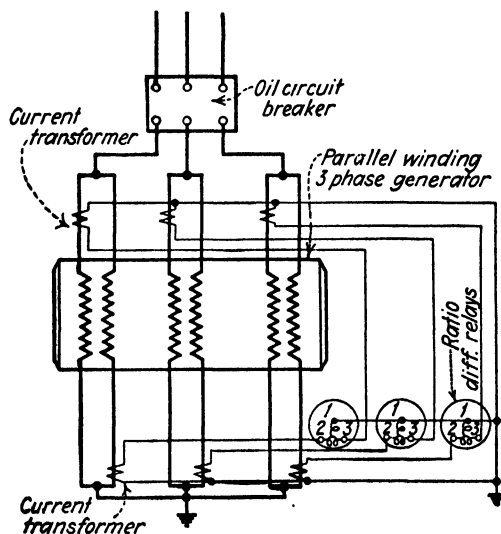


FIG. 62.

FIGS. 61 and 62.—Differential protection of parallel-winding generators.

complete split-conductor protection applied to so many parallel paths is unwarranted. One of the requirements of this scheme is that at zero load on the generator the circulating current

between the two groups of any one phase must be reduced to a value well below the pick-up current of the relay. This can be accomplished by grouping alternate circuits of any one phase together so that the circuits of any one group are evenly distributed around the stator.

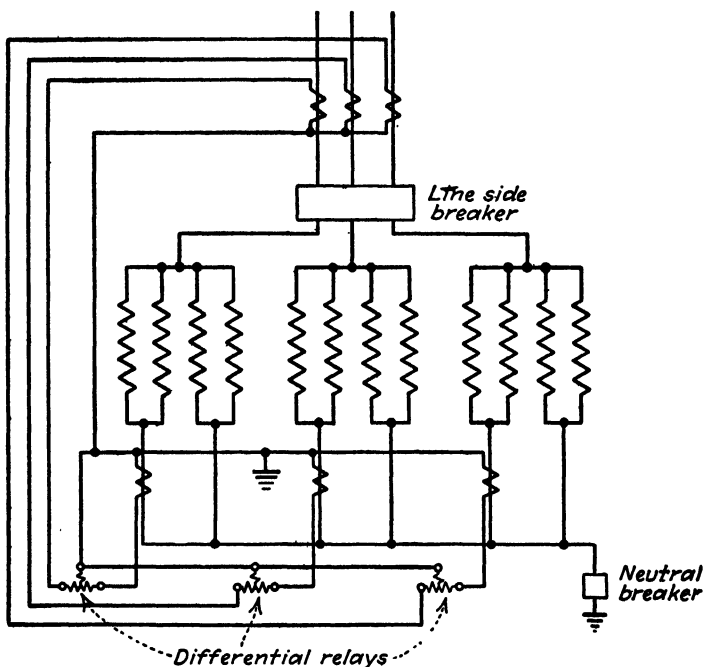


FIG. 63.—Differential protection of generator with four or more parallel paths per phase (brought out in two groups per phase) particularly adaptable to large water-wheel generators.

Grounding-bank Method.¹—Since a high percentage of electrical failures in generators are phase-to-ground faults rather than phase-to-phase faults, it is important that adequate protection be provided against ground faults. Differential protection may clear the machine from the line in case of a ground fault, but, if the generator neutral remains externally grounded, there is a possibility of damage to the machine after the line breaker and field breaker have opened. Unless a neutral breaker is opened, residual magnetism in the field may maintain sufficient

¹ HUNT, LLOYD F., "Minimizing Fault Damage Within Generators," *Elec. World*, Dec. 28, 1929.

voltage to produce an arc and perhaps cause serious damage to the machine.

In Fig. 64 is shown a system of protection which makes it unnecessary to ground the generator neutral and thereby eliminates the necessity of providing a generator-neutral circuit breaker. This method takes advantage of the ground current flowing from a grounding bank. It may be seen from Fig. 65 that the neutral of the grounding bank is the only point grounded, the generator neutral being ungrounded. The size of the grounding bank is determined by the setting of differential

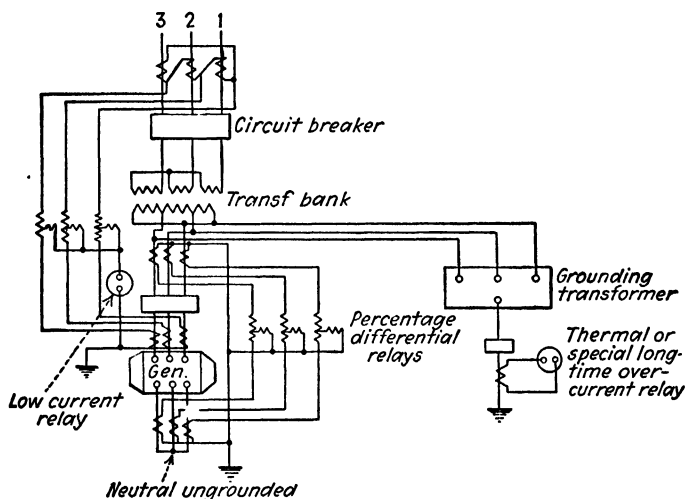


FIG. 64.—Differential protection with grounding transformer for establishing neutral.

relays. The purpose of the grounding bank, besides establishing a neutral-to-ground potential, is to cause sufficient ground current to flow in a phase-to-ground fault to trip differential relays. A low-current-range ground relay is added to the transformer-differential protection in order to make this protection more sensitive to ground faults.

The probability of ground failure in the machine near the neutral end of the winding is so small that a compromise on the size of the grounding bank must be made. Since it is possible that a ground may occur in the winding of the generator, at a voltage so low as to cause insufficient current to flow for tripping the differential relays and yet cause enough current

to flow to damage the grounding bank, it is necessary to equip the grounding bank with a neutral oil switch operated from a thermal relay or a special long-time overcurrent relay.

The grounding bank should be connected solidly to the transformer bus at a point inside the differential protection. When the generator and transformer bank are operated as a unit a ground-fault current flowing in the generator would also flow in the neutral relay of the transformer-differential protection. Therefore, if it seems desirable to keep the transformer on the line when a fault occurs in the generator, the ground relay could be set a little longer in time so as to allow the generator to clear. This will cause some little additional damage to the generator, but the ground currents are limited by the relatively small size of the grounding bank so that the damage should not be great. In the case of two generators feeding one transformer bank, this additional time given the ground relay is highly necessary.

Should this method be employed when distribution circuits are taken off the generator bus, it is necessary to connect the grounding bank on the bus. The feeders also should be equipped with a sensitive ground relay.

Ground-detector Method.¹—Instead of installing a grounding transformer bank as shown in Fig. 64, it is possible to use potential transformers connected Y-delta with a relay connected in the secondary delta with contacts wired to a bell alarm. If calculations indicate that arcing grounds are likely to occur with only potential transformers connected to ground, three 2.5-kva. transformers can be used. A resistor can be connected within the secondary delta to limit the ground current so that not more than full-load current can be delivered by the bank.

Should a ground occur on the generator bus or on the low side of the transformer, the ground detector gives an audible signal. When generator and transformers are operated as a unit, the trouble could be in the generator, transformers, or bus. Immediately upon getting a signal of a grounded condition, the operator shifts the load to another machine and drops the machine apparently in trouble from the bus. The ground detector then indicates whether the trouble is in the machine, on the bus, or in the transformers. Then, by dropping the transformer bank

¹ HUNT, LLOYD F., *op. cit.*

and picking up the bus with the machine, the trouble can be definitely located.

In either the grounding-bank method or the ground-detector method, the conventional schemes of phase-to-phase differential protection are required.

High-speed Differential Protection.—With the increasing use of high-speed relays and oil circuit breakers for transmission-line protection, it becomes essential also to provide high-speed transformer and generator protection and, in many cases, high-speed bus protection. For instance, if high-speed fault-clearing equipment such as high-speed distance relays and high-speed breakers are used on a transmission line, a bushing flashover or an internal fault occurring in a power transformer would cause the relays at the far end of the adjacent transmission line to function on the timed element, and, unless the transformer can be disconnected in approximately the same time as a line fault in the instantaneous zone of the line relays, the time of the line relays must be materially increased. It is evident that the speed of clearing line faults over part of the line section must be slowed up to accommodate selection with the transformer-differential protection.

On transmission and interconnection developments comprising large hydroelectric stations transmitting power over 200 to 300 miles of transmission line and interconnecting with steam and hydro power at load centers, stability limitations require that high-speed differential protection be applied to generators, transformers, and buses. Carrier-current pilot-wire relaying, high-speed distance relaying, and high-speed balance relaying, operating in conjunction with circuit breakers operating in a time as low as three cycles, are being considered for such installations. High-speed differential protection of equipment must therefore go hand in hand with high-speed relaying of transmission lines.

There are many existing installations of high-speed transmission-line relaying which should be made complete by adding high-speed apparatus protection, thereby getting material benefits in continuity of service from a stability standpoint and considerably reducing the damage to the apparatus.

High-speed Differential Relays—Principles of Operation.—This type of relay employs the same percentage-differential

principle as is used in the slower speed-induction type. That is, the differential current required to cause operation is always a constant percentage of the current entering or leaving the winding being protected. This means that the relay is insensitive to the unbalance caused by current-transformer errors on heavy through faults but, nevertheless, gives sensitive protection for faults within the apparatus.

Figures 65 and 66 show the schematic connections of the generator and transformer relays, respectively. Windings 1 and 2 are the restraining coils, and, for balanced conditions, current flows through these two coils and produces a torque

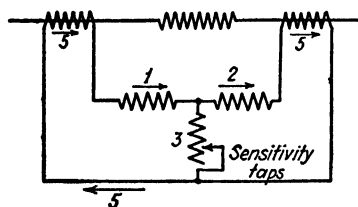


FIG. 65.—Schematic connections of high-speed generator-differential relay.

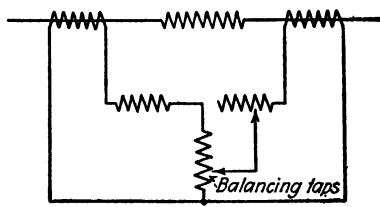


FIG. 66.—Schematic connections of high-speed transformer-differential relay.

which prevents the contacts from closing. During unbalanced conditions, the difference of the two currents flows through coil 3 which operates to close the tripping contacts. In Fig. 66 taps are arranged in coils 2 and 3 so that normally unbalanced currents can be balanced without the use of external autobalance current transformers.

This relay is of the three-phase type, employing three balanced beam-operating elements somewhat similar to the impedance elements used in the high-speed distance relay. The two restraining or sum coils are wound on one electromagnet and exert a force on the steel beam in a direction to open the contacts. The operating or difference coil is wound on the second electromagnet and exerts a pull on the other end of the beam, and when its force overcomes the restraining force, the contacts close.

The transformer-type high-speed percentage-differential relay can be obtained with a fixed percentage differential of 25 or of 50 per cent. There are available, in one type of relay, internal autobalancing taps. The generator type is available with a

percentage slope of 5, 10, 15, or 20 per cent. One type of relay includes taps for all four characteristics. The minimum pick-up for generator protection can be made as low as 0.1 amp.

When a transformer is first energized, a rush of magnetizing current flows which may be several times the full load current of the transformer. This transient decreases quite rapidly during the first few cycles and then it decreases more gradually, until finally it reaches the normal value of the magnetizing current. If the transformer is not carrying an appreciable load current, this magnetizing current produces an unbalance in the differential relay. With the slower speed induction relays, tripping on the first high magnetizing rush is prevented because the magnetizing current will generally drop below the operating point of the relay before it has had time to operate. However, with high-speed relays operating in less than one cycle, the differential element can nearly always be expected to operate when the transformer is energized.

There are two principles which can be used as the basis for preventing tripping of breakers due to the transient inrush. The first makes use of voltage-restraining coils connected in multiple and energized through a normally closed contact of a time delay, opening and closing voltage relay. Excitation of these restraining coils raises the minimum operating value of each differential relay unit from 2 up to 8 amp. The second method for preventing trip-out due to transient inrush currents makes use of voltage and current synchronous timers, with contacts in series with the main differential-relay contacts.

In the voltage-restraint relay the auxiliary-voltage time delay is energized when the transformer is energized. The auxiliary-relay contact is normally closed but, due to its time-delay feature, the contacts do not open instantaneously. This feature permits the voltage-restraining coils to remain energized during the momentary inrush of magnetizing current, to the transformer. After a short time delay the auxiliary-relay contacts open, cutting off the voltage restraint. The relay is then ready to function as a straight current differential relay.

Figure 67 gives the characteristics of a high-speed percentage-differential relay for transformer protection using the voltage-restraint principle. The upper curve gives the insensitive operating characteristics and the lower curve the sensitive

characteristics. Figure 68 is a time-current characteristic curve and indicates that the relay operates in $\frac{1}{2}$ to 1 cycle depending upon the operating current. For certain transformers it may be necessary to use a relay having a more insensitive setting than shown in Fig. 67. The ampere pick-up of the insensitive setting should not be so high that it will not adequately protect the transformer in case the voltage is too low to open the contacts of

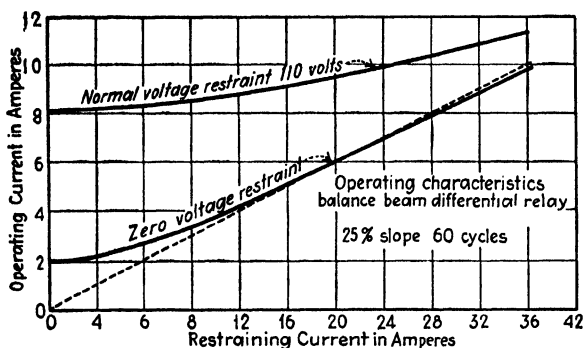


FIG. 67.

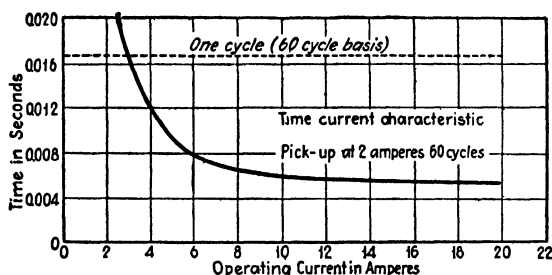


FIG. 68.

FIGS. 67 and 68.—Characteristic curves of high-speed differential relay using voltage-restraint principle.

the auxiliary relay. A schematic internal diagram is given in Fig 69.

A second type of percentage unbalance relay makes use of a synchronous timer and three instantaneous voltage-relay elements with front and back contacts all mounted in the differential-relay case. The instantaneous voltage-relay elements with normal voltage applied will open their back contacts before differential contacts can close on magnetizing-current rush. Refer to Fig. 70. The timer is adjusted so that it will not

close its contact before differential-relay contacts reset, following subsidence of magnetizing current. Also, timer on drop-out

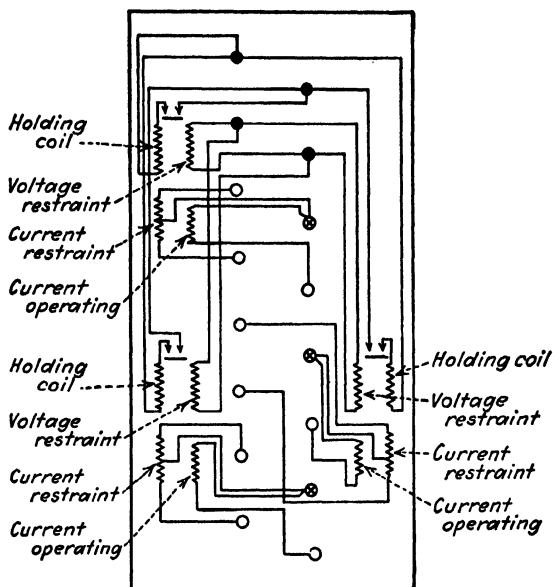


FIG. 69.—Schematic internal diagram of high-speed differential relay with characteristics as shown in Figs. 67 and 68.

should not open its contact before voltage relays travel from opening of front to closing of back contacts. Drop-out of

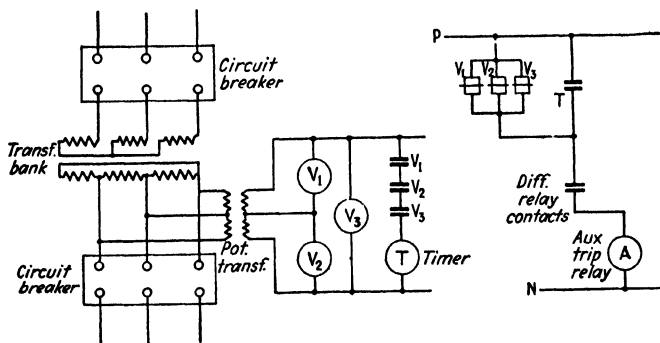


FIG. 70.—High-speed differential relay using synchronous timer and instantaneous voltage-relay elements.

voltage relays should be above drop-out voltage of timer unless timer is made direct-current operated.

On magnetizing of transformer bank, differential relay cannot trip until timer closes. Transformer faults are tripped without time delay. Closing in on severe transformer fault trips without time delay. In the case of a magnetizing surge following clearing of external fault, the timer inserts time delay depending on how far it has reset. Full time delay is not necessary as through current causes restraint.

Figure 70 is illustrative of one system of relay connections and may be varied for different applications. On voltages of 220 kv. and higher, consideration should be given to the time difference in the establishment of current in the various phases when the circuit breaker closes the circuit. A further difference may be produced by the individual voltage elements due to the voltage of all three phases not reaching a maximum at the same time. Also, magnetizing inrushes may occur during the clearing of an external fault. However, the proper arrangement of timers and voltage elements is available to take care of any particular application.

Bus-differential Protection.—Bus-differential protection is based upon the same principles as generator- and transformer-differential protection. Incoming power to the bus is balanced against outgoing power. A fault within the differential unbalances the system, and current flows in the operating coil of a relay which in turn trips the circuit breakers.

There may be many circuits connected to the bus which necessarily requires the paralleling of a number of current-transformer secondaries. In the past, secondary short-circuiting auxiliary switches have been used in order to minimize circulating current within the differential due to leakage through dead bushing-type current-transformer secondaries on line switches out of service. However, it has been found that the use of secondary short-circuiting auxiliary switches presents more hazard than their omission. In bus-differential schemes involving bushing-type current transformers, six to eight secondaries can usually be paralleled without difficulty. For such schemes standard overcurrent relays will work to better advantage than will differential relays. A typical bus-differential scheme using standard overcurrent relays is shown in Fig. 71.

A fault on one section of the high-tension bus (Fig. 71) will clear circuit breakers 6, 7, and *a*, and a fault on the other section

will clear 7, 8, and *d*. Differential-relay protection of the transformers is also shown in Fig. 71. A fault in one transformer will clear *a* and *b* and in the other transformer will clear *d* and *e*. If it is important from a system-relay-selectivity standpoint, and for purposes of continuity of service at an important substation, to isolate low-tension bus faults quickly, low-voltage bus-differential protection, not shown, can be applied in the same way as shown for the high-tension bus in Fig. 71. If such low-tension bus protection is applied, a fault on one section of the low-tension bus will clear the bus-tie breaker, as well as the feeder breaker and the low-tension transformer breaker on that section.

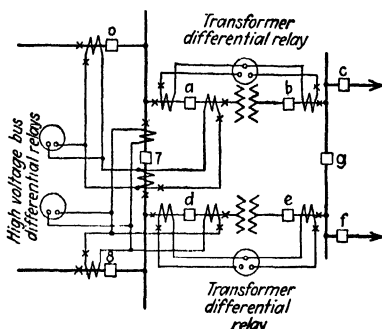


FIG. 71.—Typical scheme of connections for bus-differential protection of important substation.

Any fault which may occur in the substation will be isolated by clearing the proper circuit breakers, and only the faulty section will be removed from service. It will be noted that each circuit breaker is included in two differential loops to insure that circuit-breaker bushing flashovers or trouble inside the breaker will be properly cleared.

For generating-station bus protection, percentage-differential relays can be used to advantage, as shown in Fig. 72. The current transformers can all be of the wound type which inherently have high impedance with primary side open. The current-transformer secondaries from the feeders on the outgoing side of the bus are connected in parallel with each other and then through the percentage-differential relay to the current-transformer secondaries from the circuits on the incoming or generator side of the bus, which are also in parallel with each other. For most conditions of external faults, this relay has

an added advantage over plain overcurrent relays, due to the restraining effect of the through short-circuit current, which raises the amount of differential current required to operate the relay. Under the worst conditions the relay is reduced to the same sensitivity as a plain differentially connected overcurrent relay. It is advisable that the overcurrent setting be approximately the same as would be used for the plain overcurrent relay

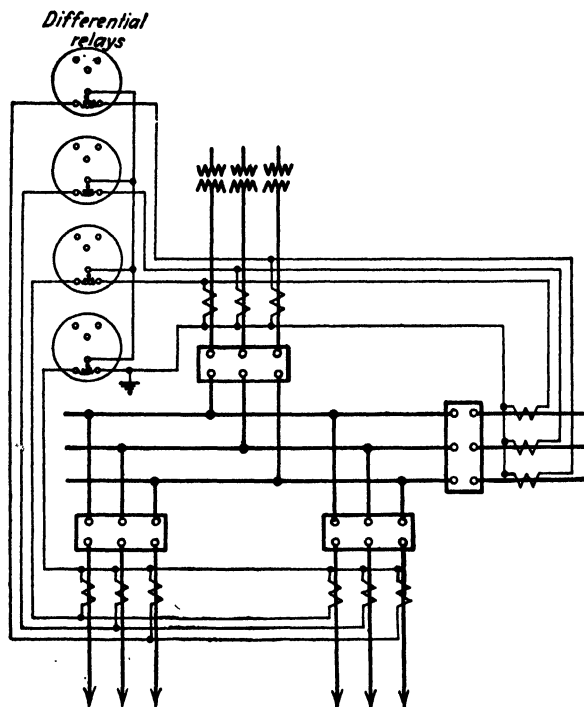


Fig. 72.—Differential protection applied to generating station.

when used in a differential scheme of connections. The transformer-type percentage balance relay is best suited for the three line differential relays.

Bus Protection for Ground Faults Only.—In many cases it is impractical to install differential bus protection for phase-to-phase and three-phase bus faults. The number of current transformers and interconnections of secondaries required makes it doubtful in many instances if the expense for complete phase protection is justified. Furthermore, it must be recognized

that the large number of cross connections for complete phase protection gives a possible source of trouble. An incorrect operation is serious in that a complete bus section will be isolated unnecessarily and a large block of power dropped. In such cases a bus-differential scheme for ground faults only can be installed at a small cost and with a minimum of cross connections as shown in Fig. 73.

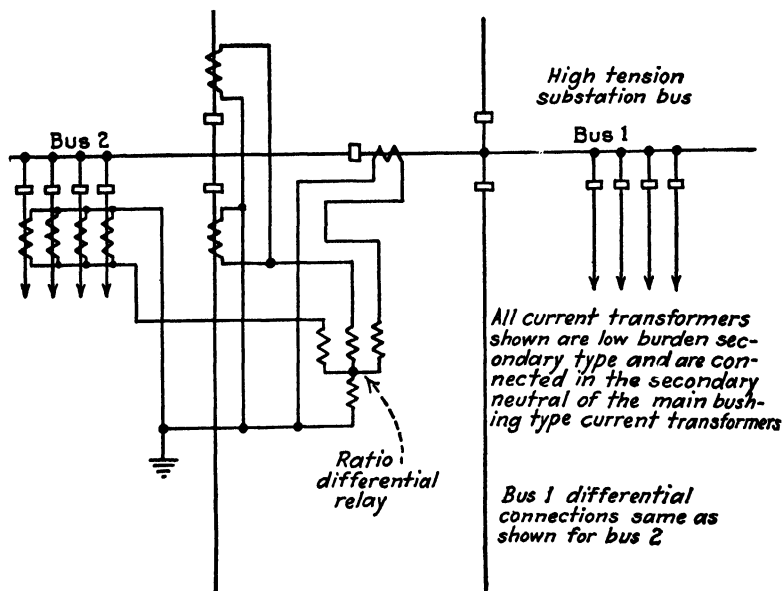


FIG. 73.—Differential bus protection for ground faults only. The current transformers shown are low-voltage, low-burden, current transformers connected in the neutral circuits of the main high-voltage bushing-type current transformers.

In Fig. 73 there are four main line breakers, one bus-tie breaker, and eight feeder circuits. The simplest means of effecting this protection is to connect an auxiliary current transformer in the secondary neutral of the bushing-type current transformers of each circuit. The secondaries of the auxiliary current transformers are differentially connected through the restraining coils of a four-element percentage-differential relay as in Fig. 73.

It is important to give careful study to the saturating characteristics of the current transformers, both main and auxiliary. A balance must be reached between the volt-ampere burden

imposed on the main current transformers and the saturating characteristics of the auxiliary transformer. The greater amount of iron and ampere turns required for a non-saturating auxiliary transformer will impose a relatively high burden on the main transformers and hence tend to break down the ratio of the main current transformers.

The main current transformers will have different ratios, but this can be compensated for by using auxiliary-current-transformer ratios which will give the proper balance. Special ratio auxiliary current transformers are not costly, and, in fact, ratio taps of plus and minus 10 per cent can be added at a cost of little more than standard transformers. The auxiliary current transformers can be built with a low-voltage primary winding, thus obtaining as low a volt-ampere burden as possible, yet having ratio curves on overcurrent better than standard 25-volt-amp., 4500-volt current transformers. This better ratio is due to the closer coupling of the windings resulting from the decrease in insulation. This burden can be made less than 10 volt-amp.

Fault-bus Scheme.¹—This scheme involves merely the placing of well-grounded buses in the vicinity of the phase conductors of the buses and connecting these grounded buses to the metal supporting and shielding parts of the switches and circuit breakers. Each ground, or fault bus, is connected separately through a current transformer to ground. The current transformer energizes an overcurrent relay, which trips the breaker on that section of bus. This type of protection is particularly well adapted to isolated phase stations. It requires that the breaker framework, disconnects, switch bases, bus supports, and also the auxiliary fault bus be well insulated from the building steel, because of the need of eliminating parallel paths to ground, which may cause incorrect operation of the relays. Figure 74 shows a typical installation. Its disadvantages are first, the added cost of fault buses and, second, the high cost of insulation of metal parts from building steel.

Overspeed Protection of Generators.—In addition to differential protection of generators, it is desirable to provide overspeed protection. This is usually obtained by the use of a centrifugal switch mounted on the end of the generator shaft. This switch

¹ NELA, "Handbook," Supplement, p. 1169.

consists of a weighted lever which is forced outward by centrifugal force and restrained by a spring. When the generator has reached a certain speed, the outward movement of the lever causes a contact to be closed. The generator speed at which these contacts close may be adjusted by changing the tension of the spring.

Generator-field Protection.—Protection against failure of the generator-field winding or against loss of field for any cause is

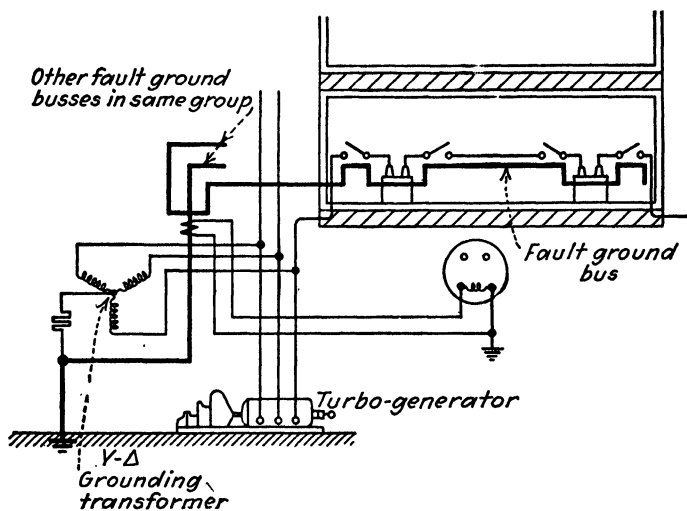


FIG. 74.—Fault-bus ground protection.

usually not considered justifiable. Failure of the field winding is of infrequent occurrence due to the low voltage applied, and even when it does occur it does not ordinarily communicate itself to the high-voltage winding on the armature. Furthermore, a comparatively complicated scheme is required to secure adequate protection against field failure, some form of overcurrent relay and automatic field switch being necessary. The introduction of such devices is likely to interfere with normal operating conditions due to incorrect or false tripping, thus needlessly dropping the load at a time which may prove highly embarrassing.

For large generators it is desirable to have an alarm and annunciator indication for "loss of field" so that the operator will know definitely that a generator has an open field.

MOTOR PROTECTION¹

Protection of motors serves three main purposes: protection against damage to the motor windings, protection to the system to which the motor is connected, and protection to the equipment operated by the motor.

In case all six leads are brought out of a motor, sensitive protection for internal faults can be easily provided by the differential-relay scheme. However, there is a large class of motors in which the six leads are not brought out and hence require some other form of protection. An economic balance between protection investment cost and relative importance of the motor must be determined and protection supplied accordingly.

If straight overcurrent relays are used with this latter class of motors, the relays must be given high settings in order to prevent their operating during the starting period of the motor. Any mechanical arrangement to short-circuit the overcurrent relays automatically during this period has the disadvantage that there is no protection on the motor during starting, and any mechanical shunting of relays is likely to give trouble and usually occurs at the time the protection is most needed.

In cases where it is considered satisfactory to provide only short-circuit protection and long-time-delay overload protection, the scheme shown in Fig. 75 may be applied. The relays are special 40-sec. overcurrent relays of the geared type to take care of the overload protection and have in addition an instantaneous

¹Space does not permit covering the subject of "Motor Protection" completely. Only the case of large motors and higher voltages is here considered. For smaller motors and lower voltages, thermally operated, totally enclosed, panel-mounted, air-break circuit breakers are now available in ratings up to 600 amp., 600 volts alternating current, 250 volts direct current. This circuit breaker is also equipped with an instantaneous trip device for short-circuit protection. These breakers are now being generally applied to 440-volt power plant, station-service, auxiliary-motor circuits, and industrial-plant motor circuits. Interrupting capacities are available up to 20,000 amp. Certain small motors may require industrial-type thermally operated motor starters, backed up by the thermal and instantaneous short-circuit trip breakers described above. These same back-up breakers can be supplied with shunt trip coils and overcurrent induction relays where it is desirable to have several breakers operate selectively. Published manufacturers' data are available giving the complete details of these circuit breakers.

element which is mounted within the relay to take care of short-circuit protection. The long-time feature of the geared induction element provides a method of giving the relay a relatively low overcurrent setting which will not operate during the starting period of the motor. The 40-sec. overcurrent element can be connected as shown in the internal-wiring diagram, Fig. 75,

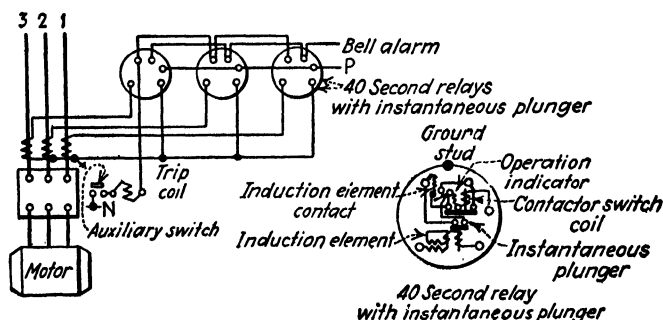


FIG. 75.

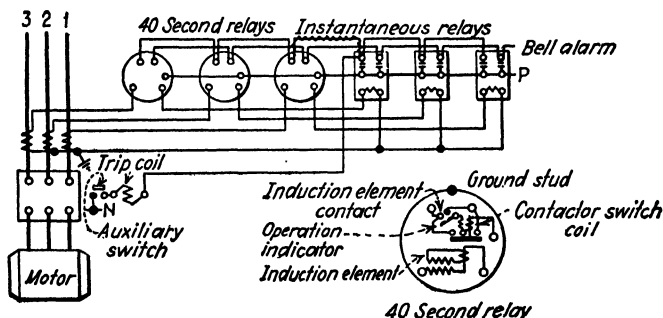


FIG. 76.

FIGS. 75 and 76.—Motor protection using special 40-sec. overcurrent relays and instantaneous short-circuit trip.

to trip the breaker and ring an alarm; or, by removing the cross-hatched connection, this element will merely ring the alarm. The short-circuit instantaneous element should, of course, trip the breaker.

In cases where the normal load current of the motor is high, so that the instantaneous short-circuit element must have a high current rating, it is desirable to use separate instantaneous overcurrent relays, as shown in Fig. 76. An internal-wiring diagram of the geared 40-sec. relay without an instantaneous

element is shown in Fig. 76. This scheme gives protection which is identical with that outlined for Fig. 75. Where it is desired to simply give an alarm in case of sustained overload, the cross-hatched connection shown in Fig. 76 should be removed.

In addition to instantaneous short-circuit protection and long-time-delay overload protection, it is often necessary to provide

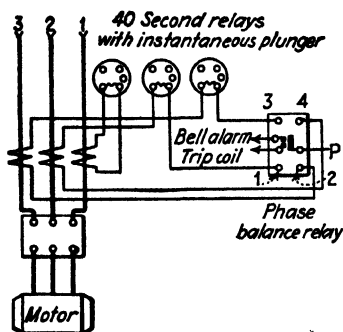


FIG. 77.—Motor protection using special 40-sec. overcurrent relays and phase-balance relay.

protection to the equipment operated by the motor. A connection is given in Fig. 77 which supplies all three of these forms. The phase-balance relay is designed to protect polyphase motors from operating under conditions of polyphase unbalance, open phase or reverse phase. The protection against reverse phase will eliminate the possibility of damage to the equipment connected to the motor due to incorrect connections at the motor.

Undervoltage Time-delay Protection for Motors.—It is generally recognized that, if undervoltage protection is applied to motors, it should be of the time-delay type; otherwise, motors will be dropping out unnecessarily on the occurrence of voltage changes which are of too short duration to affect the operation of the motor.

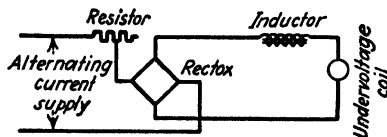


FIG. 78.—Inductor-type low-voltage time-delay trip device.

If the motor is of considerable size, the use of a time-delay undervoltage relay, with a separate direct-current control source is justified. On smaller motors, however, a cheaper and more simple form of protection must ordinarily be used. This requirement is met by the inductor-type undervoltage time-delay device.

This device, illustrated in Fig. 78, consists simply of a resistor, a "Rectox" rectifier, and an inductor coil and magnetic circuit, all

mounted in a compact metal case. It is connected between the alternating-current power source and the coil of a standard instantaneous undervoltage-release device on the circuit breaker.

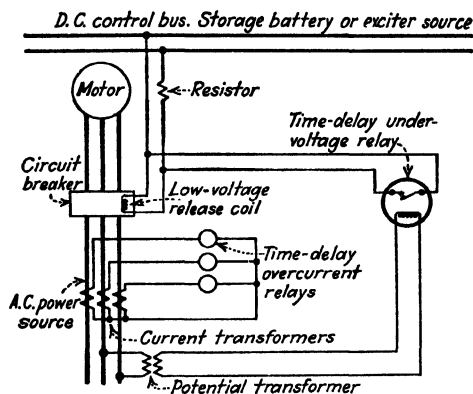


FIG. 79.—Motor protection using time-delay undervoltage relay.

When the alternating-current voltage drops, the direct current in the coil of the undervoltage-release device on the circuit breaker is sustained by the high inductance of the inductor coil.

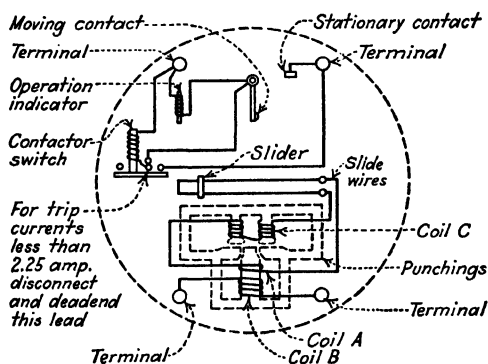


FIG. 80.—Internal connections, time-delay undervoltage relay.

The time delay of 1 or 2 sec., obtained by this means, is sufficient to prevent the motor from being disconnected during short-time voltage fluctuations. If the alternating-current voltage is recovered before the undervoltage mechanism functions, the current through the undervoltage coils is restored, thus providing full time delay for any later loss of voltage.

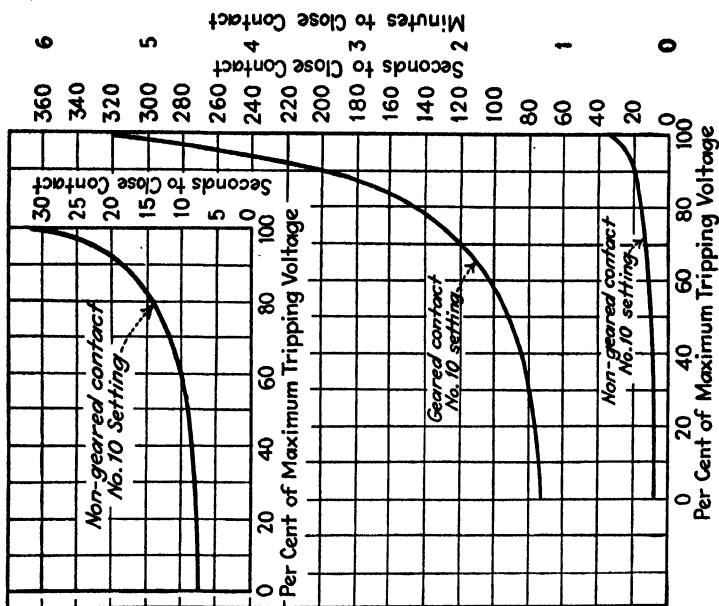


Fig. 82.—Voltage-time curves of undervoltage relay.

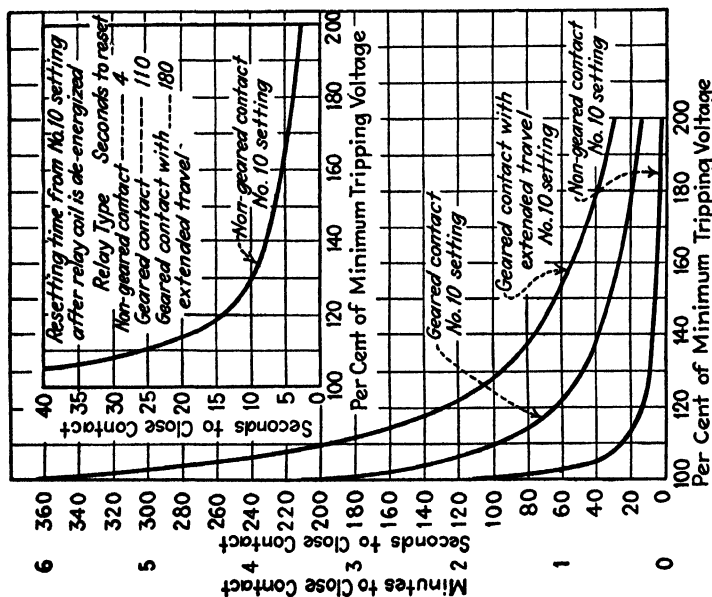


Fig. 81.—Voltage-time curves of overvoltage relay.

In Fig. 79 is shown another connection for undervoltage time-delay protection by use of time-delay undervoltage relay. An internal-wiring diagram of the undervoltage relay is shown in Fig. 80 and characteristic curves in Figs. 81 and 82.

SYNCHRONOUS-CONDENSER PROTECTION

Differential protection and overspeed protection similar to that used for generators are ordinarily applied to synchronous condensers, and, in addition, protection against low voltage and overload is generally used.

Low-voltage protection is obtained by a time-delay undervoltage relay, energized from a potential transformer connected to the source of power which supplies the synchronous condenser. The tripping contacts of the relay are normally held in the open position by the impressed voltage. During fault conditions or failure of supply voltage, the contacts of the relay will close and trip the circuit breaker for the synchronous condenser when the voltage applied to the relay drops to that of the relay setting, usually 50 per cent of undervoltage normal value. In many cases it is not possible to apply a very long delay to synchronous condensers unless automatic resynchronizing equipment is supplied.

Overcurrent protection of synchronous condensers is ordinarily obtained by the use of non-directional overcurrent relays.

SYNCHRONOUS-CONVERTER PROTECTION

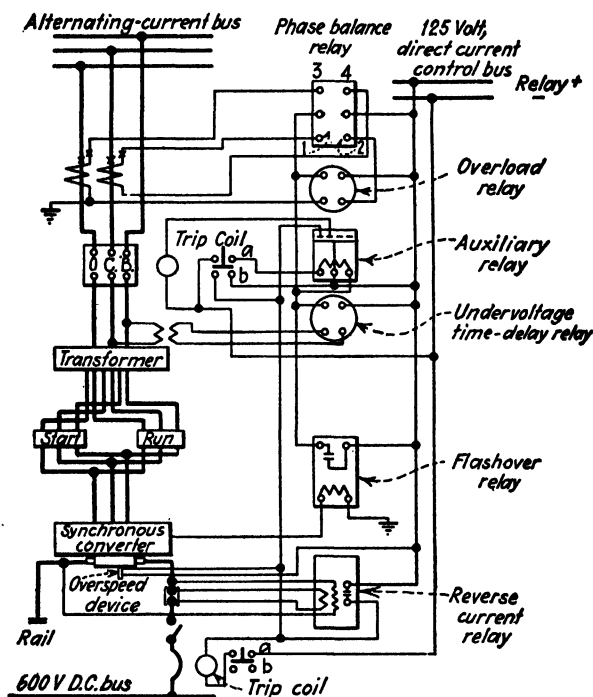
Synchronous converters should be protected against low voltage, overload, overspeed, phase unbalance, direct-current reversal, and flashover. Figure 83 shows the connections for such protection. Low-voltage, overload, and overspeed protection is obtained in a manner similar to that for synchronous condensers.

The phase-balance relay is operated from current transformers in the alternating-current leads and will protect against phase unbalance, phase reversal, and open phase.

Protection against reversal of direct current is obtained by the use of a direct-current relay. It is operated from a shunt in the direct-current leads of the synchronous converter and from the direct-current potential of the converter and will close its

contacts when current is flowing from the direct-current bus into the machine.

Protection against flashover is provided by a relay connected between the machine frame and ground. When a flashover



a - Closed when breaker is closed
b - Closed when breaker is open

FIG. 83.—Diagram of connections, synchronous-converter protection.

occurs, current flowing from the machine frame to ground will operate this relay and trip out the breakers.

MOTOR-GENERATOR PROTECTION

Motors generators are usually protected against overload, overspeed, phase unbalance, and direct-current reversal similar to the protection used in synchronous converters. If six leads are available on the motor, differential protection is recommended.

PROTECTION OF FREQUENCY-CHANGER SETS

Frequency-changer sets should, of course, be protected against all internal-winding faults, and it is also usually desirable to protect them against through short circuits. On large frequency changers, six leads are ordinarily available outside of the frames of both the motor and generator units. In such cases differential protection may be applied in the conventional way. In addition, protection against through short circuits is secured by the use of three overcurrent line relays and one overcurrent ground relay.

CHAPTER XI

PHASE AND LINE-TO-GROUND-FAULT PROTECTION OF TRANSMISSION LINES¹

Relay protection can be divided into two general classifications: first, apparatus protection and, second, transmission-line and distribution-circuit protection. Apparatus protection has been covered in detail in Chap. X.

The general factors which account for the many combinations of relays for transmission and distribution circuits are due to the inherent differences in long single circuits and parallel heavily loaded ties. Loops differ from radials. The electrical characteristics of metropolitan systems differ from lightly loaded suburban systems. System stability limits differ widely. Other determining factors are presence or absence of grounding, whether lines are overhead or cables, comparative capacities of network links, distances involved, and the rigidity of the load requirements. Extensive and frequent readjustment of load division to conform to best economy of generation and transmission of power will also influence the choice of the relay scheme.

Each power system presents an individual problem and should be studied as such. It may seem that there are so many schemes in use that these certainly must be based upon a marked difference of opinion among relay and operating engineers. This is not the case. Local modifying factors not represented on schematic diagrams are as likely to determine the protective scheme adopted as are the inherent characteristics of the system connections. It is true, of course, that some of the differences in relay practice are attributable to the holdover of past practice. Furthermore, in many cases it is not considered justifiable from the standpoint of economics to install a more complete relay

¹ Description of relays and their characteristics obtained from published data of the Westinghouse Electric and Manufacturing Company and the General Electric Company.

scheme, which admittedly would give better protection. These are problems which are local in nature and cannot be covered within the limits of a general presentation.

In this chapter only transmission-line relay systems of the normal-speed induction type will be considered, Chap. XIII being devoted to the high-speed relay systems.

The induction-relay principle used throughout the complete line of normal-speed transmission-line relays is largely responsible for the success with which transmission lines can be removed from service upon the occurrence of faults.

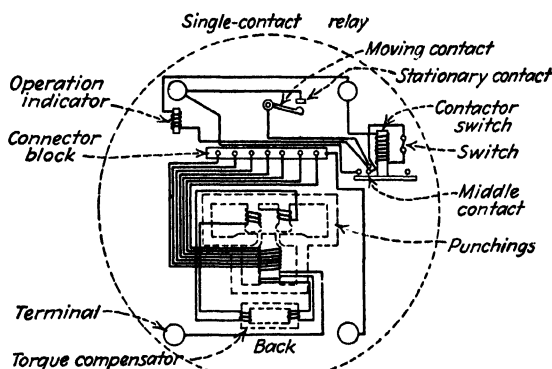


FIG. 1.—Internal wiring diagram of overcurrent induction relay.

Refer to Chap. XIV for a complete discussion of the theory of the induction relay.

In this chapter, reference to a *line relay* refers to a relay used for protection against three-phase and single-phase line-to-line short circuits. A *ground relay* refers to a relay used for protection against line-to-ground faults. Somewhat different characteristics are required for these two applications.

In Fig. 1 is shown an internal-wiring diagram of a typical overcurrent line relay having current taps from 4 to 15 amp. and time-current characteristics as illustrated in Fig. 2. The ampere turns are approximately constant for each tap, giving volt-ampere values as shown in Fig. 3. The ground relays have 0.5- to 2.5-amp. taps, with time-current characteristics as shown in Fig. 4. A comparison of the number 10 time setting for the line and ground relays is shown in Fig. 5, the low-energy

type being the ground relay and standard-energy type the line relay.

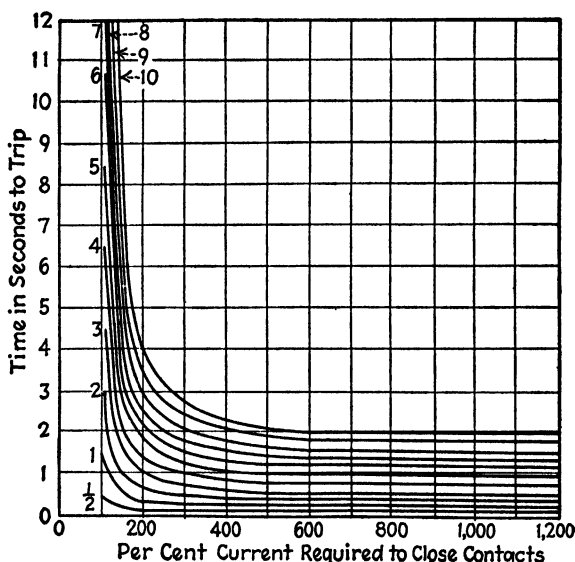


FIG. 2.—Time-current characteristics of line-type overcurrent induction relay.

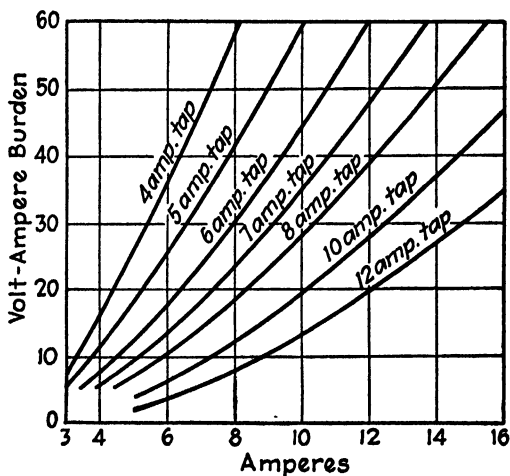


FIG. 3.—Volt-ampere characteristics of line-type overcurrent induction relay.

In Fig. 6 are shown time-current curves for a second major type of induction relay having a different mechanical design.

Among other differences, the torque is opposed by a weight instead of a restraining spring.

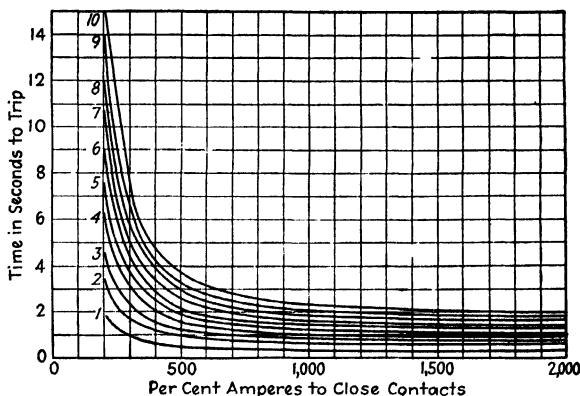


FIG. 4.—Time-current characteristics of ground-type overcurrent induction relay.

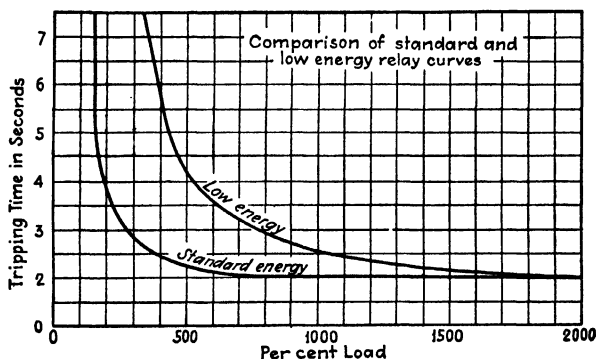


FIG. 5.—Comparison of time-current characteristics of line-type and ground-type overcurrent induction relay.

THE RADIAL SYSTEM

The simplest system of distribution is one having a single source of power with a number of feeders connected to the generator bus, each feeder in turn subdivided into a number of smaller feeders. This is commonly known as the radial system, a typical arrangement of this type being shown in Fig. 7. The protection of such a system against short circuits may be secured

by the use of ordinary overcurrent relays of the induction type. In the simple circuit shown, it is an easy matter to calculate the short-circuit current values for faults on the various buses. The current tap and time-lever settings can then be assigned to

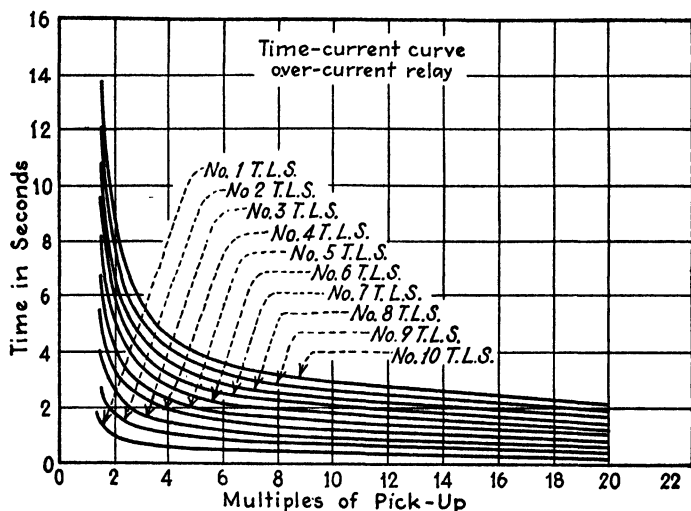


FIG. 6.—Time-current characteristics of another major-type overcurrent induction relay.

each relay, which will give selective operation. Settings of 0.5, 1, and 1.5 sec. have been designated in Fig. 7. These values are merely given to illustrate the approximate values in order to get proper selectivity. In the past it has been usual practice to

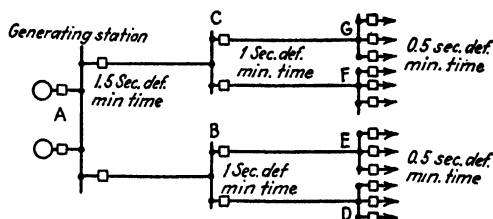


FIG. 7.—Typical arrangement of radial system.

allow 0.4 to 0.5 sec. between relay settings in order to give sufficient margin to allow for circuit-breaker opening time. The opening time of circuit breakers (from time, trip coil is energized until arc is extinguished) has been decreased materially during the last few years. The lower the opening time, the

Group-breaker Scheme.—It is often desirable to supply automatic overcurrent protection to all the circuit breakers on buses *D*, *E*, *F*, and *G* of Fig. 7, but this may require circuit breakers of such high interrupting capacity that their cost may not be considered justifiable from an economic standpoint. To overcome this, a group-breaker scheme such as that shown in Fig. 8 can be used. A group breaker of high interrupting capacity is installed and a set of overcurrent induction relays is used on the feeder breakers and also on the group breaker. The protective features are as follows:

1. Feeder overloads and short circuits, within the feeder-breaker rupturing capacity, trip the feeder breaker.
2. Feeder short circuits, exceeding the rupturing capacity of the feeder breaker, do not trip the feeder breaker until after the fault is cleared by the group breaker.

The first of these conditions, *viz.*, tripping of the feeder circuit breaker by faults within its capacity, is secured by the operation of the induction element of the feeder overcurrent relay. When the fault current exceeds this rupturing capacity, the desired operation is secured as follows:

1. The instantaneous plunger of the feeder overcurrent relay is adjusted to operate if the feeder current approaches the rupturing capacity of the feeder circuit breaker.
2. A severe short circuit picks up this instantaneous plunger of the overcurrent relays on the faulty feeder, which picks up an auxiliary relay which, in turn, opens the feeder-breaker trip circuit.
3. As soon as the contacts of the feeder overcurrent relay close, a circuit is established through the contactor switch coil, operation indicator, external resistor, and feeder-breaker auxiliary switch, thus operating the operation indicator and contactor switch, which seals itself in, as long as the feeder-breaker auxiliary switch and the feeder breaker remain closed.
4. The group-breaker overcurrent relay clears the fault by tripping the group breaker.
5. Clearing the fault drops the feeder-overcurrent-relay instantaneous plunger and the auxiliary relay, whose back contact trips the breaker of the faulty feeder.

Number of Relays Required.—On the lower voltage circuits there is a tendency to reduce the initial cost of an installation by using only two overcurrent relays and two current transformers on an ungrounded three-phase system, or three current transformers connected in *Z* with two overcurrent relays on a

grounded system. However, the use of Z-connected current transformers adds an unjustifiable complication which merely confuses the operators and test men, making it very easy for mistakes in connections to occur. The use of three overcurrent relays gives a "back-up" relay for each phase fault and is well worth the slight additional expense. With three relays in the circuit, it is possible to test one relay while the circuit is in service and still have two relays to give complete protection in case a line-to-line fault should occur. It is highly recommended that three line-type overcurrent relays and three current transformers be used to give complete protection for line-to-line and three-phase faults and, in addition, that one ground relay

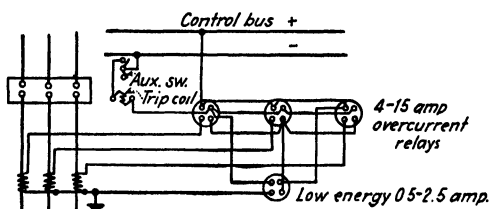


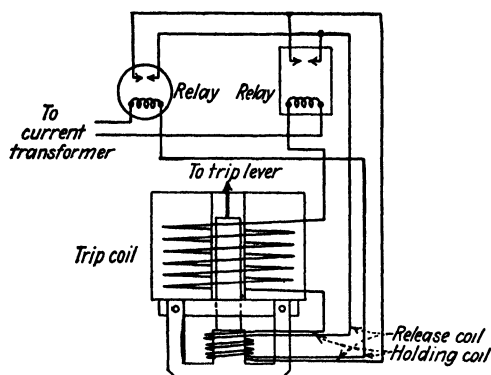
FIG. 9.—Diagram of connections of three overcurrent line relays and one ground relay applied to three-phase line.

be used where ground relaying is desirable. It is becoming common practice to use ground relays on solidly grounded systems of 11 kv. and above. The use of a resistor in the neutral ground circuit makes ground relays very desirable, even on low-voltage circuits. A ground relay having low-ampere taps provides more sensitive protection than is possible by any other scheme and has the distinct advantage that relay settings can be made irrespective of load. The connections for both line and ground relays are shown in Fig. 9. The ground relay is placed in the neutral lead of the current transformers and hence carries no load current. Ground relays are timed in the same manner as line relays.

Direct-current Tripping.—The use of overcurrent relays, as shown in Fig. 9, requires direct-current tripping of the circuit breakers. In the larger generating stations and substations, this direct current is secured by the use of a 125-volt battery and in less important substations by a 12- or 24-volt battery. On isolated feeders located at some distance from the main system, it is often considered undesirable to use even a 12- or

24-volt tripping battery, as periodic inspection and maintenance are required. With the development of the copper oxide, trickle-charge rectifier, this objection has been largely removed. Direct-current tripping gives the most positive and simple means of tripping a circuit breaker and is constantly coming into more general use.

Alternating-current Direct-trip Devices.—Where time adjustment for tripping on overload is required and a separate source of direct current is not available, protective relays capable of accurate adjustment are used in connection with an alternating-



Direct trip mechanism

FIG. 10.—Schematic diagram of alternating current direct-trip device.

current direct-trip attachment or in connection with tripping reactors.

The direct-trip attachment shown in Fig. 10 is described as follows: The tripping mechanism of the circuit breaker is equipped with an overload trip coil designed for operation through a current transformer. The tripping lever is actuated by a plunger which is moved in a vertical direction against the force of gravity by the trip coil. Attached to the bottom of the trip coil is a separate magnetic circuit on which are wound a holding coil and a releasing coil. The lower end of the tripping plunger in a normal position completes the path of this magnetic circuit and is enclosed by the releasing and holding coils. The holding coil is connected in series with the trip coil in the secondary circuit of a current transformer. The magnetic circuit of the tripping coil is open when the tripping plunger is in a normal

position, while the magnetic circuit of the holding coil is closed. This prevents any movement of the plunger regardless of the amount of current through the combination. The protective relay in closing its contacts short-circuits the releasing coil, thus demagnetizing the magnetic circuit of the holding coil, whereupon the tripping plunger is actuated by the trip coil. This device is largely used in isolated installations where direct current is not available for tripping.

A second scheme of alternating-current direct tripping is that of the tripping reactor.

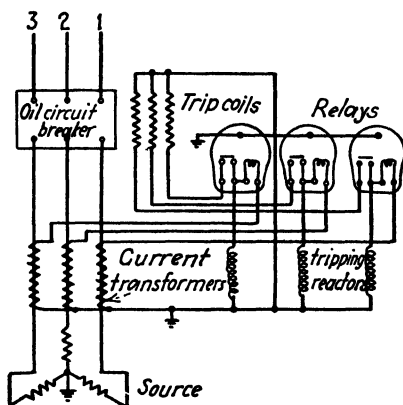


FIG. 11.—Schematic diagram of alternating-current tripping reactor.

Tripping reactors are connected in series with the operating coils of protective relays across the secondary terminals of current transformers. The circuit-breaker trip coils are connected in series with the relay trip contacts across the tripping reactors. When current passes through the circuit breaker, the current-transformer secondary currents produce a drop across the tripping reactors due to the impedance of the reactor circuits. When this current exceeds the value at which the protective relays are adjusted to trip, the relays close the trip-coil circuits and the voltage across the tripping reactors causes sufficient current to flow in the trip-coil circuits to trip the circuit breaker. A diagram of connections is shown in Fig. 11. Additional alternating-current tripping schemes are given later in this chapter.

Parallel Lines for First Line Section out of Generating Station.

The first line section out of a generating station often requires parallel lines in order to provide good regulation, reliable service, and sufficient flexibility for all service conditions. A typical radial system with parallel lines between A and B, and A and C,

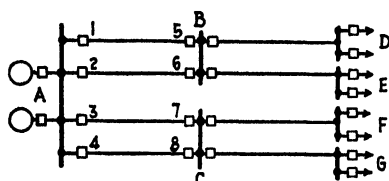


FIG. 12.—Typical arrangement of parallel-line radial system.

is shown in Fig. 12. This arrangement makes possible the use of selective-differential current balance relays at Station A to give fast clearing of faults during parallel line operation.

Three current balance relays are required for line protection and one for ground protection per pair of lines at Station A. A set of four induction overcurrent back-up relays per line are required. These overcurrent relays not only act as back-up relays during parallel line operation but give overcurrent protection when only one line is in

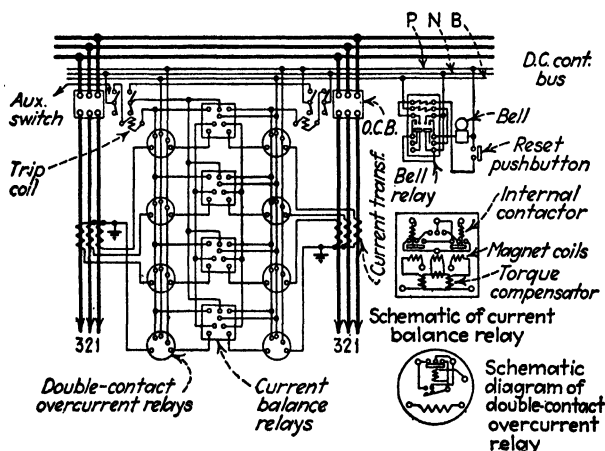


FIG. 13.—Current balance relay system.

service. The trip-coil circuit of the current balance relay is connected through auxiliary switches on both breakers in series, so that when one breaker is in the open position, the current balance relays are cut out of service. A diagram of connections is shown in Fig. 13.

The current balance relay can be applied only at a station bus where there is a source of power. If this source of power is not present a short circuit on a parallel line section will set up a condition whereby the currents in the two lines are equal and reversed. It takes an actual unbalance of current to operate a current balance relay.

Single-element Current Balance Relay.—For the protection of transmission lines operated in parallel, balanced relay protec-

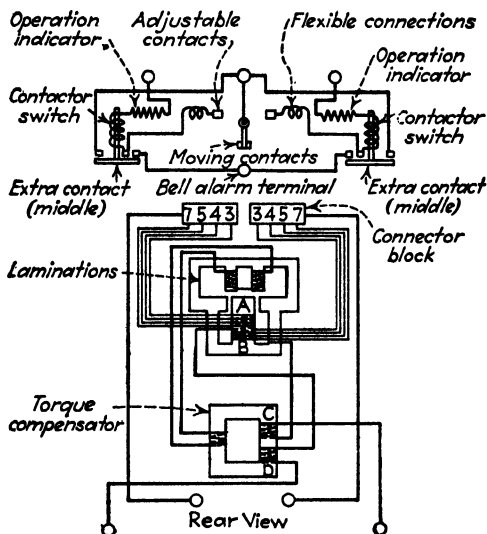


FIG. 14.—Internal diagram of current balance selective-differential relay.

tion gives very satisfactory operation. The relays can be set with low current and time settings, thereby giving fast operation. Balanced relay protection can be obtained by the use of the selective-differential current balance relay shown in Fig. 14. The relay illustrated has current taps from 3 to 7 amp. and is used for protection against line-to-line faults. A similar relay, except with lower current windings and lower volt-ampere burden, is available for line-to-ground fault protection. This latter relay has taps from 0.5 to 1.2 amp.

The selective-differential relay operates on the induction principle. As shown in Fig. 14, the lower pole of the main element has two main current windings *A* and *B*, each of which is connected in series with one of the windings *C* and *D* on the

torque compensator. Taps are provided on the main windings to permit different current settings. The two upper poles of the main element are energized from a third winding on the torque compensator, and the relation between the currents in the upper and lower poles of the main element is such as to produce rotation of the disk.

When a selective-differential relay is applied at a generating-station bus, the currents in the two lines will usually both be outward from the bus even though a fault may be present

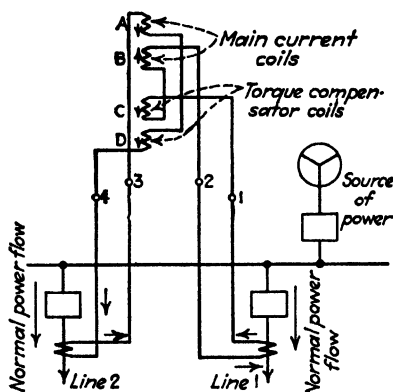


FIG. 15.—Schematic diagram of current balance relay system.

on one of the lines. At the substation end, however, conditions are usually different, and it is possible to have short-circuit current flowing in opposite directions in the two lines. The operation of the relay is somewhat different for each of these two conditions.

A schematic diagram of the relay connections when used at a generating station for the protection of two parallel outgoing lines is shown in Fig. 15. With normal balanced power flow on the two lines, the currents will flow through the coils in the directions indicated. The currents in the main current coils *A* and *B* are opposite and hence neutralize each other. The currents in the torque-compensator coils *C* and *D* are additive and induce current in the upper pole coils. However, since the effective current in the main coils is zero under these conditions, there will be no torque on the disk. Should a fault occur on line 1, resulting in a large increase in current in this

line, the current in coil *B* becomes larger than that in coil *A* and a torque is produced to close the right-hand contacts. If a fault should occur on line 2, the opposite effect will be produced and the left-hand contacts will close.

When the currents in the two lines are in opposite directions, as may occur when the relay is applied at the substation end of

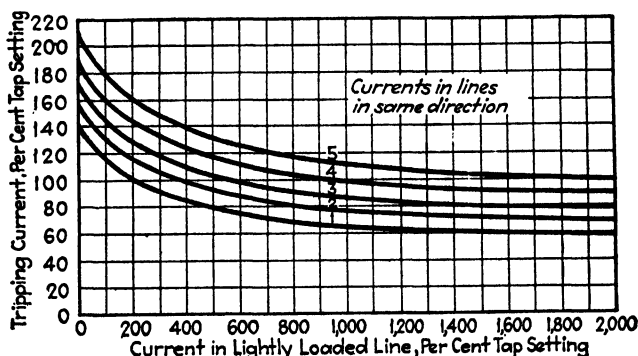


FIG. 16.

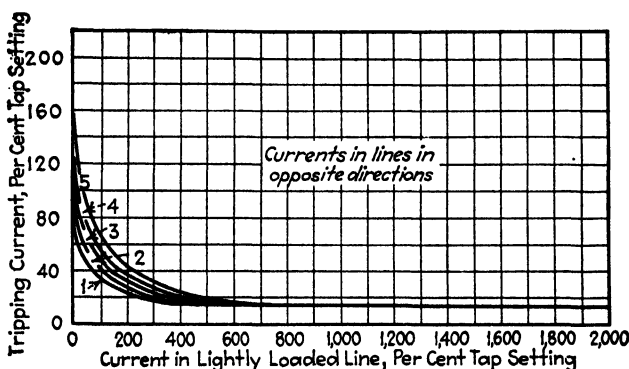


FIG. 17.

FIGS. 16 and 17.—Operating characteristics of current balance relay.

the line, the currents in the main coils are additive and the effective current induced by the torque compensator in the upper pole coils is proportional to the difference between the two currents.

The current characteristics of a line-type relay are shown in Fig. 16 for the case when the currents in the lines are in the same direction; and the characteristics of this relay when the currents in the lines are in opposite direction are shown in

Fig. 17. It will be noted that under the latter condition the relay is more sensitive.

The numbers on the curves in Figs. 16 and 17 refer to the time-lever setting, and the ordinates and abscissas are given in per cent of the current tap rating. The ordinates of the tripping current are the differences between the currents in the two current-transformer secondaries.

Two-element Current Balance Relay.—The principle of operation of the relays of this group is based upon the comparison of two torques, produced by shaded-pole electromagnets, which act in opposition on a disk, so that one, the operating

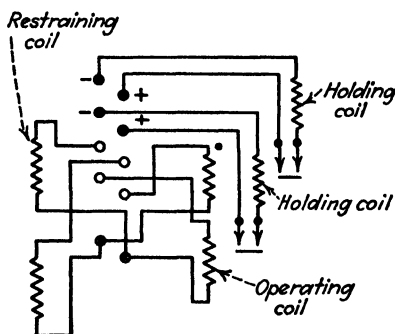


FIG. 18.—Schematic diagram of two-element current balance relay.

torque, tends to close the relay contacts; and the other, the restraining torque, tends to hold the contacts open. There are two elements each having a single-throw single-circuit contact. The elements are electrically interconnected as shown in Fig. 18. The operating coil of each element is connected in series with the restraining coil of the other element, so that the operating (or fault) current which actuates one of the elements at the same time increases the restraint in the other element, thus obtaining selective operation. Each restraining coil is usually wound with 25 per cent more turns than the operating coil. This characteristic is illustrated in Fig. 19. The connections for parallel-line protection are shown in Fig. 20; the relay characteristics in Figs. 21 and 22.

Directional Protection at Substation End of Parallel Lines.—At Substations B and C (Fig. 12), directional overcurrent relays are required on the incoming breakers of the parallel lines.

There is no source of power on buses *B* and *C*, and therefore a fault on one line of the parallel section will produce a condition of equal and opposite currents through breakers 5 and 6 at *B*

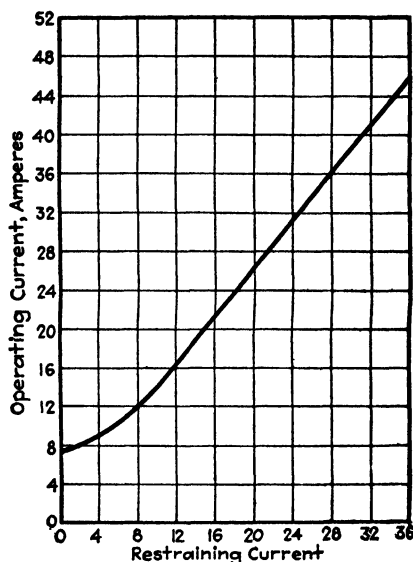


FIG. 19.—Operating characteristics of two-element current balance relay.

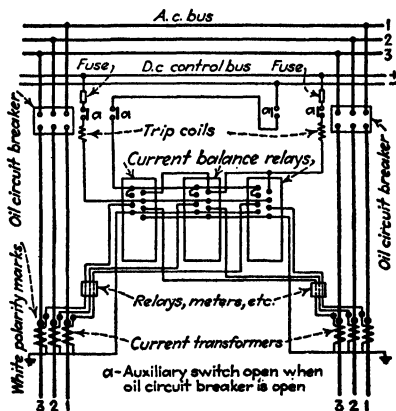


FIG. 20.—Current balance relay system.

and 7 and 8 at *C*. As already explained, the current balance relay cannot be applied to such a substation, because this relay requires an unbalance of current in the two lines during short-circuit conditions, if it is to operate.

Single-phase Directional Overcurrent Relay with Directional Control.—The single-phase directional overcurrent relay com-

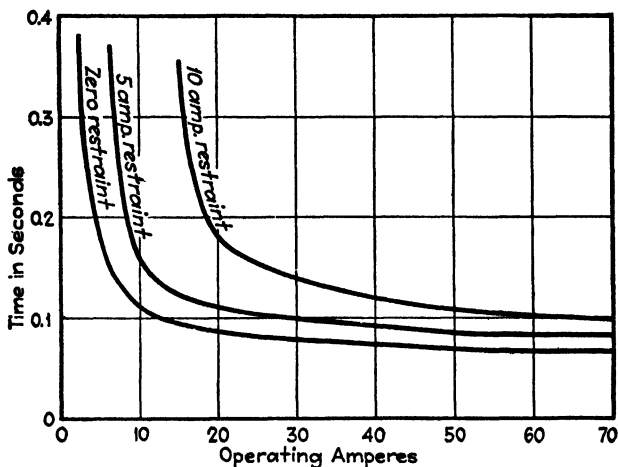


FIG. 21.

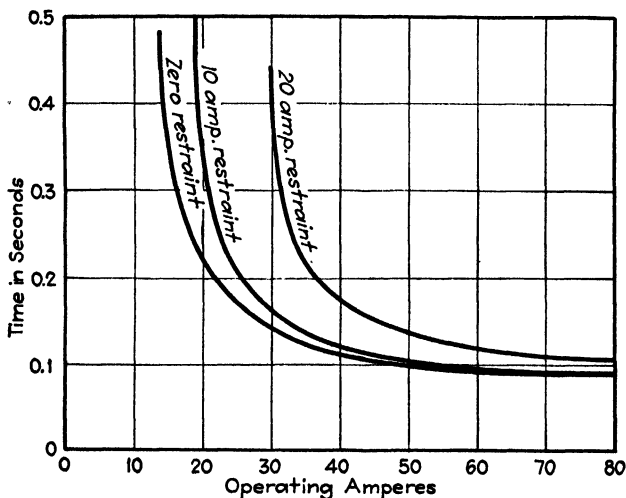


FIG. 22.

FIGS. 21. and 22.—Time-current characteristics of two-element current balance relay.

bins an overcurrent element with a directional or wattmeter element. The overcurrent element is identical with the induction overcurrent relay already described. The directional-element

electromagnet is made up of punchings similar to those used for the overcurrent element, having a potential coil on the lower pole and a current element on the upper pole.

There are two general types of single-phase directional overcurrent relays, one having the overcurrent characteristics as shown in Fig. 2 and the other characteristics as illustrated in Fig. 6. In addition to this, the directional-control feature is accomplished in each of these types in a different way.

The distinction between a *directional overcurrent relay* and a *directional overcurrent relay with directional control* should be clearly understood, or confusion is likely to result. In a simple directional overcurrent relay the overcurrent-element contacts and the directional-element contacts are connected in series in the trip circuit of the circuit breaker. If the direction of flow of short-circuit power is into the bus, the overcurrent-element contact will close, but the directional contact will remain open.¹ Therefore, the breaker will not trip. If the direction of the power flow is away from the bus, both the overcurrent-element contacts and directional-element contacts will close and trip the breaker.

However, the development of the directional overcurrent relay with directional control was necessary owing to a condition which may result where there is a multiple feed to a bus. Under these circumstances a fault on some other line may operate the overcurrent element without operating the directional element. This fault may then be satisfactorily cleared by the operation of the proper relay. However, this sequence of events may cause the directional contact of the first mentioned relay to close (due to power-load flow or system oscillating after fault has cleared) before the overcurrent element has had time to reset, resulting in an incorrect breaker operation. Directional overcurrent relays have therefore been developed which control the operation of the overcurrent element so that the overcurrent element cannot operate unless the directional-element contact is closed. The directional-element contact is made an inherent part of the operation of the overcurrent element, and only the overcurrent-element contact is connected in the trip circuit.

¹ Refer to Chap. VIII for a complete explanation of directional-overcurrent-relay connections.

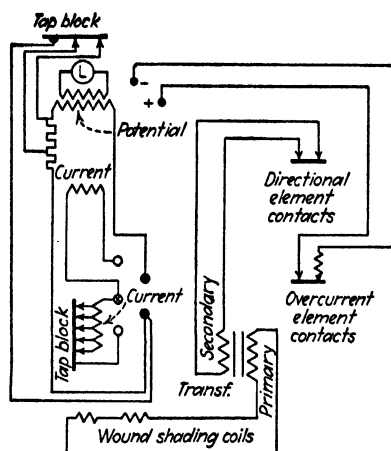


FIG. 23.

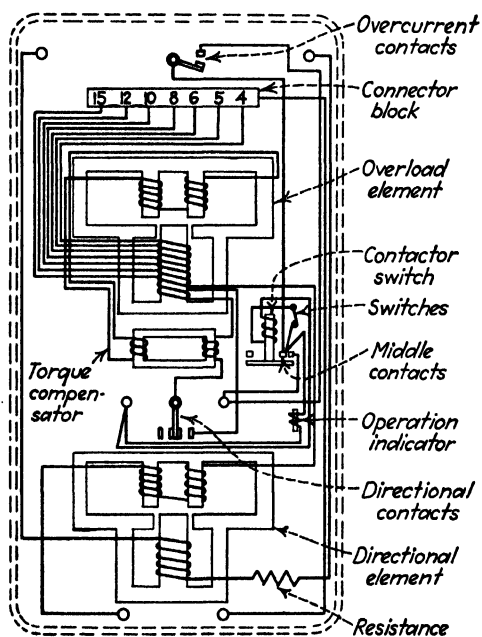


FIG. 24.

Figs. 23 and 24.—Two general types of directional overcurrent relays with directional-control feature.

The diagram of internal connections for one type of single-phase directional overcurrent relay with directional control is shown in Fig. 23. The shading coils are connected to the primary winding of a small interposing potential transformer. The secondary winding of the transformer is connected across the contacts of the directional unit so that it is normally open-circuited when these contacts are open.

A second type of directional overcurrent relay with directional control is shown in Fig. 24. In this case the directional-element

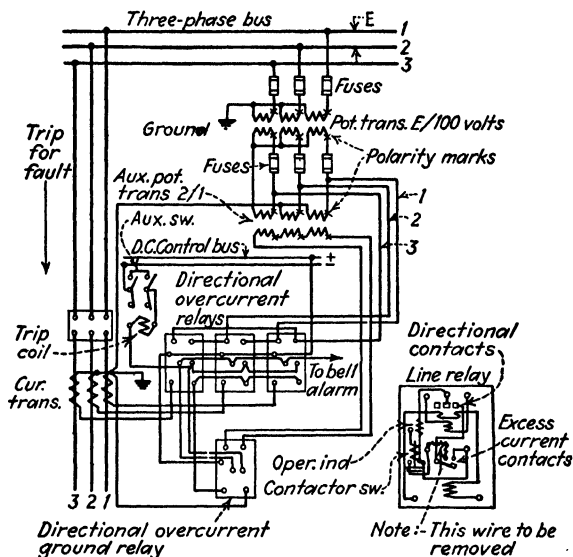


FIG. 25.—Complete connections of overcurrent directional protection using potential transformers.

contacts are connected in the primary of the torque compensator, and these directional contacts must be closed before the overcurrent element can operate.

The single-phase directional overcurrent relays are available in 4- to 15-amp., 2- to 6-, and 0.5- to 2.5-amp. ratings. A complete diagram of connections is given in Fig. 25, showing use of potential transformers and in Fig. 26 for applications where the potential device is used.

Polyphase Directional Overcurrent Relay.—In place of using three single-phase directional overcurrent relays, it is advantageous in some applications to use one polyphase directional

relay and three overcurrent relays. The polyphase directional relay consists of three directional elements mounted to operate on a common shaft. The relay contacts are therefore opened or closed in accordance with the resultant direction of current flow in all three phases. This is of special importance on systems with high transmission voltages and a multiplicity of solidly grounded neutrals, where it is possible for heavy current to

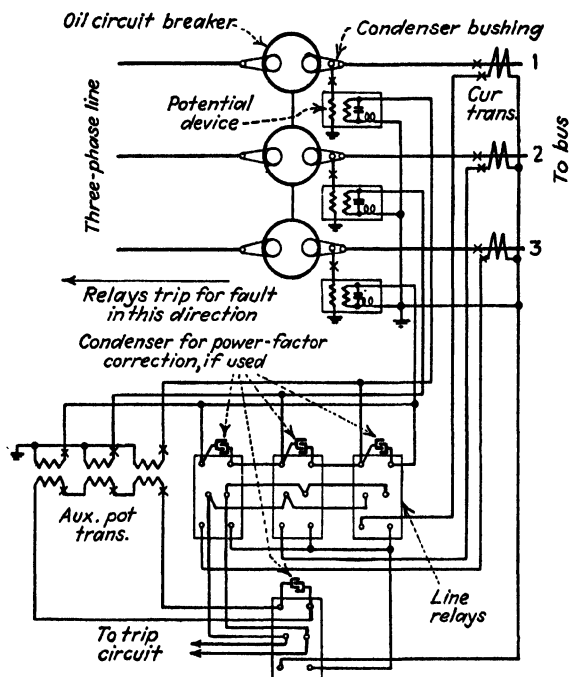


FIG. 26.—Complete connections of overcurrent directional protection using condenser potential device.

flow in all three phases during a line-to-ground fault, with the result that certain single-phase relays may trip incorrectly. Where this difficulty is experienced, the three-phase relay offers the best solution. The residual current furnishes the predominating torque during phase-to-ground fault conditions, so that the current in the good phases cannot cause faulty operation. Similarly, the fault current on such a system should be sufficiently large so that the load current will never affect the relay in such a direction as to prevent proper operation.

The polyphase directional overcurrent relay has a distinct field of application. In many cases, either the single-phase or three-phase relay will operate satisfactorily. On systems having the neutral grounded through a high resistance, the polyphase directional relay may operate incorrectly, since the torque due to the normal load current on the unfaulted phases may exceed that due to the relatively small fault current.

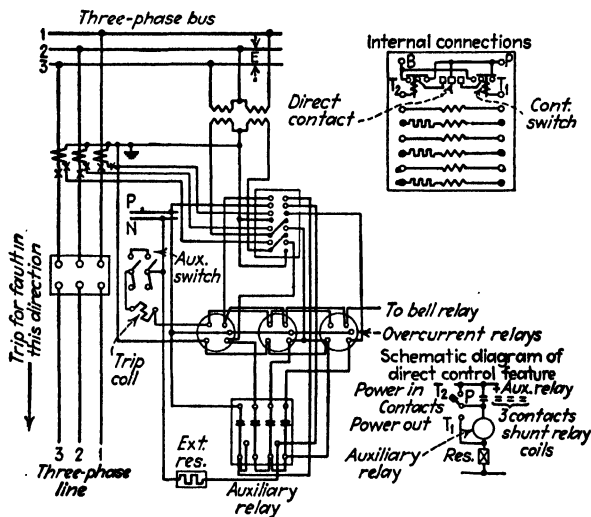


Fig. 27.—Overcurrent directional protection using polyphase directional relay.

The connections for line-fault protection, only, by means of a polyphase directional overcurrent relay and three non-directional overcurrent relays are shown in Fig. 27. This scheme also includes the directional-control feature, using an auxiliary relay for that purpose. The auxiliary relay short-circuits the current coils of the three overcurrent relays, except when the contacts of the polyphase directional relay are closed in a tripping direction. This avoids the possibility of incorrect operation on a good line immediately following a fault, which may occur if the contacts of one of the overcurrent relays have been closed by fault current and the sudden reversal of power flow also closes the directional contacts of the polyphase relay before the contacts of one of the overcurrent relays have parted. The connections giving not only line protection but also ground

protection are shown in Fig. 28, the ground relay in this case being the single-phase directional overcurrent relay. A set of typical characteristic curves for a polyphase directional relay is

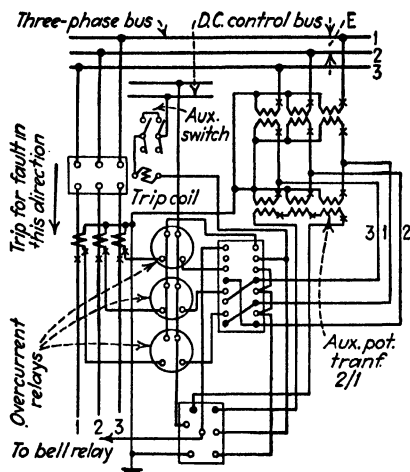


FIG. 28.—Complete overcurrent directional protection using polyphase directional line relay and single-phase ground relay.

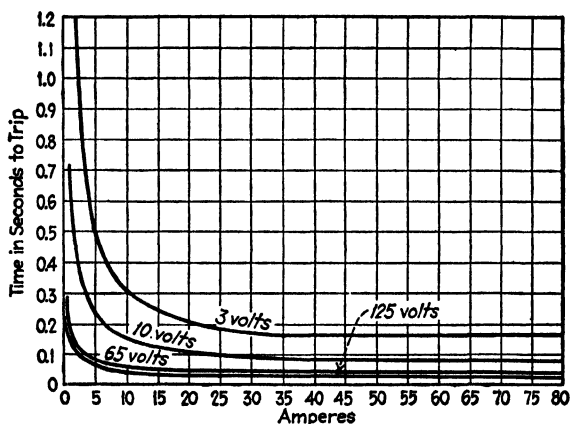


FIG. 29.—Time-current characteristics of polyphase directional relay shown in Figs. 27 and 28.

shown in Fig. 29. A second type of polyphase directional relay is shown in Fig. 30, and a complete diagram of its application to a single line is illustrated in Fig. 31.

To obtain the maximum torque of the relay at lagging fault currents, internal resistors are connected in series with the potential coils (Fig. 30). These resistors carry the potential flux of each element forward from its natural position to a point approximately 90° behind the phase position which the fault current is expected to occupy. Since the torque is proportional to the sine product of effective potential and current fluxes, the

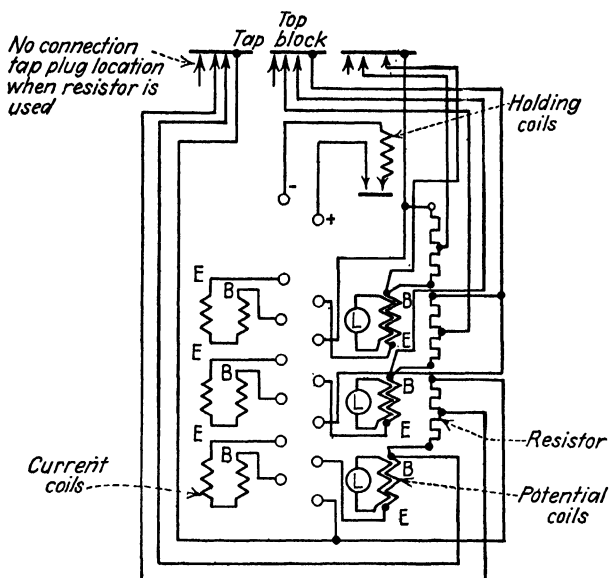


FIG. 30.—Schematic internal connections of a second general type of polyphase directional relay.

greatest torque possible with the fluxes available is thus obtained when it is most required. Taps are brought out from these resistors to three tap plates which are marked 5° , 30° , and 45° . These angles indicate the amount by which the current in the relay leads its unity-power-factor position when maximum torque occurs. They should not be confused with angular displacements between line voltages and currents.¹

Cross-connected Directional Overcurrent Relays.—If there is not sufficient current at a substation, as shown in Fig. 32, to give reliable directional overcurrent protection with relays connected at breakers *c* and *d*, as shown in Fig. 25, the cross-

¹ Refer to Chap. VIII for further details of these connections.

connected directional overcurrent relay system can be used to advantage. Additional advantages and other applications of the cross-connected relay system are given later in this chapter.

The simplest and most general arrangement consists of two sets of single-phase directional overcurrent relays with their

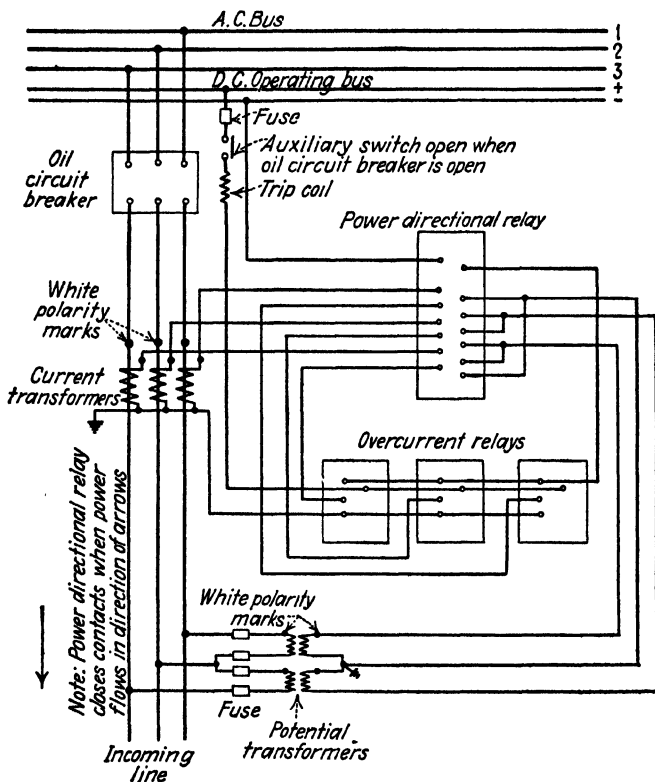


FIG. 31.—Directional overcurrent protection using polyphase directional relay shown in Fig. 30.

current coils connected in series across the cross-connected current transformers of the two lines, as illustrated in Fig. 33. The potential connections of one set of relays are reversed from that of the other set so that only the trip circuit of the breaker carrying the greater current will be closed by the unbalanced circuit. The same results can be obtained by using one set of duo-directional overcurrent relays as shown in Fig. 34.

The use of cross-connected current transformers with directional relays permits unusually rapid clearing of faults. Using the circuit-breaker designations of Fig. 32, the current relations for one phase are shown in Fig. 33 for a line fault and in Fig. 35 for a bus fault. When a line fault occurs, assuming no backfeed

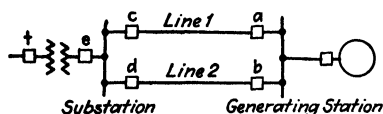


FIG. 32.

FIG. 32.—Typical parallel line section.

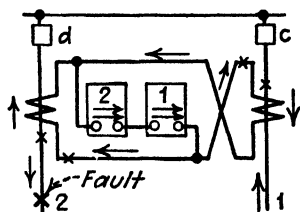


FIG. 33.

FIG. 33.—Schematic connections of cross-connected overcurrent directional-relay system.

from the substation, the currents in the two lines at the substation are equal and the current through the relay is twice the current in the individual lines. For a bus fault, if the currents in the two lines are balanced, the current through the relay is zero. This characteristic of the cross-connected scheme

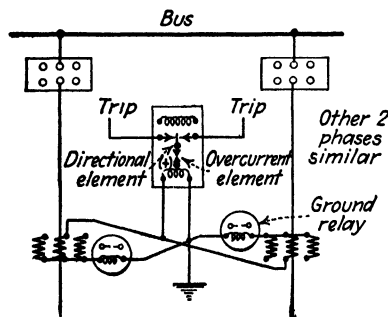


FIG. 34.—Cross-connected relay system using duo-directional over-current relay.

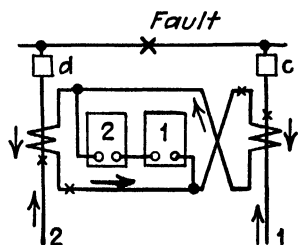


FIG. 35.—Flow of current in cross-connected relay system assuming bus fault.

allows the use of low current and low time settings on the directional relays. The potential connections to the directional elements are made so that, with the current flowing as in Fig. 33, the directional contacts of relay 2 will close and those of relay 1 will open. For a fault on line 1, the current flow through the

relays is in the opposite direction, so the directional contacts of relay 1 will close and those of relay 2 will open. Unless the relays are equipped with the directional-control feature, both overcurrent contacts will close if sufficient current is flowing to operate them.

If the relays at the substation are set for less than the total load current in the two lines, trouble will be encountered in restoring a circuit once it is tripped out. For example, if line 1 is opened at the substation, the total load must be carried over line 2. If the current setting of the relays on line 1 is less than the total current, both directional and overcurrent contacts will

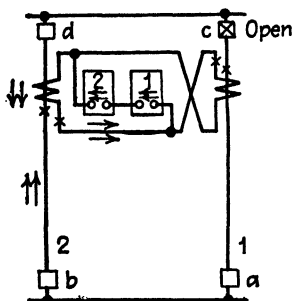


FIG. 36.—Current flow in cross-connected relay system with one line open.

close, as the direction of current flow in the secondaries of the current transformers on the relays of line 1 is such as to cause the tripping contacts of relay 1 to be closed while the breaker is open. This will be clear from an inspection of Figs. 33 and 36. In Fig. 33 the current is in such a direction as to trip relay 2 and open the directional contacts of relay 1. In Fig. 36, for the condition where line 1 is open, it is apparent that the total current flows through the relays and its direction is opposite to that in Fig. 33. Therefore, the directional and overcurrent contacts will close on relay 1. As a result of this, if line 1 is closed, it will immediately trip out, since the trip circuit is already closed at the instant the circuit breaker is closed.

Also, if cross-connected relays are used at the generating station, the opening of line 1 at the substation will tend to trip the relays on line 2 at the generating station. The total current flows in line 2, and, since it is flowing away from the bus, the direction is such as to trip relay 2 at the generating station. The obvious remedy for this condition at either the substation or the generating station is to use a higher current tap setting. This, however, partially defeats the purpose of the cross-connected scheme. The maximum and minimum short-circuit currents and the load currents determine the current tap setting that must be used, and a complete analysis must therefore be made for each case.

One method which can be employed to allow the use of a lower setting is to connect mechanically operated auxiliary switches across the relay coils. For example, auxiliary switches on the circuit breaker of line 1 may be so connected that, when the breaker is in the open position, the auxiliary switch short-circuits the current coil on relay 1. If the circuit breaker of line 1 trips, the secondary current from line 2 passes through the relays on line 2 but is by-passed by the auxiliary switch on relay 1. When the breaker on line 1 is reclosed, it will then trip only if the fault is present on the line. If this scheme is used, it is possible to set the relays for any desired value of current.

Fault-detector relays may be used to accomplish the same results and at the same time avoid the disadvantage of introduc-

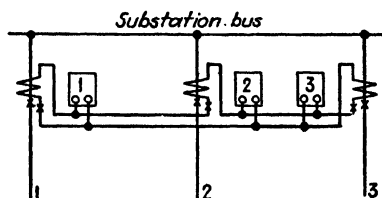


FIG. 37.—Multiple parallel-line protection.

ing an additional hazard in the current-transformer circuit. Using this scheme, the trip circuit from the relay cannot be completed through the detector relay unless a fault actually exists on the system. For the usual types of low-voltage circuits, it is unnecessary to go to any of these elaborate schemes in order to provide low-current tripping.

Multiple Parallel Lines.—To get proper voltage regulation at an important substation, it is common to have three, four, and even up to six and eight sections of line operating in parallel. Theoretically, any number of parallel cables can be provided with balanced protection, but the large number of relays and complications and possibilities of leakage currents makes this inadvisable.

Three parallel lines are often provided with balanced protection using either current balance relays or directional relays. A schematic diagram of directional relays applied to three lines is shown in Fig. 37. In some cases, two lines are balanced against each other and the third operates with single-line protection. With four lines, it is advisable to balance the lines in pairs

rather than to attempt a single complete balance of the four lines.

Limitations of Radial System.—Reference to Fig. 7 will indicate that the radial system has distinct limitations from the standpoint of service security, regulation, and economical use of copper. The opening of the breaker at Station A interrupts service to Substations C, F, and G. In most radial systems, many more feeders are affected by the opening of a single breaker than are shown in the simple arrangement of Fig. 7. Modern service requirements and regulation standards require inter-connection of feeders which give continuity of service under practically all fault conditions.

THE LOOP SYSTEM

The addition of one line as shown in Fig. 38 will serve to form a simple loop when operated in parallel with a sectionalized

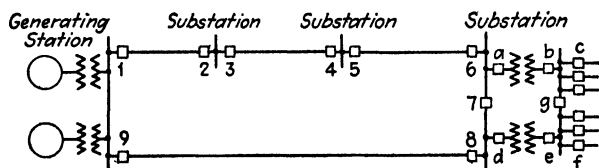


Fig. 38.—Typical arrangement of loop system.

radial line. It is necessary to use overcurrent directional relays on breakers 2, 3, 4, 5, 6, and 8. In Fig. 38, there are but three substations, only one of which is shown in detail. The relays on the substation circuits must act selectively with the relays on the high-voltage circuits. Since the substation and line relaying are not independent, speed of operation of the high-voltage circuit relaying must be sacrificed; this may lead to more or less serious difficulties depending upon the characteristics of the particular system. If the transformer capacity in the substation is small, the short-circuit current on the low-tension side may be so low that the transmission-line relays may be set above this current value. However, if the transformers are of large capacity, this would not be possible because the maximum current setting of the transmission-line relays is limited by the minimum fault current on those circuits.

It will be assumed that the substation transformers in Fig. 38 are of large capacity, and we will consider, first, the transmission-

line relays only. Relay 8, being at the farther end of the first section of line out of the generating station, can be given a minimum setting of 0.2 sec. Relay 5 is then timed for 0.7 sec., relay 3 for 1.2 sec., and relay 1 for 1.7 sec. In the opposite direction around the loop, relay 2 is timed for 0.2 sec., relay 4 for 0.7 sec., relay 6 for 1.2 sec., and relay 9 for 1.7 sec. The feeder relays in the substation, such as relays *c* and *f*, can be given a time setting of 0.2 sec., relay *b* 0.7 sec., and relay *a* 1.2 sec. Obviously, for a fault within the station, relays *b* and *a* will not operate selectively with relays 5 and 3. The latter must therefore be given a longer time setting. This is not desirable, because the time for relay 5 would be 1.7 sec., for relay 3 it would be 2.2 sec., and for relay 1 it would be 2.7 sec. This, however, is too long for a large system.

The conditions described are on the basis that with a short circuit on the low-tension bus, the current flowing in the substation and transmission relays is of such value that all relays will operate on the flat part of their characteristic curves. Actually on most systems, this would not be the case, and improvement in timing could be secured by properly grading the current tap settings of the various relays. The low-voltage feeder relays *c* and *f* are normally set for 200 to 300 per cent of the circuit rating. The total feeder capacity fixes the setting of relay *b*, and the transformer capacity fixes the setting of *a*. If *a* is given a high-current setting, no protection will be afforded for heavy, sustained overloads, and, furthermore, a failure within the transformer may not produce operation of overload relays.

The problem of selective timing between substation and transmission relays is considerably simplified when distance relays are used on the high-voltage circuits. When a fault occurs on the low-tension side of the transformers, the distance relays at the far end of the high-voltage sections adjacent to the substation tend to trip for a fault distance equal to the full section length plus the reactance of the substation transformers. Due to the high-voltage drop through the transformers and transmission circuit, the restraint on the voltage coils of the distance relays is of a high value, so that the operating time is delayed. In some cases this characteristic allows independent timing of substation and transmission relays. This will, of course, depend upon the transformer capacity, line reactance, and the short-circuit current and its distribution.

The most satisfactory method to eliminate the timing problem is to apply transformer-differential and bus-differential protection in the substation, as explained in Chap. X. The low-voltage-bus-differential protection can be made the same as the high-voltage-bus-protective schemes. A fault on one section of the high-tension bus will clear circuit breakers 6, 7, and *a*, and a fault on the other section will clear 7, 8, and *d*. A fault in one transformer will clear *a* and *b* and in the other transformer will clear *d* and *e*. A fault on one section of the low-tension bus will clear the bus-tie breaker, as well as the feeder breaker and the low-

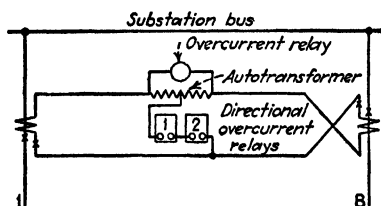


Fig. 39.—Cross-connected relay system with overcurrent relay and autotransformer added to give bus protection.

tension transformer breaker on that section. Any fault which may occur in the substation will be isolated by clearing the proper circuit breakers, and only the faulty section will be removed from service. Each circuit breaker should be included in two differential loops to insure that circuit-breaker bushing flashovers or

trouble inside the breaker will be properly cleared.

The differential relays may be given a low time setting and thus obviate the necessity of increasing the time setting of the transmission-line relays. If proper current transformers and relays are used in the differential protection, there will be no tendency for operation on through faults. Overcurrent relays for back-up protection may be installed for operating circuit breakers *a*, *b*, *d*, *e*, and *g*.

An inspection of Fig. 35 will show that the cross-connected relay scheme does not offer protection to the substation bus. Several schemes are applicable for securing bus protection and all require the use of additional relays. One scheme is to install a 2:1 ratio current balance autotransformer in series with each pair of cross-connected current transformers as shown in Fig. 39. An overcurrent relay is connected across each of the three autotransformers. This scheme is unique in that the current in the overcurrent relay for a given load is the same, irrespective of whether one or two lines are in service. When two lines are in service and carry equal currents, only magnetizing current will

flow through the autotransformers, since their impedance is high in comparison to that of the overcurrent relays. Therefore, with 5 amp. in the secondary of each circuit, the total current through the overcurrent relay is also 5 amp. For the same load conditions with only one line in service, there will be 10 amp. flowing in the secondaries of the current transformers on that circuit. Because of the high impedance of the secondaries of the current transformers on open circuit, this 10 amp. will all flow through the directional overcurrent relays. This current will divide in the autotransformer, 5 amp. flowing through one-half and 5 amp. flowing in the opposite direction through the other

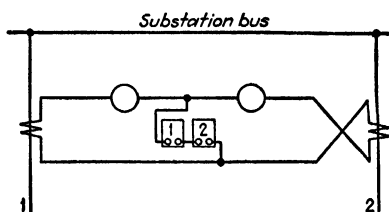


FIG. 40.—Cross-connected relay system with overcurrent relays added for bus protection.

half and through the overcurrent relay. It is necessary when using this scheme to check carefully the characteristics of the current transformers in order to be sure that the burden is not sufficiently great to break down the ratio at high values of current.

The connection for one phase of an alternate scheme which requires the use of two sets of overcurrent relays to provide bus protection is shown in Fig. 40. The scheme is not so satisfactory, because with only one line in service the overcurrent relay on that line carries twice the current that flows when both lines are in service. This necessarily means that the current setting of the overcurrent relay must be higher than the maximum load current. The characteristics of the particular system determine the selection of the proper scheme of bus protection.

A typical loop system consisting of one generating station and four substations is shown in Fig. 41. Straight overcurrent relays are applied at the generating station, and overcurrent directional relays are applied at each substation. Proceeding in a counterclockwise direction around the loop, the time settings of the breakers at *E*, *D*, *C*, and *B* are increased in steps of 0.5 sec.,

starting with the incoming breaker at *E*. Proceeding in the clockwise direction, the settings of the incoming breakers at *B*, *C*, *D*, and *E* are likewise increased in 0.5-sec. increments. In each case, the lowest setting on the loop is on the first breaker out from the generating station. A check of the circuit-breaker

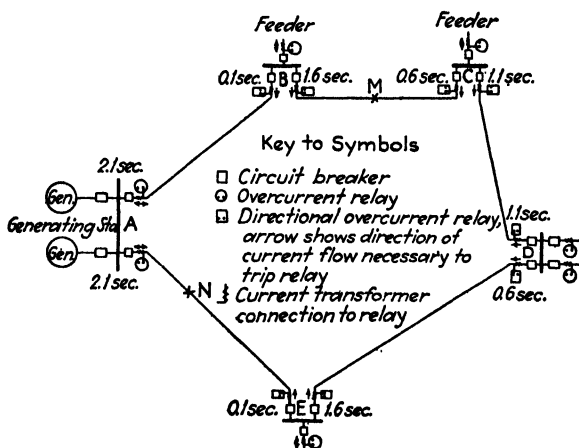


FIG. 41.—Loop-system relay protection.

opening time in many cases makes it possible to cut down the 0.5-sec. increment, allowing five substations to be protected selectively without exceeding 2 sec. at the generating station.

Loop System with More than One Source of Power.—A loop system containing two sources of power is shown in Fig. 42. Let

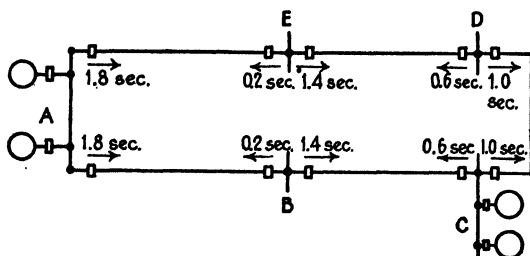


FIG. 42.—Loop system with two sources of power.

it be assumed that the overcurrent directional relays are timed selectively for clockwise power flow by starting from Station A and proceeding counterclockwise around the loop. In a similar manner the relays set for protection against short-circuit flow in the counterclockwise direction are set selectively in the clockwise

direction. Assume further that a fault occurs between Station A and Substation E. The circuit breaker of this section at Substation E opens in a relay time of 0.2 sec. The circuit breaker at Station A has a relay time of 1.8 sec. It is obvious that after the breaker opens at Substation E it is possible that the short-circuit power from source *C* may be of sufficient magnitude to operate the relays timed for 0.6 sec. at *C* and the relays

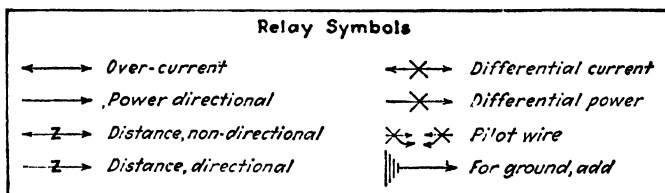


FIG. 43.—Relay symbols.

timed for 0.2 sec. at *B*, before the station breaker at *A* on the faulted section can operate. It is for this reason that impedance-type relays may be required for a system plan shown in Fig. 42 if adequate protection is to be effected.

A more convenient system of relay symbols than indicated in Fig. 41 is given in Fig. 43. This scheme of symbols is used in designating the relays for Figs. 44 and 50.

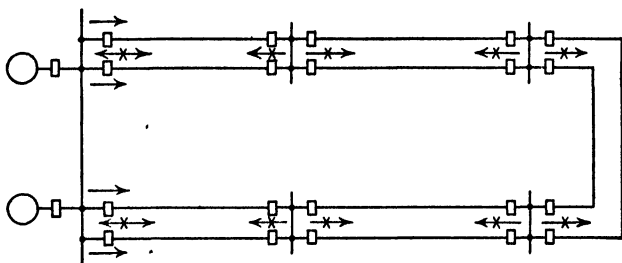


FIG. 44.—Double line loop.

Double Line Loop.—Where the load density justifies a double loop, an arrangement such as Fig. 44 affords a very effective and reliable system plan. It gives a flexible circuit layout which can be relayed very satisfactorily. Current balance relays can be used at the generating station to give fast relay operation when both lines of a section are in service. Complete protection at the substations can be obtained by the use of a cross-connected overcurrent directional-relay system. This can be of the simple

cross-connected scheme shown in Figs. 33, 35, and 36, but a more effective scheme is shown in Fig. 45.

The voltage and current connections to the relays are so made that the relays of each set close their directional-element contacts when unbalanced power is flowing out over the line which those relays protect. Each relay is provided with a tap between the directional-element contacts and the over current-element contacts, and these taps are joined to a common bus for each pair of lines, so that any overcurrent element can trip either

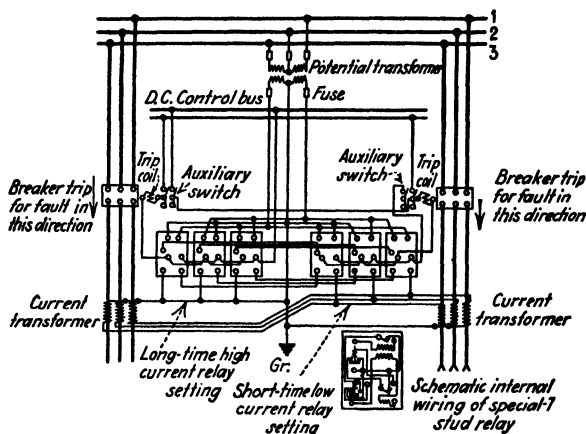


FIG. 45.—Cross-connected relay system with special trip circuit interconnection.

circuit breaker by acting through the directional-element contacts of the corresponding set of relays.

With the feeders carrying equal loads, there is no current flowing through the current windings of the relays. As soon as a fault develops on one line, an unbalanced current circulates through the relays, causing the directional-element contacts of the faulty-line group to close and those of the good-line group to open. The faulty-line circuit breaker is tripped by the closing of the contacts of the overcurrent element having the short time setting. As soon as the faulty-line circuit breaker has opened, the control circuit to the short-time overcurrent element is broken by the series connection through the circuit-breaker auxiliary switches. The remaining line then has directional overcurrent protection only, the time and current settings of the overcurrent element being determined by operating conditions

on the loop. A schematic of the trip connections is shown in Fig. 46.

Two additional cross-connected relay systems are shown in Figs. 47 and 48. In Fig. 47 one polyphase duo-directional relay

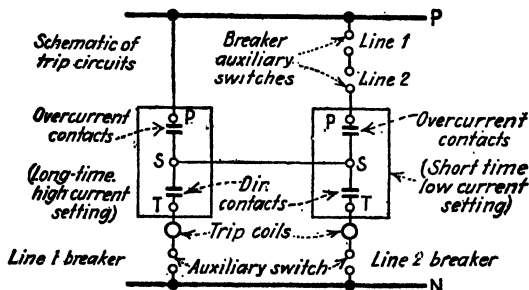


FIG. 46.—Schematic diagram of trip circuit interconnection for relay system, Fig. 45.

and three overcurrent relays are used to give single-phase and three-phase fault protection. An internal schematic of the polyphase relay is shown in Fig. 27. In Fig. 48 this same protec-

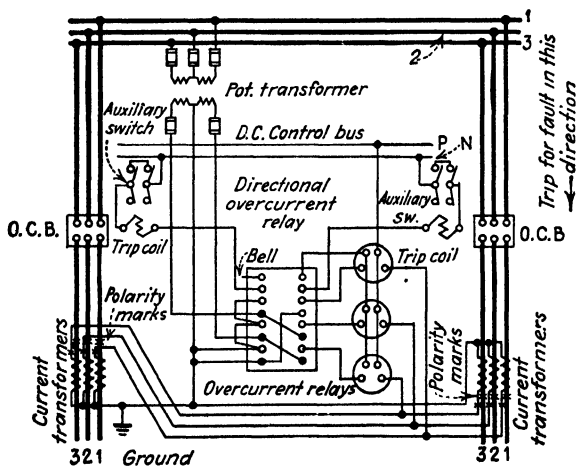


FIG. 47.—Cross-connected relay system for parallel lines.

tion is accomplished by the use of three single-phase duo-directional overcurrent relays.

On certain extensive high-tension systems, there may arise a problem of timing which cannot be taken care of by the cross-connected schemes so far discussed. The relay system illus-

trated in Fig. 45 may have the disadvantage that the high-current, long time setting has to be high enough to allow two breakers to

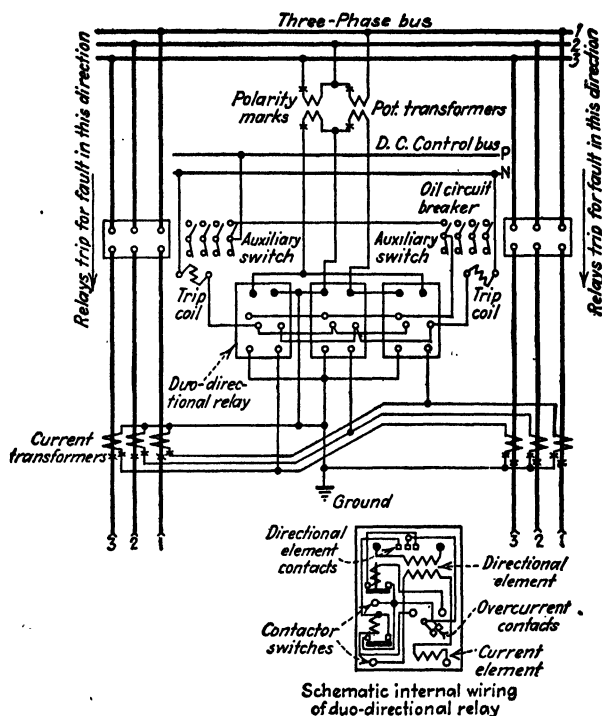


FIG. 48.—Cross-connected relay system for parallel lines.

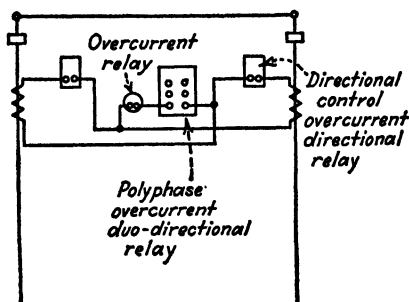


FIG. 49.

open, or, in other words, the time must be at least 1 sec. In Fig. 49 the polyphase overcurrent duo-directional relay and overcurrent relay take care of parallel-line protection, and the

directional-control overcurrent directional relays supply single-line and back-up protection. The advantage of this particular scheme is that the directional-control relays need be set only long enough to allow one breaker to open, about 0.5 sec.

It is also possible to use directional-control relays across the autotransformer shown in Fig. 39 to take care of single-line bus and back-up protection against phase-to-phase faults. Directional-control relays so used have to be set a little higher than the directional-control relays shown in Fig. 49, but not so high as the high-current, long time setting of Fig. 45.

In cases where there are too many line sections in series to apply overcurrent directional relays, it is possible to apply pilot-

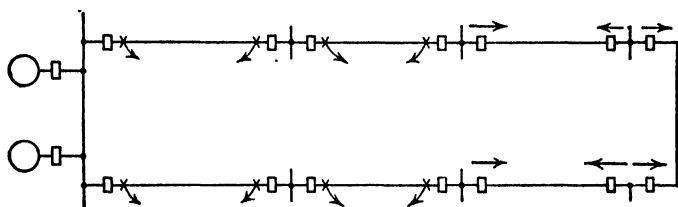


Fig. 50.—Loop system with seven line sections showing application of pilot-wire protection on four line sections.

wire relaying to one or more sections, as shown in Fig. 50. This system of relaying is covered in detail in Chap. XII. The impedance relay is also applicable to system connections of Fig. 50.

The Induction-type Impedance Relay.—The impedance relay is of the same type as the ordinary overcurrent relay but with the addition of a voltage-restraining coil. Its operation is therefore dependent on both excess current and low voltage.

The pull of this voltage coil, which opposes the closing of the contacts, is directly proportional to the applied voltage. During a short circuit the voltage at the point of fault will, of course, be approximately zero, and the voltage applied to the relay-restraining coil will therefore be the voltage drop from the point at which the relay is applied to the point of fault. The time of operation of the relay is inversely proportional to the short-circuit current and directly proportional to the voltage drop. Since the impedance from the relay to the point of fault may be expressed as voltage drop divided by short-circuit current, it can be seen that

the impedance relay has an operating time that varies directly as the impedance to the fault. This relay uses no definite selective lever settings, but automatically adjusts its time according to the location of the short circuit.

Its principal application is for protection against line-to-line faults rather than against line-to-ground faults, owing to the difficulty of calculating the proper relay settings for the latter case. Sufficiently accurate calculations can be made for three-phase and single-phase line-to-line faults to determine the time setting of the relay and this time will be practically constant for a given fault location. Ground-relay settings can be only approximate because of variations in the zero-sequence impedance of a transmission line and variations in earth resistance and in contact resistance at the point of ground.

Instead of operating the contacts directly, the movement of the disk winds up a spring, one end of which is geared to the disk shaft. The other end of the spring is connected to a rocker arm pivoted at its center and mounted directly above the disk. This rocker arm carries the contact on one end, and the core of restraining coil is suspended from the other end. A special construction of the voltage-restraining element is used to overcome the effect of residual magnetism and to eliminate friction.

There are many induction-impedance relays in service, and they have fulfilled a real need in cutting down the time of operation on systems where a number of sections in series have made selective definite minimum time settings too high for faults close to the generating sources. Also it has made possible the relaying of loops with more than one source of power.

Satisfactory operation of induction-impedance relays is being obtained, but it requires careful and frequent maintenance in order to keep the relays accurately set.

The high-speed reactance or impedance relay is now available and will give simpler and faster protection. Refer to Chap. XIII, for a complete analysis of high-speed relaying.

GROUND RELAYING

It is becoming common practice to ground all transmission and distribution systems either solidly or through resistance or reactance. This is due to many important factors which serve to improve materially the continuity of service. When a system

is grounded, it is highly desirable to supply ground-relay protection.

An ungrounded system is likely to be often subjected to the arcing-ground disturbance. Arcing-ground disturbances are hazardous to insulation and act in such a way as to cause flashovers at points far removed from the location of the initial flashover to ground. This causes relay operations which are difficult to locate and explain.

An arcing-ground disturbance takes the nature of a high-frequency oscillatory voltage between line and ground. It occurs when one phase of a transmission line flashes over to ground. Severe oscillatory voltages then appear on the other two phases. Briefly stated, these voltages are caused by the negative-resistance characteristic of arcs in combination with the fact that the current passing through the arcing ground is a capacity current and therefore 90° out of phase with the voltages. Hence, when the current approaches zero and the arc goes out, the voltage on the lines is close to its maximum. This leaves a charge trapped on the system when the arc extinguishes. When the arc restrikes at a peak of the voltage wave, it discharges the condenser formed by the lines and the earth. High-frequency oscillations are generated which may reach several times the normal voltage between lines and ground.

Ground-relay systems offer the most satisfactory and positive means of clearing short circuits. The ampere ratings of the relay coils are designed for more sensitive pick-up than can be accomplished by line-type relays. Ground relays can be set irrespective of load currents in that, under normal load conditions, there is no current flow through the ground relay. In other words it is a system of relaying which can be specifically designed for fault currents and does not have to be a compromise relay system such as is the case where settings must be used that will take care of load currents. Additional advantages can be listed as follows:

1. Ground relays are independent, for all practical purposes, of surges due to switching, synchronizing, motor starting, and condenser starting.
2. Ground protection is based on zero current in the relay under normal conditions, and therefore such protection can be made very sensitive.
3. Ground relays can easily be made to operate at high speed.
4. The high attenuation of ground-fault current with distance gives an effective means of providing selectivity.

5. Ground relays operate fast enough to clear up faults with a minimum amount of damage to lines and equipment.

6. In many cases systems of different voltages will not interchange ground current and make possible the setting of ground relays on one voltage system independently of those on another voltage system.

Non-directional Ground Protection.—The connection for residual or zero-sequence overcurrent protection is illustrated in Fig. 51. The ground relay may be of the instantaneous-plunger type or of the induction-overcurrent type. The instantaneous-type relay is described in Chap. XIII. The induction-over-

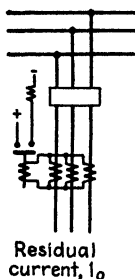


FIG. 51.

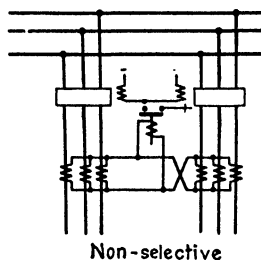


FIG. 52.

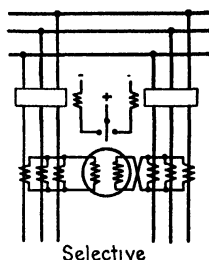


FIG. 53.

Figs. 51, 52, and 53.—Schematic connections of overcurrent ground-relay systems.

current relay is described in detail in the first few pages of this present chapter. (Refer to Figs. 1 to 6, inclusive.) The ampere tap values are 0.5 to 2.5 amp. and a characteristic load-time curve is shown in the upper curve of Fig. 5. The complete connections for three overcurrent line relays and one overcurrent ground relay are shown in Fig. 9.

Non-selective Ground Relaying.—Figure 52 illustrates non-selective relaying of two circuits for protection of ground faults. Any unbalance of ground current in the two lines will operate the ground relay and trip both breakers. For through faults (equal ground current in each line) no relay operation will result.

Selective Ground Relaying.—Figure 53 illustrates selective ground relaying. The selective-differential relay has already been discussed and its connections are shown in Figs. 14, 15, and 18. In Fig. 13 are shown the complete connections for three line relays and one ground-relay current balance selective relay applied to parallel lines. In Fig. 54 one ground balance relay is used in conjunction with cross-connected line relays.

In Fig. 55 cross-connected duo-directional phase relays and a duo-directional ground relay are used in conjunction with auto-transformers and overcurrent relays to give complete parallel-line protection.

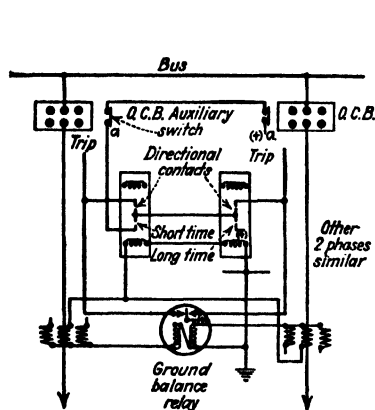


FIG. 54.

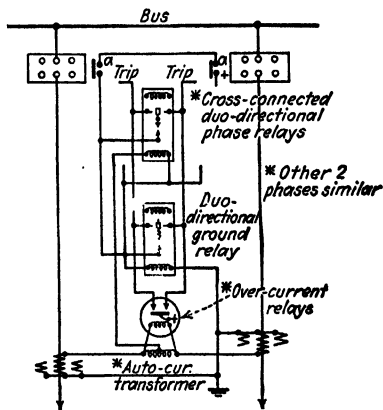


FIG. 55.—Application of duo-directional ground-relay to cross-connected line-relay system.

When loads are tapped to paired feeders with neutral current balance protection, a fault on the branch circuit does not affect the balanced neutral relays on the main circuit provided the tapped loads are taken through delta transformation as shown

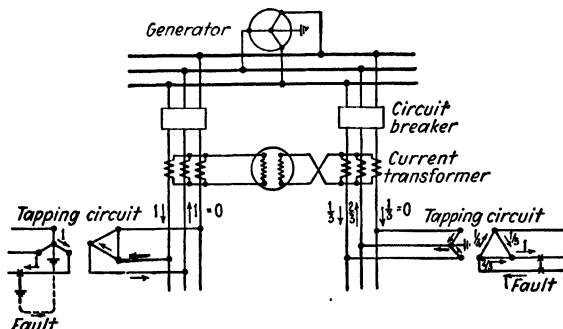


FIG. 56.—Current flow set up by faults on tapped circuits through delta-Y and Y-delta connected transformers.

in Fig. 56. The vector sum of such currents is zero in the main circuit. A single-phase fault through a Y-delta bank is illustrated on the right-hand feeder and a ground fault through a delta-Y bank is illustrated on the left-hand feeder.

Directional Ground Relaying.—In Fig. 57 is shown the connection for obtaining residual or zero-sequence voltage E_0 , which can be used in conjunction with residual current I_0 , to give directional ground protection. Internal connections for two types of overcurrent directional ground relays are shown in Figs. 23 and 24. Complete circuit connections for three line relays and one overcurrent directional ground relay are shown in Figs. 25 and 26. This scheme requires three potential trans-

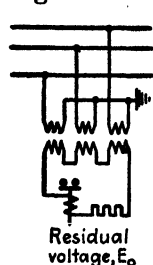


FIG. 57.—
Schematic connections for obtaining zero-sequence voltage.

formers connected star-star together with a set of small auxiliary potential transformers connected star-delta. The directional-relay potential is connected inside the auxiliary-potential-transformer delta. The three main star-star connected potential transformers must be grounded and connected directly to the circuit being protected. In other words, it is not possible to use low-tension potential transformers for ground relays on the high-tension line. The auxiliary transformers are used in order to eliminate unbalanced delta voltages from interfering with line-type relay connections which are usually connected to the secondary of

the main potential transformers.

Effect of Solid Grounding of System Neutral on Relays Utilizing the Residual Voltage.¹—It is important to give careful consideration to the transformer-neutral grounds or unsatisfactory relay operation is likely to result. Residual voltage may be very limited in case of a ground fault on a system with neutral solidly grounded, provided the fault resistance is of such magnitude as to have appreciable voltage drop.

As already explained, the standard means of securing the residual voltage (resultant of the phase-to-ground voltages) is to open one corner of a delta-connected secondary of three potential transformers whose primaries are connected in star with grounded neutral. This is further illustrated in the right portion of Fig. 58. It will be noted that the potential-transformer-secondary voltages are proportional to the phase-to-ground voltages of the system, and on account of the delta connection any voltage appearing across the opened corner of the

¹ Acknowledgment for this analysis is made to E. H. Klemmer.

delta is the vector sum $V_{1g} + V_{2g} + V_{3g}$ or the resultant of the three phase-to-ground voltages.

Under normal conditions the phase-to-ground voltages V_{1g} , V_{2g} , and V_{3g} form a balanced system of three-phase vectors and hence their resultant voltage is zero. Figure 59 illustrates vectorially this condition of no fault and applies whether the system neutral N is grounded or ungrounded.

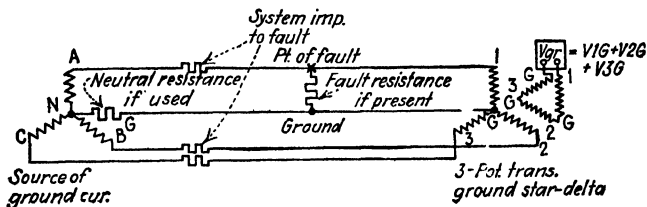


FIG. 58.—Schematic system connections used as basis for vector diagrams, Figs. 59 to 64.

In the accompanying vector analysis ABC represents the generated e.m.f. triangle, and 123 the terminal-voltage triangle at the point of fault. N represents the neutral point of the system, and G designates ground potential, which is used as a reference. Therefore, E_{NA} , E_{NB} , and E_{NC} are the generated star voltages; V_{1N} , V_{2N} , and V_{3N} are the phase-to-neutral voltages; and V_{1g} , V_{2g} , and V_{3g} are the phase-to-ground voltages whose resultant is designated V_{GR} . From the connection diagram, Fig. 58, it can be seen that V_{A1} is the voltage drop across the system impedance to the point of fault, V_{1g} is the voltage drop across the fault impedance, and V_{GN} is the voltage drop across the neutral impedance. In this analysis all impedances are considered as pure resistances for simplicity. This simplification makes all the voltage drops in phase and, therefore, gives a clearer picture of their relative effect. Actually, reactance is present, which will cause considerable skewing of the voltage triangle with respect to the e.m.f. triangle.

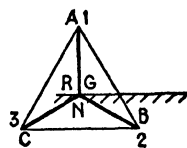


FIG. 59.—Normal condition considering no fault.

Figure 60 shows the vectors for a solidly grounded neutral system having a zero-resistance fault on phase 1 to ground. In this case phase 1 assumes ground potential and hence $V_{1g} = 0$, while V_{2g} and V_{3g} remain almost normal. The resultant voltage V_{GR} is therefore equal and opposite to the voltage V_{1g} before it

was collapsed. Similarly, if phase 2 were grounded, the residual voltage would be equal and opposite to the normal phase 2 to ground voltage V_{2g} , and, if the ground fault were on phase 3, the residual voltage would be equal and opposite to the normal V_{3g} .

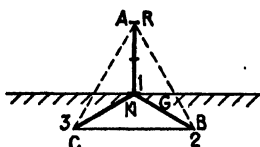


FIG. 60.—Source neutral solidly grounded. Zero resistance ground fault on phase 1.

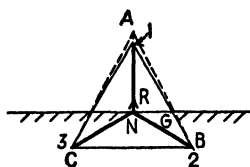


FIG. 61.—Source neutral solidly grounded. High-resistance ground fault on phase 1.

In case of resistance in the ground fault, the voltage drop across the fault resistance keeps the voltage V_{1g} from collapsing. If the fault resistance is high, so as to be the chief factor in determining the magnitude of the fault current, the voltage drop across the fault will keep the voltage V_{1g} practically normal. Consequently, the resultant of the phase-to-ground voltages is practically zero, which makes it impossible to secure satisfactory relaying. Such a condition is illustrated by Fig. 61, which, it

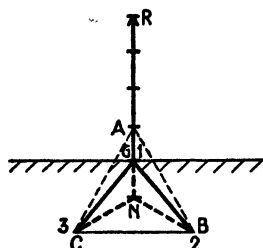


FIG. 62.—System neutral grounded through moderate value of resistance. Zero-resistance ground fault on phase 1.

will be noted, approaches the normal condition shown in Fig. 59. The residual voltage in this case is only equal and opposite to V_{A1} , the voltage drop in the faulty phase, which is small since most of the e.m.f. E_{NA} is used up as drop across the fault resistance, shown as V_{1g} .

The next three figures illustrate how the residual voltage in case of a ground fault can be materially increased if the system neutral N is caused to shift with respect to ground potential G , by connecting the ground-current source neutral to ground through a resistor. In this case any ground current through the resistor causes a voltage across the neutral resistor with the result that the source neutral N must shift with respect to ground potential G .

Figure 62 shows the vectors for the case of a solid ground fault encountering only resistance in the system and in the neutral

in which the point N shifts along the line of the faulty phase 1 to ground voltage. The shift in the system neutral, represented by the voltage drop V_{GN} for the case illustrated, is one-half the normal voltage V_{1G} , which means that the neutral resistor has ohms equal to the remaining ohms in the ground circuit, whose voltage drop is represented by the line $A-1$. Since this figure also considers a solid ground fault, $V_{1G} = 0$ and the resultant of the phase-to-ground voltages $V_{GR} = V_{2G} + V_{3G}$. This residual voltage is of appreciable magnitude, being double V_{GR} of the solidly grounded-neutral case for a solid ground fault illustrated in Fig. 60.

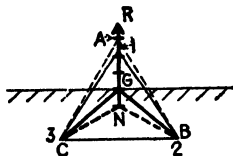


FIG. 63.—System neutral grounded through moderate value of resistance. High-resistance ground fault on phase 1.

Figure 63 considers a case similar to Fig. 62 except that the fault itself has high resistance so that most of the e.m.f. E_{NA} is used up across the fault, shown as V_{1G} , but little across the neutral resistor which is shown as V_{GN} , and still less within the system which is shown as V_{A1} . Owing to the shift of the system neutral with respect to ground, which is represented by V_{GN} , the resultant of the phase-to-ground voltages V_{GR} is still appreciable, assuring good relay action. The reason for this can be better understood by considering Fig. 64, the case of a phase-to-ground fault on a system with ungrounded neutral, where the voltages to neutral hold up; but the neutral N takes the extreme shift as a result of phase 1 assuming ground potential. The residual voltage $V_{GR} = V_{1G} + V_{2G} + V_{3G}$, but, since $V_{1G} = 0$,

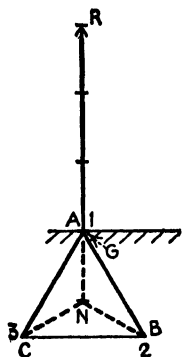


FIG. 64.—System neutral ungrounded or grounded through high resistance. Ground fault on phase 1.

$$V_{2G} = V_{2N} + V_{NG},$$

and $V_{3G} = V_{3N} + V_{NG}$, it will be found that

$$V_{GR} = 0 + V_{2N} + V_{3N} + 2V_{NG}.$$

Also, since in this case V_{1N} , V_{2N} , and V_{3N} are equal, respectively, to E_{AN} , E_{BN} , and E_{CN} , which are a balanced system of three-phase vectors, $V_{2N} + V_{3N} = V_{N1}$. Therefore, $V_{GR} = V_{N1} + 2V_{NG}$, and since in this case $V_{N1} = V_{NG}$, it follows that $V_{GR} = 3V_{NG}$. Hence, in case of a ground fault on an ungrounded system, the resultant

of the phase-to-ground voltages is equal and opposite to three times the shift in the system neutral with respect to ground potential. It has previously been shown in connection with Figs. 60 and 61 that, if the system neutral is tied solidly to ground so that it cannot shift with respect to ground, the residual voltage is equal and opposite to the voltage drop V_{A1} in the system's faulty phase-to-neutral voltage. Returning now to Figs. 62 and 63, it can be shown in a similar manner that if there is a shift in the neutral as well as a drop in the faulty phase-to-neutral voltage, $V_{GR} = 3V_{NG} + V_{1A} = -(3V_{GN} + V_{A1})$. Therefore, the residual voltage in case of a ground fault is equal and opposite to three times the system-neutral shift with respect to ground plus the drop in the faulty phase-to-neutral voltage between the source of e.m.f. and the point of fault.

Since the grounded star-delta potential scheme inherently multiplies any shift in the system neutral by three, it is especially effective in producing residual voltage if the system neutral is caused to shift with respect to ground at the time of ground faults. This can be readily accomplished by inserting an impedance between the system neutral and ground. A comparative analysis of Figs. 60 and 61 with Figs. 62 and 63, respectively, will bear this out.

Of course, in case of very high resistance in the fault, such as approximated by Fig. 63, the ground current may be so small as to produce very little shift in the system neutral, in which case the normal condition shown in Fig. 59 is again approached. High resistance as here used may be defined as one which is almost solely responsible for the magnitude of the ground current which can flow, so that almost the entire normal voltage to neutral appears as voltage drop across it. Similarly, moderate resistance as here used is such that only approximately one-half the normal voltage to neutral appears as drop across it.

Since in the lower voltage systems the normal voltage to neutral is correspondingly low, it follows that ordinary fault resistances such as are encountered on wood-pole lines may appreciably limit the magnitude of the fault current, the shift in the system neutral, and the magnitude of the residual voltage. The adequacy of the residual voltage in case of high-resistance ground faults is, therefore, an important consideration where residual voltage is used for directional relaying of ground faults. This is

because the directional ground relays which should respond may not function on account of insufficient voltage, and, consequently, the non-directional ground relays nearer the source of ground current, which function on current alone, may trip out a large portion of the system. To obviate this, the magnitude of the residual voltage should be checked for the smallest current which will operate the non-directional ground relays.

Since the cost of neutral resistors increases rapidly with decreasing resistance, there is no advantage in going to the lower ohmic value unless it is to allow more reactive current to flow to neutralize residual charging current of the system so as to avoid excessive voltage stress to ground on the good phases. In this event, though, it may be preferable not to reduce the ohmic value but to make the neutral impedance partly or entirely reactive. Introducing reactance in the neutral increases the angle of lag to the ground-fault current behind the residual voltage, and, if the neutral impedance is entirely reactive, the angle of lag approaches 90° . Hence, if residual current and residual voltage are applied to a relay directional element, having true wattmeter characteristics, low torque may be expected if the system neutral is grounded through reactance alone, due to the current and voltage being nearly at right angles. Moreover, phase-angle errors in the instrument transformers may actually lead to reversed torque that would result in incorrect relaying. Therefore, in studying the effectiveness of grounding the system neutral through an impedance which is partly or entirely reactive, or of a system which has its neutral solidly grounded, consideration must be given not only to the magnitude of the residual voltage but its phase relation with the fault current as well.

This is in general the same situation encountered in line relaying but is of even more importance in that the relatively high value of zero-sequence reactance of a transmission line produces a decided lagging fault current with respect to the voltage, if the fault occurs any distance at all out on the line. To take care of this condition, ground-overcurrent directional relays have an inherent compensation of 15° . In other words, maximum torque is produced in the relay when the current in the current coil lags the voltage of the potential coil by 15° .

In cases where the 15° compensation is not sufficient, additional compensation can be secured by the use of an external

phase shifter. This consists of a resistor and reactor connected in series, as in Fig. 65. The compensator is connected inside the delta of the auxiliary transformers, and the relay is connected across the resistance portion of the compensator. In Fig. 66 is shown a simple vector diagram illustrating the use of the external phase shifter. Voltage B_X is shown vertical. It is the resultant voltage produced within the delta of the auxiliary potential transformers when a fault occurs between line 3 to ground. The voltage B_X is made up of B_Y and Y_X . The resistance and reactance of the phase shifter are approximately equal, so that there will be an angle of 45° between B_X and B_Y . For 100 per

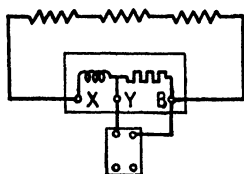


Fig. 65.—Schematic diagram of phase shifter for ground relays.

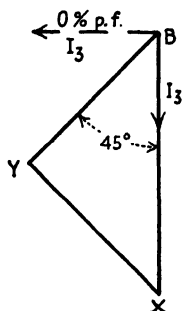


Fig. 66.—Vector diagram of phase shifter shown in Fig. 65.

cent power factor, I_3 is in phase with B_X . Hence, a 45° power factor will put I_3 in phase with B_Y . The relay itself is compensated for 15° , so that a lag of 60° will produce maximum torque.

On low- and medium-voltage systems, where fault resistance may produce considerable voltage drop across the fault, it is desirable to ground the system neutral through impedance rather than solidly, in order to insure ample residual voltage for relaying purposes. So-called solid grounding of the neutrals of high-voltage systems is permissible in that the attending heavy ground currents produce considerable drop in the faulty phase-to-ground voltage. Moreover, since it is impracticable to make a neutral connection devoid of resistance, some shift of the system neutral with respect to ground will occur, due to the heavy ground current passing through the neutral contact resistance. Both the drop in the faulty phase-to-ground voltage and the shift in the system neutral with respect to ground insure residual voltage in

case of ground faults. The shift in the system neutral is especially effective in producing residual voltage since any shift is inherently multiplied by three.

Directional Ground Relaying without the Use of Potential Transformers.—An overcurrent directional ground relay without potential windings may be used at the stations where the transformer high-tension neutrals are grounded and sufficient ground

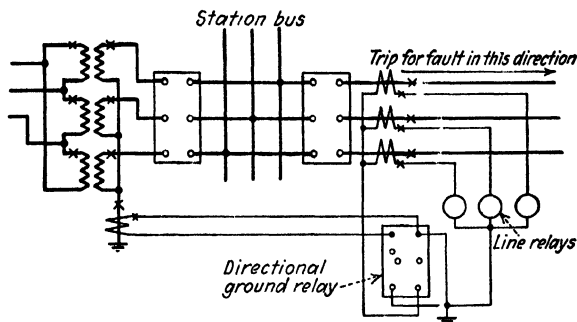


FIG. 67.—Overcurrent directional relay polarized by current transformer in neutral of power transformer.

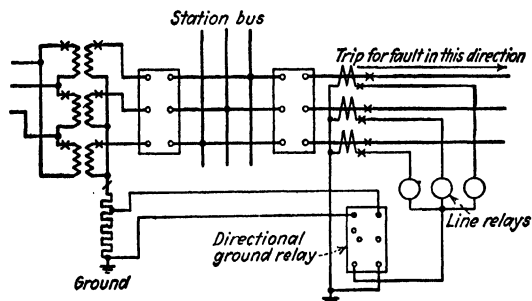


FIG. 68.—Overcurrent directional relay polarized by voltage drop across neutral resistor.

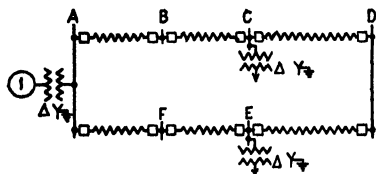
current will flow under all conditions to operate the directional element. This directional element consists of two current coils, one connected in the neutral of the line current transformers and the other to the current transformer in the transformer-bank neutral, as shown in Fig. 67. The use of high-tension potential transformers is thereby eliminated. As the residual line current and transformer-neutral current are very nearly in phase, the relay is designed so that the directional element develops maxi-

imum torque under these conditions. The relay element connected to the transformer-bank-neutral current transformer polarizes the relay, similar to the potential-transformer connection shown in Fig. 25.

An alternate scheme, shown in Fig. 68, may be used where the neutral is grounded through a resistance. The drop across the

resistance when a fault occurs supplies potential for the operation of the directional element.

The application of relays to systems having more than one grounded neutral point requires a careful consideration of ground-fault currents. In many loop systems the transformers at the substations are connected in delta on the low-tension side and in star with grounded neutral on the high-tension side. Each neutral may be solidly grounded or



Generator 1-50,000 kva, Transient reactance 20%
Transformer A-50,000 kva, 12,132 kv, reactance 8%
Transformers C and E-20,000 kva, reactance 8%

Transmission line section	Miles	Ohms reactance	Zero-sequence reactance
AB	20	15.6	46.8
BC	20	15.6	46.8
CD	30	23.4	70.2
DE	30	23.4	70.2
EF	20	15.6	46.8
FA	20	15.6	46.8

FIG. 69.—Loop system with three points solidly grounded.

grounded through resistance or reactance, a system with three solidly grounded neutral points being shown in Fig. 69.

Ground-fault Currents.—The ground-fault currents on such a system are radically different than for a system with only a single ground. The greater portion of the fault current flows through the grounded neutrals of the transformers adjacent to the fault, only a smaller part flowing in the grounded banks beyond the faulted section. The magnitude and distribution of the ground current for a particular fault depend upon the relative magnitude of the impedances of grounded transformer banks and the zero-sequence reactance of transmission circuits.

In Fig. 70 are shown the magnitude and distribution of the positive-, negative- and zero-sequence currents for a line-to-ground fault at Station B on the system shown in Fig. 69. Positive- and negative-sequence currents can flow only from the Generating Station A, while zero-sequence currents can flow through the transformer neutral at Stations A, C, and E. Consequently, the distribution of the zero-sequence currents is entirely different from that of the positive- and negative-

sequence currents. It is assumed that the positive- and negative-sequence networks are identical. In general, very little error is introduced by this assumption and the numerical calculations are considerably reduced.

The total current in the grounded phase, and the residual line currents and transformer-neutral currents are also shown in Fig. 70. It should be remembered that the total current in the grounded phase at a particular point in the system may be considerably different from the residual current. The residual current, equal to the vector sum of the three phase currents, is also equal to three times the zero-sequence current. Since

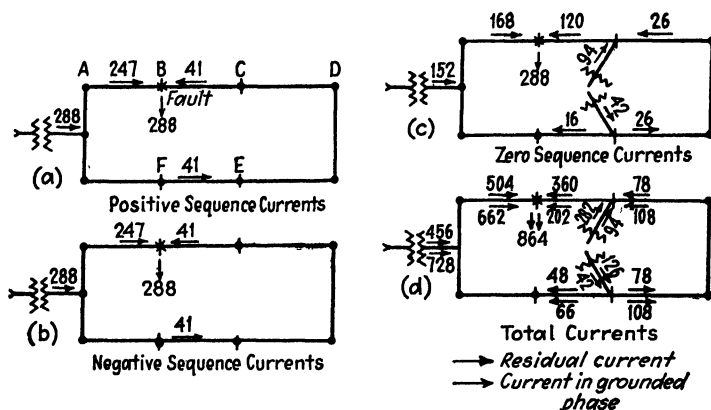


FIG. 70.—Fault currents of loop system with short circuit at Station B.

the total zero-sequence current divides equally through the three phases, the total current in the neutral of star-connected current transformers is three times the zero-sequence current in one phase. The distribution of the zero-sequence current depends entirely upon the relative impedance of the various branches of the zero-sequence network and is independent of the distribution of the positive- and negative-sequence currents. The total current in the grounded phase at any point in the system is equal to the summation of the positive-, negative-, and zero-sequence currents at the same point in each network. An inspection of Fig. 70 shows that, since the positive- and negative-sequence currents are quite different from the zero-sequence currents, in certain branches of the circuit, the total current in the grounded phase and the residual current at corresponding points

will therefore be different. To clarify these current relations, an example of the calculations for a fault at *B* is given under Calculation of Ground-fault Currents, page 413.

The total ground current and the current in the neutral of each transformer are shown in Fig. 71 for faults at various points, Fig. 72 showing the current in the grounded phase and the residual currents. It will be noted that the current supplied by any transformer drops off quite rapidly as the fault moves farther away from the transformer.

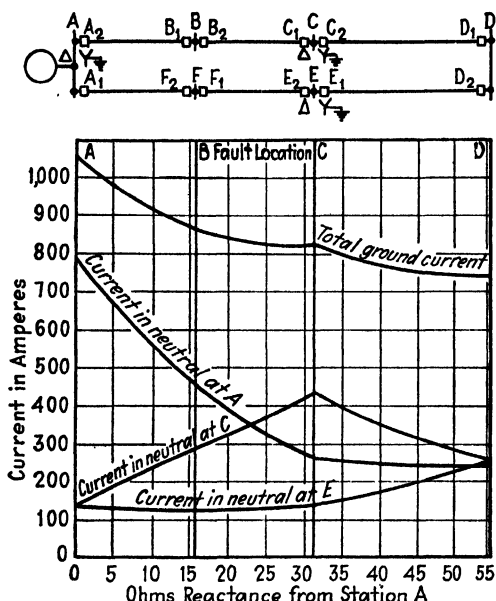


Fig. 71.—Total ground current and neutral current for faults at various locations of loop system, Fig. 69.

In Fig. 73 are shown the phase relations of the currents and voltages for a fault at *B* and with the distribution of sequence currents shown in Fig. 70. These relations are based on the assumption that the system impedance is pure reactance.

Several types of ground relays are applicable to the loop system with more than one grounded neutral point.

The most simple type of protection is secured by the use of the overcurrent directional ground relay, as shown in Figs. 25, 67, and 68. This relay, however, is not entirely satisfactory for

this system because the time setting of the relays nearest the generating source must necessarily be high in order to secure

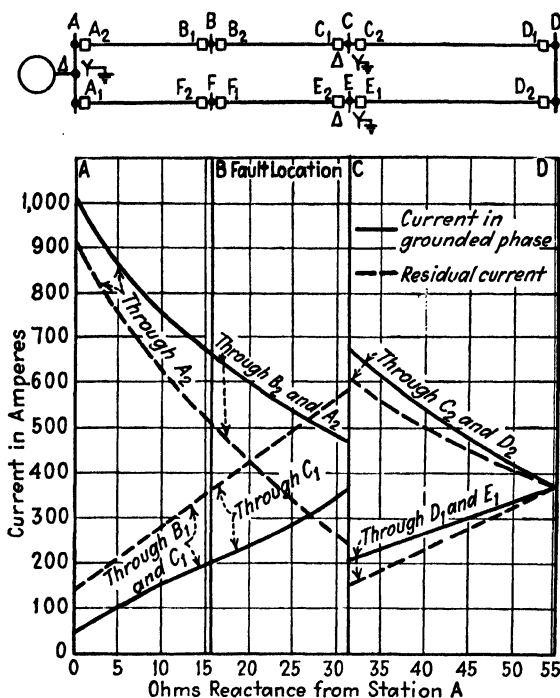


FIG. 72.—Current in grounded phase and residual current at various locations in loop system, Fig. 69.

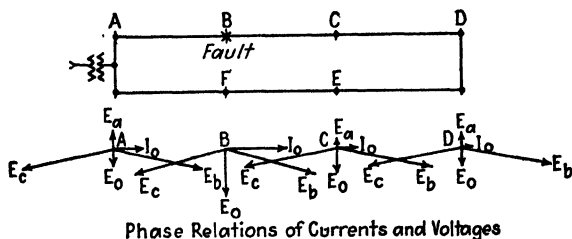


FIG. 73.—Phase relations of currents and voltages with fault at Station B of loop system, Fig. 69.

selectivity around the loop. Because of this, a severe system jolt would result from heavy faults near the generating source.

Directional Residual Volt-ampere Relay.—Satisfactory ground-fault protection can be secured on the system by using a residual

volt-ampere relay connected as shown in Fig. 74. This relay is actually a watt relay but may be compensated to have its maximum torque when the residual current is lagging the residual voltage by the phase angle of the most common system faults. It is inherently directional and its characteristics combined with those of the system are such as to give an action somewhat similar to the impedance relay. The residual current and voltage, each equal to three times the zero-sequence current and voltage, respectively, are applied to the relay coils. Since the zero-sequence currents and voltages are greater the closer the fault

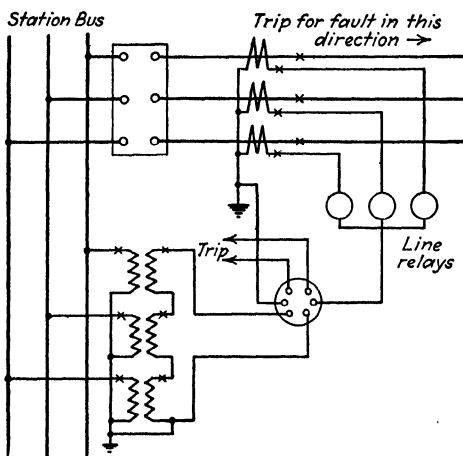


FIG. 74.—Residual volt-ampere relay connections.

is to the relay, the relays nearest the fault have the fastest operation.

The zero-sequence voltage increases from zero at the grounded neutral points, through the transformers and transmission lines, to a maximum at the fault. The transformer neutral having the lowest impedance path to the fault carries the greatest amount of ground current. The operating force on the relay, equal to the product of the residual current and voltage, is shown in Fig. 75 for various fault locations. The current- and potential-transformer ratios were assumed to be 40:1 and 1200:1, respectively. An inspection of Figs. 73 and 75 indicates that the greatest product is always nearest the fault. Figure 73 is based on the assumption that there is no resistance in the system. Actually

the zero-sequence current and voltage will be more nearly in phase than shown, due to the fact that some resistance is always present

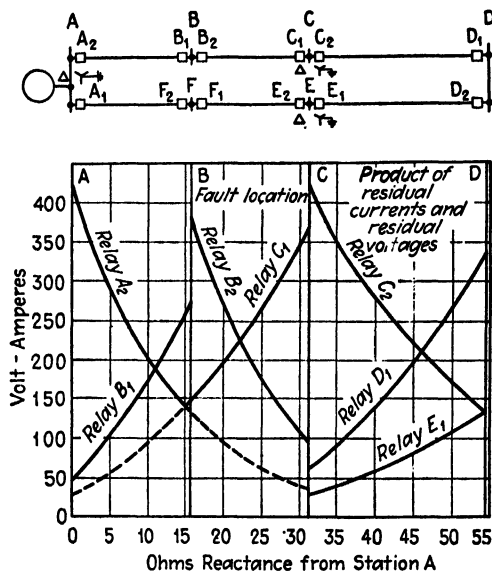


FIG. 75.—Product of residual currents and residual voltages for various locations of fault in loop system, Fig. 69.

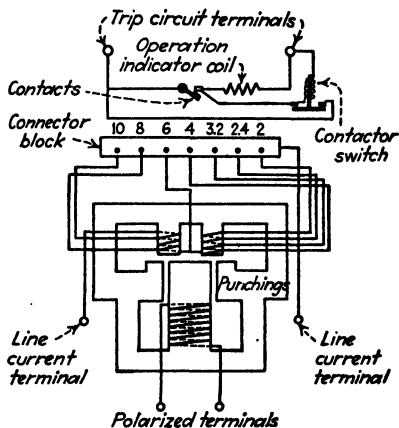


FIG. 76.—Internal connections of residual volt-ampere relay.

It will be noted in Fig. 75 that the operating forces on relays B_1 and C_1 approach each other for faults in section AB . In

this case proper selectivity is secured by means of the time-lever adjustment.

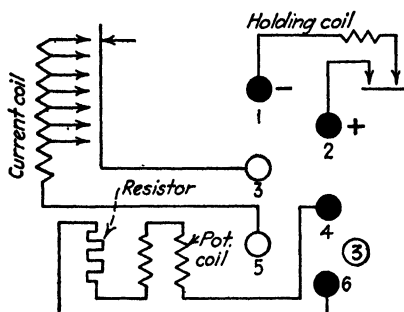


Fig. 77.—Internal connections of a second general type of residual volt-ampere relay.

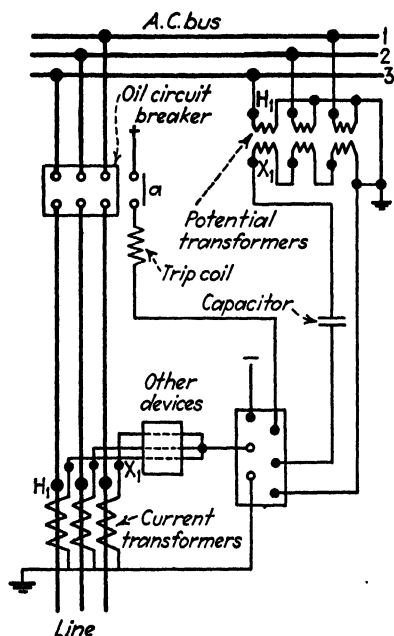


Fig. 78.—Volt-ampere relay system using relay illustrated in Fig. 77.

The internal connections of one type of directional residual volt-ampere relay, for application as shown in Fig. 74, is illustrated in Fig. 76. The lower pole is polarized with residual voltage, the upper pole being connected in the residual-current

circuit. The internal connections of a second type of directional residual volt-ampere relay is shown in Fig. 77, with complete connections illustrated in Fig. 78. The characteristics of the relay (Figs. 77 and 78) are given in Fig. 79. The time-current curves are based upon rated potential with 45° phase angle between voltage and current. The residual potential as well as the residual current should be known for fault conditions. The abscissa values in Fig. 79 are in terms of multiples of rated tap

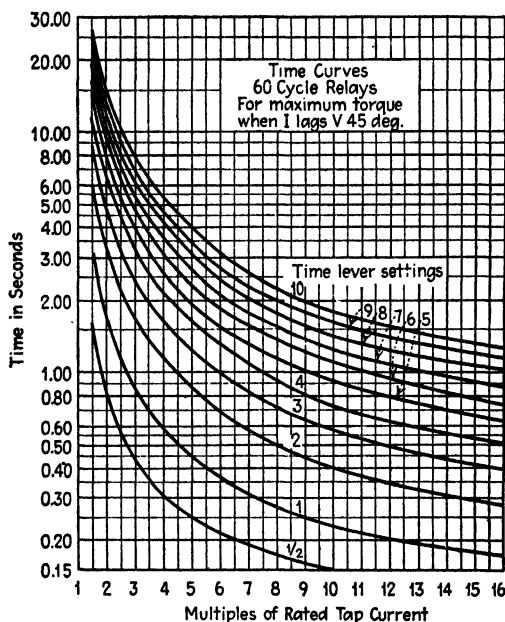


Fig. 79.—Operating characteristics of residual volt-ampere relay shown in Figs. 77 and 78.

current at 115 volts and 45° phase angle. One-half normal voltage at the relay with 45° phase angle would cut the abscissa value of Fig. 79 in half. Corrections for phase angle other than 45° angle should likewise be made.

Second Generating Station.—A transmission system identical with that previously discussed, except for the addition of a generating station at *D*, similar in rating and reactance values to that at *A*, is shown in Fig. 80. The distribution of positive-, negative-, and zero-sequence currents for a fault at *B* is shown

in Fig. 81, as well as the current in the grounded phase and the residual current in each branch of the circuit. The total ground current and the current in each transformer neutral are shown in Fig. 82 for various fault locations, and in Fig. 83 are given the currents in the grounded phase and the residual line currents.

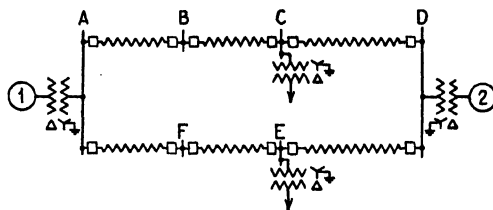


FIG. 80.—Loop system with two generating stations and four grounding points.

Directional Residual Current Relay.—The directional residual current relay is especially applicable to systems where all transformer neutrals are solidly grounded. The connections for this relay are shown in Fig. 84, no high-tension potential transformers being required. Although the connections are the same as for

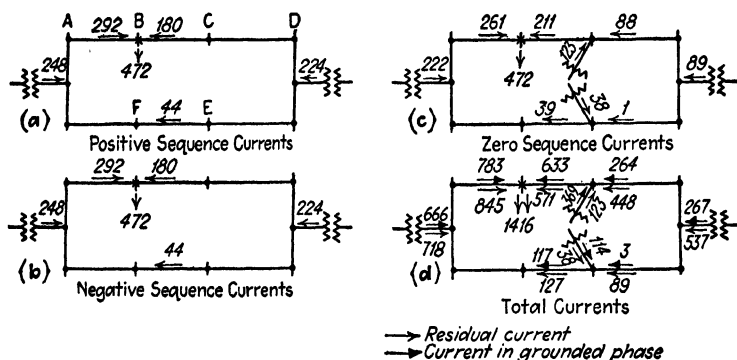


FIG. 81.—Distribution of short-circuit current with a line-to-ground fault at Station B, Fig. 80.

the directional overcurrent relay (Fig. 67), its characteristics are considerably different, as it operates on the product of the residual line current and the transformer-neutral current. The relay is directional and has an inverse characteristic, the ratio of the current transformer in the neutral connection being chosen so as to give the correct operating time.

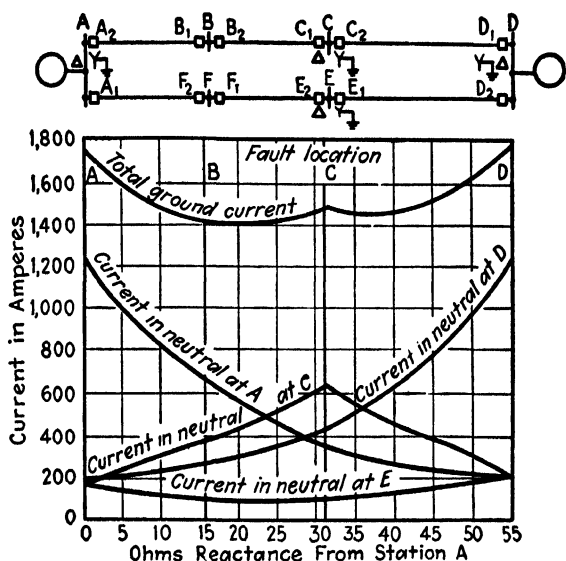


Fig. 82.—Total ground currents and neutral currents for various locations of fault of loop system, Fig. 80.

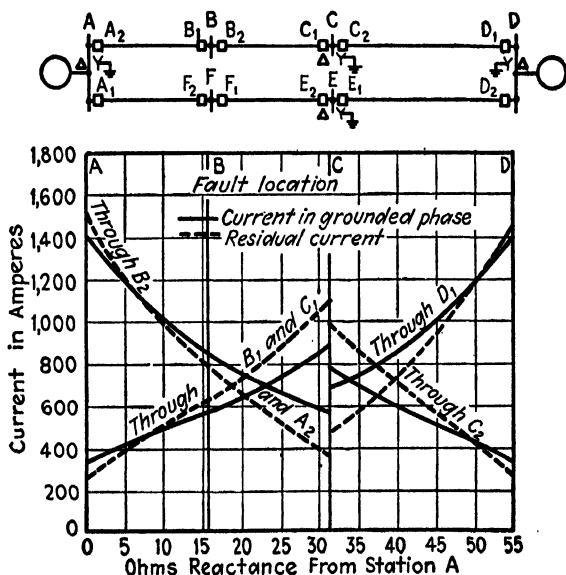


Fig. 83.—Currents in grounded phase and residual currents for various locations of fault of loop system, Fig. 80.

The internal connections for this relay are the same as shown in Fig. 76, with the exception that the lower pole is a current winding and not a voltage winding. The lower winding is

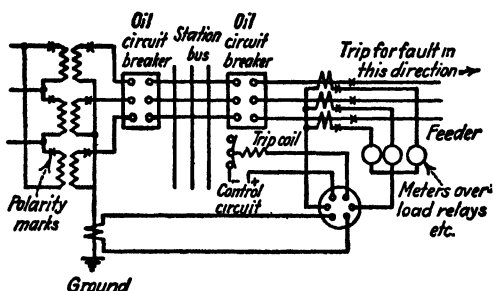


FIG. 84.—Directional residual-current relay system.

polarized by current from the neutral of the power-transformer bank. Typical time-current curves are shown in Fig. 85.

The operating torque of the relays applied at Stations A, C, E, and D are shown in Fig. 86 on the assumption that all current-

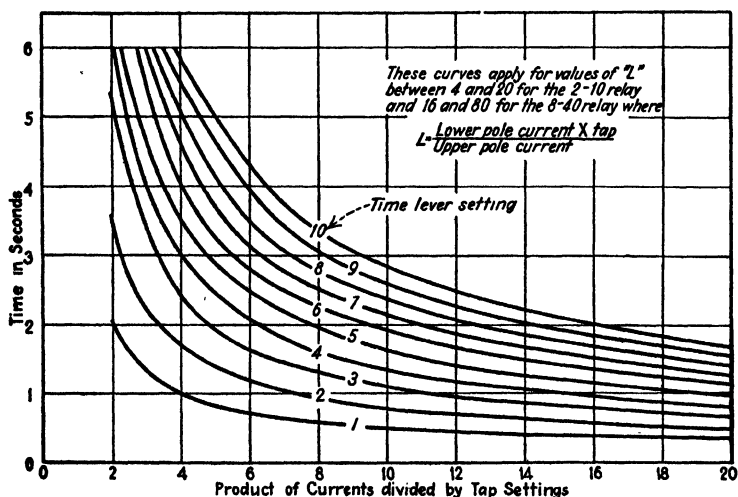


FIG. 85.—Time-current characteristics of relay system, Fig. 84.

transformer ratios are 40:1. The relay nearest the fault operates in the shortest time, as the relay torque increases rapidly as the fault location approaches a grounded neutral point.

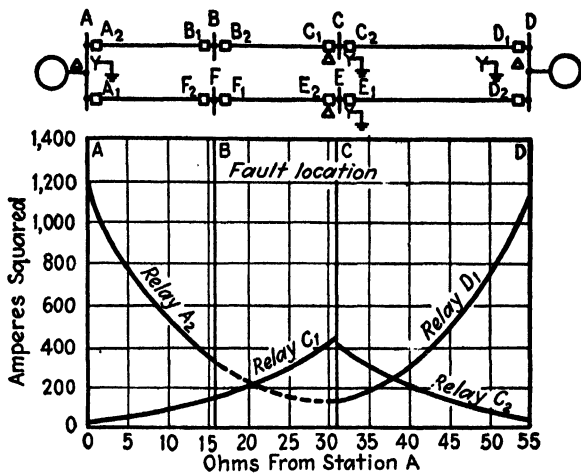


FIG. 86.

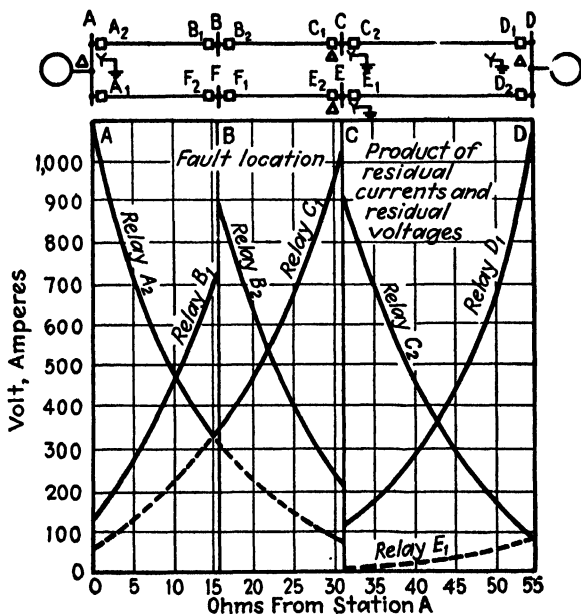


FIG. 87.

FIGS. 86 and 87.—Operating torque characteristics of relays applied to loop system, Fig. 80.

There are no grounded neutrals at Stations B and F, and at these two points the residual volt-ampere relay (Fig. 74) may be applied. The volt-ampere operating torque of these relays is shown in Fig. 87 for various fault locations. It will be noted that the system characteristics are such as to produce a good operating torque for each relay.

The relay of Fig. 84 may be used as a simple residual overcurrent relay where the directional feature is not necessary, the connections for this arrangement being shown in Fig. 88. The inverse characteristic of the relay gives better selective timing in some cases than a definite-time overcurrent relay. An example of such a case is where a radial circuit is fed from a substation in a loop or a station in a double-circuit line. The loop circuit

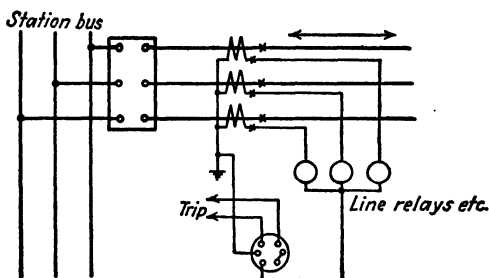


FIG. 88.—Residual overcurrent relay system.

may be protected by inverse time relays, and these must afford back-up protection for the relays in the radial feeder. Using the relay connected as in Fig. 88, it is possible to secure selectivity with the loop relays because of the similarity of the characteristics of all relays. The use of definite-time relays on the radial feeder would necessitate increasing the time settings of the loop relays which afford back-up protection.

On systems where the neutrals are solidly grounded, it may be found that there is little difference between maximum and minimum ground-fault currents. It is sometimes desirable on systems of this type to use instantaneous overcurrent relays in addition to the inverse-type relays. In these cases the instantaneous overcurrent relay must be set sufficiently high so that it will operate only for heavy faults within its own section. The majority of heavy faults within that section will then be cleared by means of the instantaneous relay.

The principal objection to the use of a residual-current ground relay is the difficulty in determining the proper setting. In applying this type of relay, a very careful study of the ground currents must be made to determine the proper transformer ratios at the various stations. Since a residual-current ground relay is seldom used on any but important lines, such a study is well worth the time required.

Local conditions will so influence the system layout that it is improbable that any two loops will be the same. However, all loops originate from the same fundamental plan and the principles of relay application developed above may be readily expanded to cover any case. A loop network which might appear difficult to relay, after careful analysis and determination of the limiting operating conditions, generally reduces to a form to which standard relaying schemes are applicable.

Calculation of Ground-fault Currents.—The calculations for the currents and voltages shown in Figs. 70 and 73 for a line-to-ground fault on phase *a* at Station B are given in the following example.

The equivalent impedance of the positive-sequence network from the generating source to the fault is 111.38 ohms and the equivalent impedance of the negative-sequence network is assumed to be the same. The equivalent impedance of the zero-sequence network from the three grounded neutrals to the fault is 42.2 ohms.

The positive-, negative-, and zero-sequence currents, respectively, in the grounded phase at the fault are given by the expression

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_n}{Z_1 + Z_2 + Z_0} \quad (1)$$

where E_n is the voltage to neutral; and Z_1 , Z_2 , and Z_0 are the positive-, negative-, and zero-sequence impedances, respectively, to the fault.

Since we are dealing only with pure reactance, if the phase of the reference voltage is taken as imaginary, the currents I_{a1} , I_{a2} , and I_{a0} will be real. The choice of the phase position of the voltage is arbitrary, but by making the voltage imaginary the negative imaginary terms for currents are eliminated.

For the particular case, the sequence currents at the fault are

$$I_{a1} = I_{a2} = I_{a0} = \frac{+j76,200}{j(111.38 + 111.38 + 42.2)} = 288 \text{ amp.} \quad (2)$$

The total short-circuit current in phase *a* at the fault is

$$I_a = I_{a1} + I_{a2} + I_{a0} = 288 + 288 + 288 = 864 \text{ amp.} \quad (3)$$

The current distribution in the individual networks can be found in the usual manner by working back through the original network reduction which was made in finding the equivalent impedance of each network. The total current and the current in each branch in phases *b* and *c* can be found from the currents in phase *a*:

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} \quad (4)$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} \quad (5)$$

The formulas give correct results if the positive direction of current flow chosen in the three networks is the same.

The total ground or residual current at the fault is equal to the total short-circuit current in phase *a* and is also equal to three times the zero-sequence current:

$$I_R = 3I_{a0} = 3 \times 288 = 864 \text{ amp.} \quad (6)$$

The sequence components of phase-to-neutral voltage and the line voltages may be found after determining the sequence currents and their distribution through the networks. The positive-sequence voltage at any point is equal to the generator voltage minus the positive-sequence drop from the source to the point. For example, the positive-sequence voltage at Station C is equal to the generated voltage to neutral minus the drop through the generator and transformer and around the loop to the right to C. The positive-sequence voltage at C is then, referring to Fig. 70a.

$$\begin{aligned} E_{a1} &= +j76,200 - (288 \times j98 + 41 \times j78) \\ &= +j44,600 \text{ volts} \end{aligned} \quad (7)$$

The negative-sequence voltage at C is equal to the generated negative-sequence voltage minus the negative-sequence drop from the source to C. The generated negative-sequence voltage is zero. Therefore, referring to Fig. 70b, the negative-sequence voltage at C is

$$\begin{aligned} E_{a2} &= 0 - (288 \times j98 + 41 \times j78) \\ &= -j31,400 \text{ volts} \end{aligned} \quad (8)$$

In like manner for the zero-sequence network (Fig. 70c), the voltage at *C* is equal to the negative of the drop from the source to *C*. The source in this case is the transformer at *C*, having a zero-sequence impedance of 70 ohms:

$$E_{a0} = 0 - (94 \times j70) = -j6580 \text{ volts} \quad (9)$$

The voltage of phase *a* at Station *C* is equal to the summation of the sequence voltages:

$$\begin{aligned} E_a &= E_{a1} + E_{a2} + E_{a0} = +j44,600 - j31,400 - j6580 \\ &= +j6620 \text{ volts} \end{aligned} \quad (10)$$

The residual voltage is equal to three times the zero-sequence voltage:

$$E_R = 3E_{a0} = 3(-j6580) = -j19,740 \text{ volts} \quad (11)$$

The line-to-neutral voltages of phases *b* and *c* can be found from the sequence components of voltage for phase *a* using the expressions

$$E_b = E_{a0} + a^2 E_{a1} + a E_{a2} \quad (12)$$

$$E_c = E_{a0} + a E_{a1} + a^2 E_{a2} \quad (13)$$

Substituting the numerical values:

$$\begin{aligned} E_b &= -j6580 + (-0.5 - j0.866)(+j44,600) + (-0.5 + j0.866) \\ &\quad (-j31,400) = 65,840 - j13,180 \end{aligned} \quad (14)$$

$$\begin{aligned} E_c &= j6580 + (-0.5 + j0.866)(+j44,600) + (-0.5 - j0.866) \\ &\quad (-j31,400) = -65,840 - j13,180 \end{aligned} \quad (15)$$

Voltage Relay.¹—Overvoltage relays connected in the corner of the delta of three star-delta connected potential transformers may be used to indicate these unbalanced voltage conditions. Under normal operating conditions there is no current flowing in the delta, and the relay potential is zero. In case of a ground on the line, this voltage becomes quite large. Harmonics and large charging currents may seriously upset such a scheme, but their effect can usually be eliminated by adding a load across the secondary of each potential transformer. This arrangement only indicates a ground fault but does not locate it. It is applicable, then, for tripping breakers only on single feeders, or a series

¹ A part of these data have been obtained from W. W. Edson, Transmission System Relay Protection, Pt. III, *Trans. A.I.E.E.*, vol. 49, p. 1213, 1930.

of radial feeders, where selective timing may be used. (Refer to Fig. 89.)

High-impedance Grounded Systems.—The ground protection of high-voltage systems using a relatively high-impedance ground presents a difficult relay problem. It is particularly pertinent that such faults be relieved, since dynamic current actually flows to ground and not, as in the case of isolated systems, only a charging current. Moreover, high voltages may appear on the system during such a ground fault, and simultaneous faults may be avoided if the ground fault is relieved quickly.

The ground-relay system must obviously operate on a small amount of current to ground, thus requiring sensitive relays.

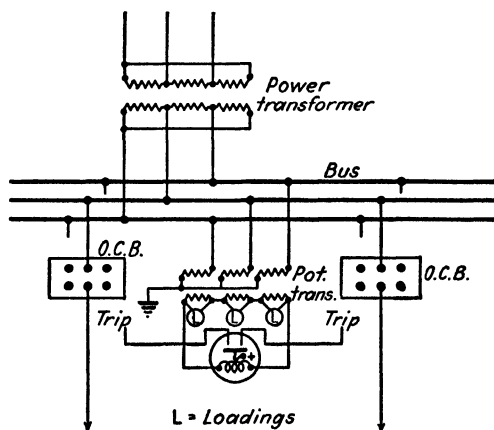


FIG. 89.—Overvoltage relay system.

Current-transformer circuits connected to such relays must be accurately balanced, as an improper unbalanced secondary loading will pass load current through the sensitive ground relays and operate them incorrectly. Ordinarily the relay volt-ampere burden is high, and care must be taken that the current transformers are not overburdened.

Another problem of such relays is to take care of simultaneous faults on opposite phases on separate parts of the system. Power-factor conditions over an extreme range may be presented to the relays, and it is impossible for the relays to detect correct current-operating conditions from incorrect ones, under such circumstances, at certain locations on the system. The most

practical solution for such a condition has been to lock out the ground relays during simultaneous faults and allow phase relays to function to relieve the disturbance. This is accomplished by utilizing the short-circuit current which flows in the neutrals under these conditions to operate lockout relays which render the directional ground relays non-operative.

If the ground current is sufficient to produce a difference in voltage drop between stations, impedance ground relays which are introduced by fault-detector relays during a disturbance may be used. (Refer to Fig. 90). These fault detectors are usually a combination of overcurrent and undervoltage relays.

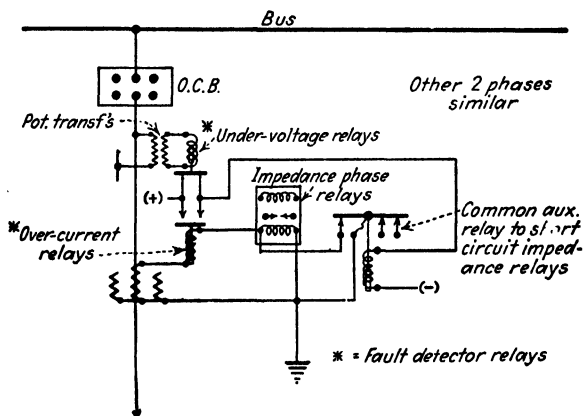


FIG. 90.—Fault detector-impedance relay system.

Overcurrent directional relays of 0.5- to 2.5-amp. type in some cases can be used with fault-detector relays to isolate line sections with short-circuit currents less than full-load-current rating.

Charging Currents.—When one phase of the system is grounded, there immediately appears on all conductors a charging current to ground. On a large system, this charging current is of sufficient magnitude to operate relays. It increases in magnitude as the point of fault is approached. It appears in the residual circuit of three star-connected current transformers and may be used to operate directional overcurrent relays. The potential for the relays is usually taken from the corner of star-delta connected potential transformers, as described above, if high-tension potential transformers are present, but potential may be supplied from the low-tension side of the power transformers if necessary.

There are but few high-voltage systems operating ungrounded so that the details of this form of relaying are not here included.

AUTOMATIC RECLOSING OF OIL CIRCUIT BREAKERS

In an attended station, the usual operating instructions direct that, when a feeder breaker opens due to faults, the operator wait a definite length of time and then reclose the breaker. If after this first reclosure the breaker fails to remain closed, the operator follows a definite schedule for closing the breaker a predetermined maximum number of times. The advantage gained by automatically reclosing feeder breakers became apparent about 10 years ago, with the result that a large number of automatic reclosing equipments have been installed.

Most of the installations have been made on stub or radial feeders on which lighting load predominated. In general, it was found that a cycle whereby a breaker is closed 15 sec. after its first opening, 30 sec. after its second opening, 75 sec. after its third opening, and locked out on a fourth opening was a desirable standard. The 15-sec. interval was an arbitrary choice and represented a marked improvement over previous manual reclosing, sufficing as a minimum value up until recently when developments started on instantaneous reclosing. The remaining intervals also were arbitrarily chosen, with some consideration toward lessening the duty on the oil circuit breaker in case of successive operations.

Immediate Initial Reclosing.¹—Operating engineers have for several years recognized the advantages of reducing the initial reclosing time to a minimum, since the 15-sec. time interval commonly used is after all an arbitrary figure.

In the past, one of the main limitations to shortening the reclosure time intervals has been that of the circuit breaker. Circuit breakers of the plain break design inherently have a long arcing time, thereby causing considerable carbonizing of the oil and formation of gas and high pressures within the breaker tank. With the advent of improved arc-extinguishing devices in modern breakers, this objection has been eliminated. This has been recognized by the joint committee of the Edison Electric Institute, Association of Edison Illuminating Companies, and the

¹ ANDERSON, A. E., Automatic Reclosing of Oil Circuit Breakers, *Elec. Eng.*, January, 1934.

National Electric Manufacturers Association, in the recommendations shown in Table I.

TABLE I.—OIL CIRCUIT-BREAKER PREFERRED INTERRUPTING RATINGS

The duty cycle of an oil circuit breaker shall consist of a specified number of unit operations at stated intervals.

Each unit operation shall consist of a closing of the circuit breaker, followed immediately by its opening without purposely delayed action.

The standard interrupting rating (standard duty cycle) of an oil circuit breaker shall be based on two unit operations with a 15-sec. interval between operations.

Interrupting ratings for other duty cycles shall be as specified below.

Number of groups	Total number of unit operations	Percentage of standard interrupting rating at currents specified				
		0 to 5,000 amp.	5,001 to 10,000 amp.	10,001 to 20,000 amp.	20,001 to 40,000 amp.	Above 40,000 amp.
Successive unit operations with 15-sec. intervals						
	2	100	100	100	100	100
	4	95	95	90	85	80
	8	85	80	70	55	50
	16	75	65	50	35	
Groups of 2 unit operations with zero ¹ second interval between operations and 2-min. intervals between groups						
1	2	90	85	75	65	60
2	4	80	75	60	45	
4	8	70	65	50	35	
8	16	60	55	40	25	
Groups of 4 unit operations with 0-15-60-sec. intervals between operations and 2-min. intervals between groups						
1	4	85	80	65	50	
2	8	75	70	55	40	
4	16	65	60	45	30	

NOTE: It is intended that satisfactory performance of breakers will be obtained under the rating schedule suggested at rates of recovery voltage normally encountered in practice.

¹ Zero second shall be interpreted to mean no intentional time delay.

Under a new schedule, oil circuit breakers are to be rated upon two unit operations, each consisting of a closing of the breaker, followed immediately by its opening without purposely delayed

action, with a 15-sec. interval between the unit operations. Any other number of unit operations or groups with other time intervals shall involve derating down to as low as 25 per cent for eight groups of two unit operations at 2-min. intervals. Provision is also made in the table for no delay between operations and for 0-15-60 sec. between them, with 2-min. intervals between groups of operations.

Standard oil circuit breakers and mechanisms are available having minimum operating times (from energizing the trip coil to closing the breaker contacts) in the order of 30 to 75 cycles, on a 60-cycle basis, depending on the type and rating of the oil circuit breaker and the form of mechanism; the shorter times being obtained on lower voltage ratings operated by solenoid mechanisms.

Trip-free mechanisms utilize a latch that must reset before the closing force is applied to the breaker and in most cases require a latch-checking function, such as a switch in the reclosing circuit actuated by the latching mechanism. Trip-free mechanisms incorporating latches with suitable resetting speeds do not require the latch-checking function.

A great percentage of the faults on overhead alternating-current distribution systems are of a transitory nature and can be cleared by removing the voltage long enough to extinguish the arc, allowing only sufficient time for the arc path to recover its dielectric properties before being reclosed. A swinging fault may persist longer than the initial reclosing period and require a second reclosure. Short circuits of a permanent nature will, of course, cause a lock-out.

A typical reclosing cycle for equipment arranged for instantaneous first reclosure is 0-15-120 sec. The zero time refers of course to a reclosing connection which allows the breaker to reclose as soon as it has reached its open position, eliminating the inserted time delay of the reclosing relay. The second reclosure has an inserted time delay of 15 sec. and the third reclosure 120 sec.

A number of other combinations of second and third reclosure time intervals have been tried in the field with satisfactory results. In some cases the third reclosure is omitted. Field data are not as yet sufficiently comprehensive to state definite advantages of one combination of second and third reclosure as

against another. Local system conditions may influence the choice of reclosing cycle to a considerable extent.

Generally speaking, induction motors will return to normal speed from any reduced value, inasmuch as their load usually is connected directly from standstill to normal operating speed. However, the longer the deenergized period the greater the decrease in speed, resulting in higher inrush current and subsequent delay in returning to normal operating voltage.

Synchronous motors will not necessarily pull back to normal speed after a voltage interruption. This depends largely on the pull-in torque and load characteristics. Some motors can be brought back into step by automatic field removal and resynchronizing equipment, but others will require in addition suitable unloading features. There are system and load conditions such that synchronous motors have remained connected, after an interruption, without the use of resynchronizing equipment.

Under full load conditions, synchronous motors will usually remain in synchronism during a complete loss of voltage for a period of only three to five cycles. The restoration of normal operating voltage within this short period is not available with present equipment, and if such a rapid reclosure could be performed it is doubtful if the fault path would have had sufficient time to recover its dielectric properties. Further field tests and operating data are required in order to determine more definitely the limits of synchronous-motor operation during loss of voltage.

There are several solutions of the various problems that enter into the design and application of alternating-current reclosing devices. Each of these devices has certain limits of application. Space does not permit a discussion of the various circuit connections. Manufacturers' data are available giving complete data for a particular-type breaker and circuit connection.

Modification of Existing Reclosers.¹—Most of the reclosers furnished until recently could be adjusted for a minimum delay of 1 to 3 sec. for the initial operation. Usually this is not fast enough where immediate reclosure is desired. Some existing reclosures can be modified so as to be suitable for immediate reclosing duty. In one case, the first of the sequentially operated contacts is latched closed at the normal position. With this

¹ANDERSON, A. E., Automatic Reclosing of Oil Circuit Breakers, *Elec. Eng.*, January, 1934.

arrangement the opening of the breaker through its *b* switch immediately energizes both the release and the antipump relay. The release magnet starts the timer and simultaneously the contact latch is released, permitting the closing circuit to be made.

One-shot Immediate Reclosing for Attended Stations.¹—In attended stations a great improvement in service is possible by applying a reclosure for one operation and leaving resetting and subsequent reclosure to the operator.

A simple and inexpensive relay giving one immediate reclosing impulse is used for this relay system. The closing of the breaker *b* auxiliary switch energizes the reclosing relay coil, and the armature picks up to close the contacts which in turn close the breaker. At the same time the relay latch is released. When the breaker closes, its *b* switch opens and the reclosing-relay armature falls to the fully open position. From this point it cannot be picked up electrically but must be returned by hand to its normal position, thus providing the required antipump feature. A second set of contacts is closed with the relay in the fully open position, giving an alarm indication.

Protective Relays.¹—Circuit-closing protective relays must open their tripping contacts with some safety factor during the deenergized period. Protective relays of recent manufacture have resetting times which are satisfactory. The older types which have too long a resetting time can be modified readily to give the desired quick-resetting feature. It is important also to check the selectivity of the relay system, so that it is not upset in the case of immediate reclosure on a persisting fault.

Self-contained Reclosing Equipment.—A single mechanical recloser is available for certain low-voltage low-capacity circuit breakers, requiring no separate relay housing. When the breaker closes, a pawl in the closing lever of the solenoid engages with a notch on a dial and mechanically revolves the dial in the opposite direction from which it rotates by spring action. If the fault on the feeder has been removed, the breaker will remain closed. If the fault still exists, the series-trip coils will again trip the breaker, and, when the mechanical interference is removed, the dial will be revolved in the proper direction by spring action to close the circuit-breaker contacts. This action is repeated except

¹ANDERSON, A. E., Automatic Reclosing of Oil Circuit Breakers, *Elec. Eng.*, January, 1934.

that on the second reclosure the dial has moved far enough so that the pawl on the closing lever engages a lug. If the disturbance or fault still exists and the breaker opens a third time, a lock-out lever falls over the pin on the dial and mechanically locks the breaker open. The first reclosure is made in approximately 1.75 sec. and the second in approximately 3 sec. In order to reset the mechanism, it is necessary then to close the breaker by hand.

Motor Outages on Voltage Dips.¹—Reduced voltage on a motor, if non-synchronous, causes it to start to slow down. If the mechanical load is constant, or nearly so, the motor will draw a higher current so as to maintain full power output. This may start the time-delay overcurrent device to operate, but, if the dip does not last too long, the overcurrent device will not complete its operation. Undervoltage devices, however, normally operate immediately, since a great percentage of these devices are of a plunger or armature arrangement which operates instantaneously at a predetermined value of undervoltage and trip-out the starter switch. Most of these devices operate on voltage drops of 10 to 25 per cent below normal.

It is becoming common practice to add a time-delay feature so that a dip lasting only a second or two will not disconnect the starter switch. In addition, the voltage drop-out point of the undervoltage device has been lowered to values of 50 or 60 per cent below normal, so that many dips leave the control equipment entirely unaffected.

The time-delay undervoltage release is not a perfect solution to this problem. In certain cases the slight reduction in motor speed, when the dip occurs, results in a flaw in the completed product. The continuity of the process is not affected, however, and the cost of scrapping the impaired material is usually less than the averted shutdown.

ALTERNATING-CURRENT TRIPPING FOR SMALL UNATTENDED SUBSTATIONS²

Adequate and reliable relay protection for small unattended alternating-current substations has always presented somewhat

¹ SLEEPER, H. P., Bulk of Motor Outages on Voltage Dips Avoidable, *Elec. World*, July, 1933.

² These data were taken from R. L. DeLoache, Circuit Breaker Tripping, *Elec. Jour.*, August, 1933.

of a problem. Some disadvantages incident to the use of storage batteries in unattended stations have created a demand for a different system of tripping, the reliability of which is not dependent upon the close supervision and maintenance required of a storage battery.

The use of a standard circuit-closing induction overcurrent relay in conjunction with a direct-trip attachment or tripping reactor imposes a heavy burden on the current transformer tending to throw the current transformer off ratio, rendering it unsuitable for use with instruments or watt-hour meters and militating against accurate relaying. This high burden often prevented the use of bushing-type current transformers. It often required the installation of two sets of wound primary

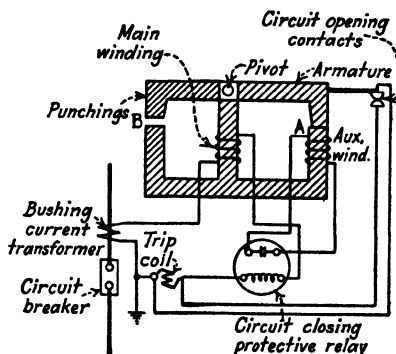


FIG. 91.—Alternating-current tripping scheme using current-transfer relay having low-energy consumption.

transformers, one set exclusively for tripping and the remaining set for the relay and instrument load.

Recent developments in relaying make it possible to obtain accurate overcurrent protection by means of circuit-closing relays in conjunction with a new auxiliary current-transfer relay having low-energy consumption. This is illustrated in Fig. 91. A three-legged laminated magnetic circuit has a pivoted armature and two windings, as shown. The main winding is connected in the current-transformer circuit while the auxiliary winding, which is normally open-circuited, is connected across the contacts of the circuit-closing protective relay. The circuit-opening silver contacts which, under normal line conditions, shunt the breaker trip coil, are held closed by gravity, assisted by the flux set up by

the current in the main coil. In the normal position, the gap at *A* is closed, as shown, while the gap at *B* is open. Most of the flux produced by the main winding follows the low-reluctance path through the closed gap at *A*, and only a small portion of the total flux takes the high-reluctance path through the normally open gap *B*. Under fault conditions the auxiliary winding is short-circuited by the protective relay, causing the reluctance of the path *A* momentarily to increase, thereby forcing the major portion of the flux from the main coil to cross the air gap *B*. The heavy flux crossing gap *B* causes the pivoted armature to move, so as to close *B* and open *A*. The contacts are opened by

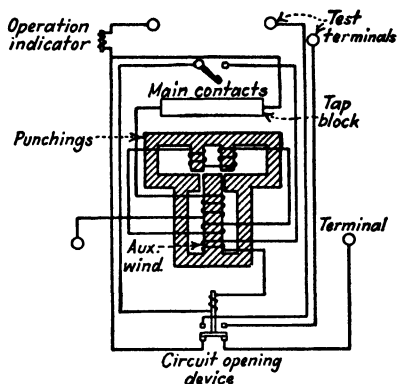


FIG. 92.—Circuit-opening overcurrent relay.

the movement of the armature with a quick, positive action and are reset by gravity when the breaker trips.

This auxiliary relay will operate positively on a minimum of 3.5 amp., and, when used with a current-transformer trip coil of average impedance, it can be conservatively applied to transfer currents up to 50 amp.

Circuit-opening Overcurrent Relay.—A second scheme of protection is based on the use of a special circuit-opening induction relay. The circuit-opening feature incorporated in the overcurrent induction relay is obtained by replacing the standard “seal-in” contactor with a coil-operated circuit-opening device, actuated through the main circuit-closing contacts by energy from an auxiliary winding placed on the main magnetic circuit of the relay (Fig. 92). When the relay operates, the main contacts are closed by the rotation of the induction disk, thereby

completing the circuit from the auxiliary winding to the operating coil of the circuit-opening device. Sufficient energy is available in the auxiliary winding, due to the transformer action by virtue of the fault current, to cause the contacts to open with a positive, quick action, efficiently transferring the secondary current to the breaker trip coil. The opening of the breaker allows the contacts to reset by gravity.

The circuit-opening contacts consist of a self-aligning silver disk bridging two ball-shaped silver stationary contacts. The

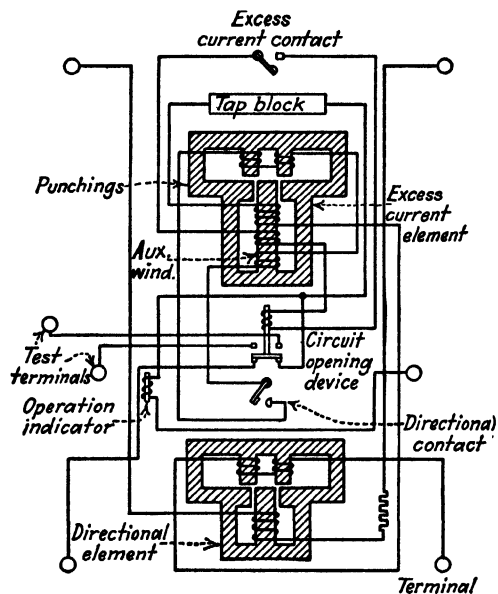


FIG. 93.—Circuit-opening overcurrent directional relay.

contact resistance is low enough to prevent any appreciable leakage of current through the breaker trip coil, even under extreme short-circuit conditions.

This scheme is applicable where the maximum secondary current is 50 amp. or less.

The directional overcurrent relay can also be made circuit opening, as illustrated in Fig. 93.

A special tripping attachment can be applied to a breaker-solenoid-operating mechanism to replace the standard shunt-trip assembly and provides space for four trip coils, any or all of which

can be wound for either current or potential. The operation of any one of the trip coils actuates a sensitive toggle mechanism which effects the tripping of the breaker. The device is fast and positive in operation and is protected from vibration by a flexible mounting.

This attachment has been successfully applied to breakers up to and including the 110-kv. rating, as well as to various types and makes, although slight mechanical modifications are generally necessary in order to adapt it to certain breakers. The volt-ampere burden at the tripping point is sufficiently low to allow positive tripping from most standard bushing-type current

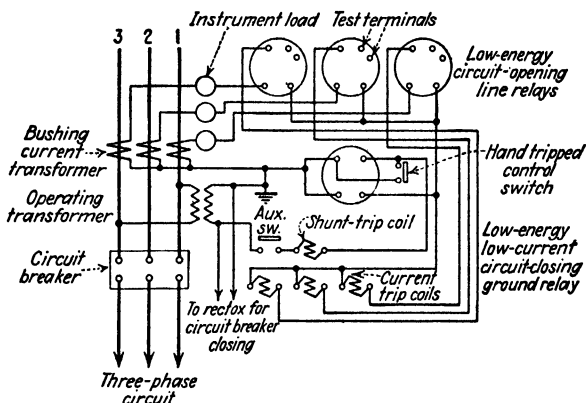


FIG. 94.—Overcurrent protection using three circuit-opening line relays and one circuit-closing ground relay.

transformers as low as 20:1 ratio when used in a circuit with any of the relays described.

A simple application of overcurrent relays is shown in Fig. 94 consisting of three circuit-opening line relays and one circuit-closing ground relay. The trip attachment is applied to the oil circuit breaker, three of the trip coils being wound for current and the fourth coil for potential. The current coils are designed to trip on a minimum of 4 amp., while the potential coil is wound to trip on a minimum of 30 per cent of normal line voltage. The energy for shunt tripping on ground faults is obtained from the station-operating transformer.

This method of alternating-current tripping is termed *overlapping current and potential*. By this method, ground faults

are cleared by potential tripping up to and even beyond the point where the ground fault becomes heavy enough to produce sufficient current to operate the line relays.

Automatic Switching for System Ties.—Space does not permit a discussion of this phase of the protection problem. Reference should be made to Automatic Switching for System Ties, by E. V. Ellett, in the *Electric Journal* of July, 1933.

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CHAPTER XII

PILOT SYSTEMS OF RELAYING FOR TRANSMISSION-CIRCUIT PROTECTION

The increase in size and complexity of power systems and the resulting problems of maintaining stability during system faults have necessitated the rapid clearing of faults. High-speed relays of the balanced and distance type, while very effective, fall short of the ideal of providing overall protection with instantaneous operation at both ends of a faulted section for any fault location. Faults near the ends of the sections protected by distance relays are cleared by sequential operation, that is, the relay nearest the fault operates at high speed, but at the other end the slower speed back-up element operates. This also applies to the operation of balanced relays on parallel lines for faults near the ends of each section. Furthermore, in complex systems, it is frequently difficult to coordinate the timing of distance and balanced current relays with relay equipment on other sections without sacrificing high speed of operation on the important sections.

It should be appreciated that it is physically impossible to secure ideal characteristics of speed and selectivity in relays for transmission-line protection that are actuated by factors existing at some one point on the system, and yet are not responsive to oscillations. System oscillations may occur which produce reactions in the relays similar to short circuits. In the methods of relaying described only three electrical dimensions and time are available at any one point as factors for the detection and location of faults. The three electrical dimensions are current, voltage, and the angle between them. Many novel combinations of these factors have been devised which have resulted in improved relaying systems, but the ideal has not and cannot be secured using factors existing at one point. Thus it would appear that to secure overall protection of a circuit, it is necessary to use quantities at two points, such as both ends of a section of

line, rather than one end. The logical outgrowth of this thought is the use of differential protection where quantities existing at two points are compared. This type of protection when applied to transmission lines is commonly called *pilot-wire protection*, because wires must be used between the circuit ends to bring the quantities together for comparison.

The principle of operation is similar to the differential systems for protection of generators and transformers, but a somewhat different set-up is required because the breakers at the line ends are so far apart that only one set of relays actuating two trip circuits cannot be used.

This method of protection is ideal from a theoretical standpoint since both line ends may be tripped instantaneously for faults at any point, and the system is non-operative for faults outside the section and therefore is inherently selective. The element time obviously is not involved since the operation is determined by the unbalance between similar quantities at the line ends. This method of relaying transmission circuits, while quite popular in Europe, has not been used extensively in this country due to the cost and complication of providing alternating-current pilot wires. Many operating difficulties arise from induced currents, where the pilot wires parallel transmission lines and the possibilities of short circuit or open circuit in the wires have discouraged the use of pilot-wire systems.

A brief description of the general types of alternating-current pilot-wire schemes will be given before considering the more important directional-comparison method in which the trip circuits of directional relays at the line ends are in effect connected in series through a pilot channel.

Opposing Voltage Alternating-current Pilot-wire Schemes.—In one very simple pilot-wire scheme, special high-voltage current transformers are connected in opposition with one relay per phase at each end of the protected section. During normal conditions, if the characteristics of the transformers on opposite ends of the same phase are similar, no current will flow. When a fault occurs, a current flows through the pilot wires and the relays at both ends.

Differential relays can be used in the same scheme, as shown in Fig. 1. There are two parallel circuits across the secondary of each current transformer; one consists of the restraining ele-

ment of the relay in series with a resistor and the other contains the operating coil of each relay connected together by a pilot wire and in series with the parallel circuit of the restraining element and other transformer secondary. Under normal conditions, equal currents flow through the restraining elements and no current flows through the operating coils since no potential difference exists between ends of the pilot wires. If a short circuit occurs, the magnitude and phase relations of the currents at the two ends will be different and a difference of potential will exist between the two ends of the pilot wire. If the flow of current in the secondary circuit is sufficient to overcome the torque of the restraining coils, the relays will operate.

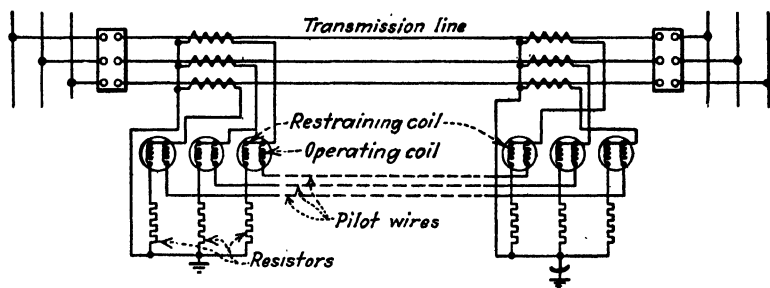


FIG. 1.—Opposing-voltage pilot-wire scheme with differential relays.

Three pilot wires are required for this scheme. With a ground fault on one phase, the current returns through the pilot wires on the other two phases. The current-transformer burden is high for both of these schemes. The resistors in Fig. 1 provide a voltage drop for operation of the relay. The rating is determined by the requirements for a particular installation. The current transformers at one end of the line must be grounded solidly and at the other end grounded through a film cutout or low-voltage arrester. The cutout prevents circulation of current through the relays, owing to difference in potential of the ground between the ends of the pilot wires, and grounds the pilot wires in case of any high potential due to induction or other effects.

The opposing-voltage schemes described are susceptible to false operation from induced voltages, due to short circuits. An open circuit in the pilot wires makes the relays inoperative, and a short circuit causes the relays to trip. Heavy through short circuits may cause false operations due to charging currents.

The length of circuit that can be protected is limited by the burden on the current transformers.

The pilot wires may be shielded by means of a metallic tube surrounding the pilot wire,¹ cut in two midway between ends. The capacity between the pilot wires and shields is in such relation to the capacity between the shields of the three phases that the charging current flows between the pilot wires and shields and not between pilot wires. This method prevents charging current from appearing in the relay windings. The same results may be obtained by using a compensating wire in parallel with each pilot wire and extending one-half the circuit length from each end of the cable. The relay coil is made in

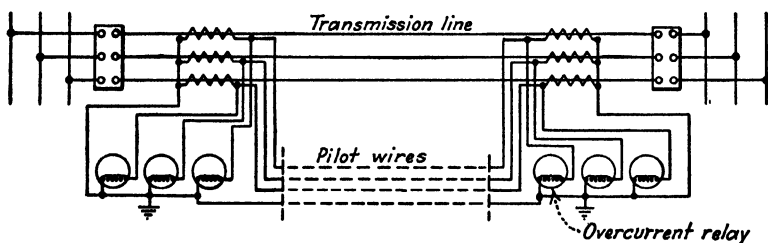


FIG. 2.—Circulating-current pilot-wire scheme.

two parts, connected differentially. When charging current flows, equal currents appear in the pilot wires and compensating wires, if the potentials of these wires are equal at all points. If the charging currents are equal, no operating torque is produced in the relay. This scheme requires a special current transformer with an air gap in the magnetic circuit to bring the line current and transformer-secondary voltage in proper relation.

Schemes of this nature have been used extensively in Europe but have found no favor in America owing to the high initial cost and apparent lack of justification for the high degree of sensitivity obtainable.

Circulating-current Pilot Wire.—In the circulating-current scheme the secondaries of the current transformers are connected in series through the pilot wires as shown in Fig. 2. This connection is the same as that used for the differential protection of generators and transformers. Under normal conditions the current circulates through the pilot wires. When a fault occurs

¹ N.E.L.A., "Handbook and Supplement," p. 697..

in the protected section, the current difference flows in the relay thus causing operation.

This scheme can be used where the potential drop in the pilot wires is not excessive under through-short-circuit conditions, and if high-peak voltages do not occur when the circuit breakers are opening due to momentary open-circuit effects. The current transformers used in this scheme generally have secondary ratings of 0.5 amp. in order to minimize pilot-line drop. A high-impedance relay is used to reduce the current leakage through the relay during through-fault conditions.

The circulating-current pilot-wire scheme is subject to false operation due to induced voltage and charging current. A

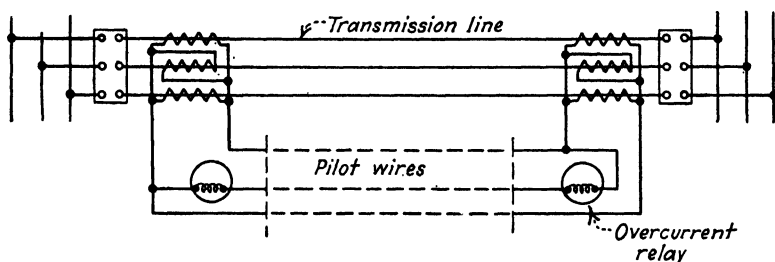


FIG. 3.—Circulating-current pilot-wire scheme for phase and ground faults.

short circuit on the pilot wire prevents, and an open circuit causes, protective relay operations.

A scheme employing three pilot wires¹ and providing line- and ground-fault protection is shown in Fig. 3. The three current transformers at each end of the circuit are connected in parallel with one phase reversed. The vector sum of the secondary currents is twice the current in any one secondary under balanced conditions, and this current circulates through two of the pilot wires. Two relays are connected in series by a third pilot wire between points of equal potential at opposite ends of the circuit. No current passes through the relay circuit under normal conditions, but, when an internal fault occurs, the current from one set of current transformers reverses and the current from both sets of transformers flows in the relay circuit.

This scheme provides protection against three-phase faults, single line-to-ground faults on any phase, and line-to-line faults

¹ N.E.L.A., "Handbook and Supplement," p. 1187.

between the phase with reversed current transformer and either of the other two phases. Protection is not provided for a line-to-line short circuit between the two phases which do not have reversed current transformers. The length of line that can be protected is limited, because twice the usual pilot-wire current flows normally.

A differential relay with two coils, so connected that under normal conditions the currents through the two coils are such that equal and opposite torques are produced,¹ is shown in Fig. 4. If the two coils have equal turns, the resistor is of such value as to cause the current to divide equally between the

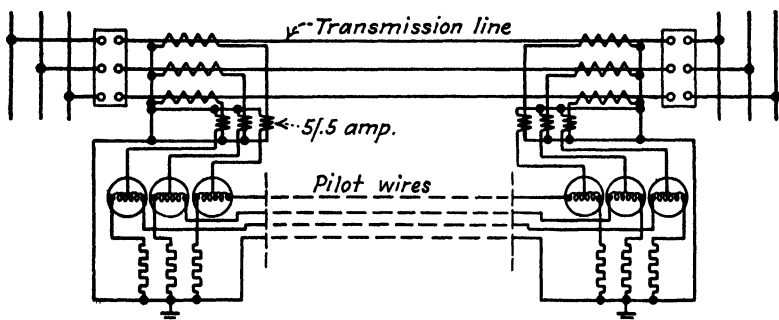


FIG. 4.—Circulating-current pilot-wire scheme with differential relays.

pilot wire and resistor. When a fault occurs, the current from one end of the line is in opposition to that at the other and the larger portion flows through the relay coil and resistor at each end, causing operation.

This scheme is suitable for short lines and has the advantage that no excessive potentials are induced in the line-current transformers.

A modification² of the scheme of Fig. 4 is shown in Fig. 5. Differentially connected, 2:1 ratio, three-winding pilot transformers are used instead of the differential relays. A high-speed sensitive relay is connected in the secondary of each pilot transformer. If a fault is external, the m.m.fs. of the two primary windings oppose, and, if the fault is internal, the m.m.fs. are additive and a relay operation results.

¹ N.E.L.A., "Handbook and Supplement," p. 692.

³ N.E.L.A., *op. cit.*, p. 1190.

In Fig. 6 is shown a similar scheme¹ in which only two pilot wires are required. The current transformers connected in the secondaries of the line-current transformers have unequal ratios. The unequal-ratio current transformers are so connected that

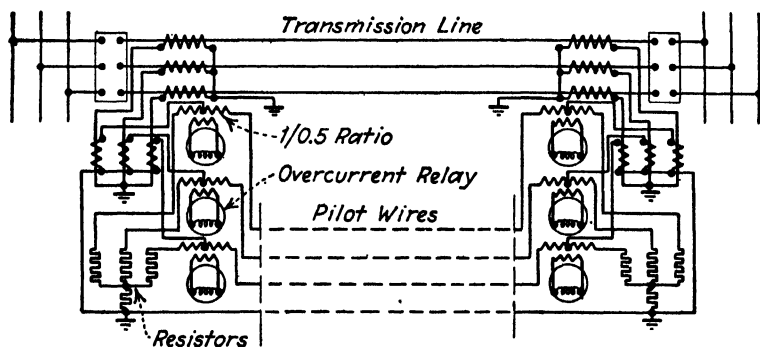


FIG. 5.—Circulating-current pilot-wire scheme with pilot transformer and four wires.

their secondary currents add vectorially to their primary currents, and the resultant flows in the primary of a 5:1 ratio isolating transformer. The secondary current of this transformer is applied to the three-winding pilot transformer. This scheme

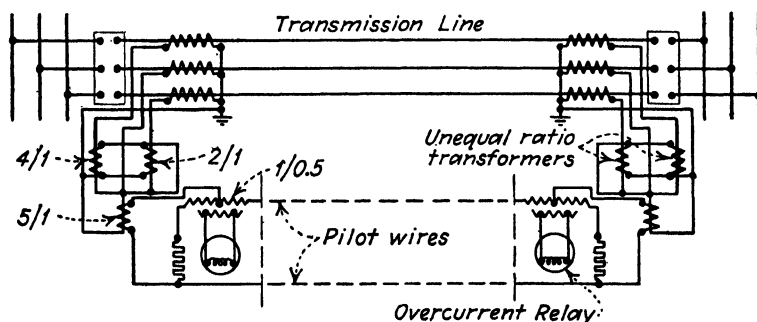


FIG. 6.—Circulating-current pilot-wire scheme with pilot transformer and two wires.

provides protection against all types of faults with a minimum amount of equipment.

The schemes shown in Figs. 5 and 6 both give a high degree of sensitivity even for comparatively long lines.

¹ N.E.L.A., *op. cit.*, p. 1190.

A circulating-current pilot-wire scheme¹ using balanced current relays is shown in Fig. 7, which imposes a low burden on the current transformers. Four balancing resistors are used at each end to maintain balanced conditions during through faults, whether line to line or line to ground, thus permitting a low current setting on the ground relay. Where sensitive ground-fault protection is not required, the ground relay is omitted but not the neutral balancing resistor R_n . This scheme may be applied to long lines if 10:1 ratio insulating current transformers are used to give a low effective pilot-wire impedance. For short lines the insulating transformers may be omitted

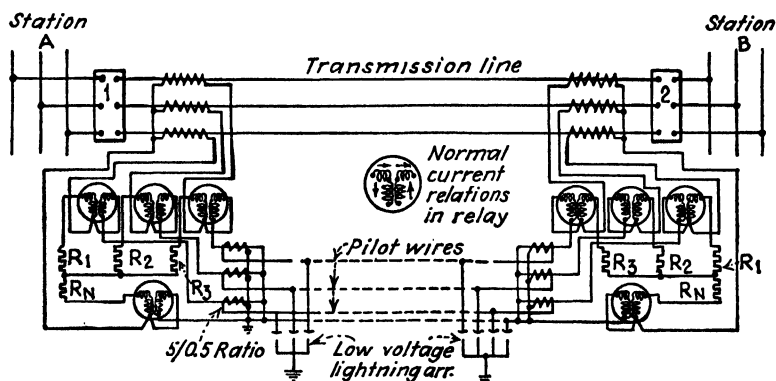


FIG. 7.—Circulating pilot-wire scheme using balanced current relays.

unless required for protection against high voltage. With 10:1 ratio current transformers, the values of balancing resistors R_1 , R_2 , and R_3 should each be $\frac{1}{200}$ of the pilot-wire impedance plus the impedance of the insulating transformers, and the resistance of R_n should be $\frac{1}{200}$ of the pilot-wire impedance only.

Pilot-wire Schemes for Ground-fault Protection Only.—It is sometimes considered desirable to protect underground cable circuits against ground faults only, when the system neutral is grounded. In most cases, cable faults either develop to ground or if from line-to-line almost immediately fail to ground. The nature of the initial failure is, of course, governed by the

¹ CRICHTON, L. N., and H. C. GRAVES, The Application of Relays for Protection of Power System Interconnections, *Trans. A.I.E.E.*, vol. 47, p. 259, 1928.

design of the particular cable, *i.e.*, whether single or three conductor, and the relative amount of insulation to ground and between phases. One method of protection is to use a sheath transformer at each end of the cable, having three primary and one secondary winding. The primaries of the sheath transformer at each end are connected to the three phases, and normally under balanced conditions the resultant flux is zero. In case of a ground fault, the flux is no longer zero and a current flows in the secondaries at the two ends of the cable and through the two pilot wires thus operating the relays.

The same results can be accomplished by connecting a 5:0.5 current transformer in the secondary neutral of current trans-

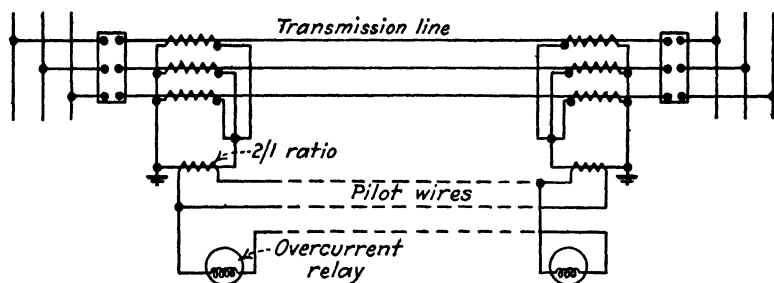


FIG. 8.—Circulating-current pilot-wire scheme for ground-fault protection.

formers which may be used for other purposes. The secondaries of the 5:0.5 current transformers are connected in series with the relays and two pilot wires. The burden on the current transformers is low with this scheme.

A scheme for ground protection¹ is shown in Fig. 8 which uses the same relay and pilot-wire connections as Fig. 3. The relays are connected between points of equal potential at opposite ends of the circuit. No current appears in the relay circuit during through faults, but, in case of an internal fault, the residual currents become unbalanced and a reversal occurs at one end so that the sum of the currents from the 2:1 ratio residual-current transformers flows through the relays.

All of the alternating-current pilot-wire schemes described have many objections among which are: The testing is complicated; the pilot wires are expensive and are subject to induced voltages and charging currents; the reliability of protection

¹ N.E.L.A., *op. cit.*, p. 1186.

depends upon the reliability of the pilot wires. Generally, the pilot wires are less reliable than the circuits protected.

DIRECTIONAL-COMPARISON SYSTEMS¹

A somewhat different scheme of overall protection has become prominent in recent years, known as the *directional-comparison method*, which has many advantages over the alternating-current pilot-wire systems. In this scheme, the relative instantaneous direction of the fault current at the two ends of line sections is compared by means of some communication channel. The communication circuit is used to establish a tripping operation on the breakers at both ends when a fault is present in the protected section. This scheme has the advantage over the alternating-current pilot-wire schemes in that a reliable communication circuit can be provided more readily and with less expense than alternating-current pilot wires. The system is also less complicated, more flexible, easier to maintain, and not so susceptible to false operations as the differential schemes.

The fundamental principle of operation of the directional-comparison method is that the presence of a fault may be determined from the relative instantaneous directions of the currents at the two ends of a line. For example, under normal conditions, with power flowing in either direction in a circuit, the current will be flowing in at the sending end and out at the receiving end. If directional relays are so connected at each end of the line that their contacts close only when current flows into the line, then, under normal conditions, the contacts of the relays at the sending end will be closed and those on the relays at the receiving end will be open. If a fault occurs on the line and a source of power supply is available at both ends, the currents will flow into the line at both ends and the contacts of the directional relays at both ends will close, thus indicating the presence of a fault. By means of some communication system the fact may be established that the directional relays at both ends are closed at the same time and a tripping operation initiated. The essential element necessary is that each end of the line receive the information that the directional relay at the

¹ Many of the essential data on the directional-comparison systems in this section were obtained from J. H. Neher, *The Use of Communication Facilities in Transmission-Line Relaying*, *Trans. A.I.E.E.*, vol. 52, p. 595, 1933.

other end is closed before a tripping operation occurs. It is evident that the element time is not involved in the operation.

The directional-comparison scheme in its simplest form consists of a set of directional relays at each end of the line with a conventional direct-current telegraph circuit extending from one end of the line to the other as shown in Fig. 9. It can be seen that the trip circuit cannot be established through either circuit-breaker trip coil unless both sets of directional relays are closed. The trip coils and the directional contacts at each end of the line are in effect connected in series.

The directional-comparison method cannot accomplish the results of a percentage-differential scheme, particularly for

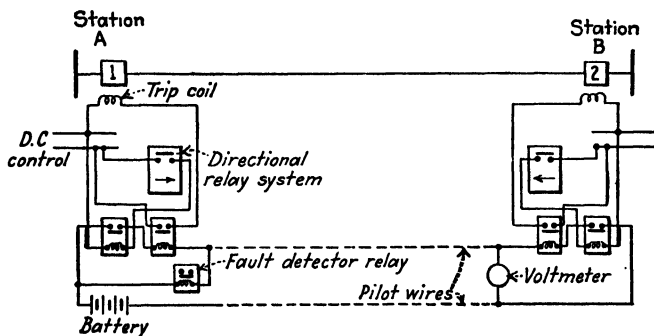


FIG. 9.—Fundamental circuit for directional-comparison method of relaying.

partial-fault conditions such as turn-to-turn failures in transformers. With the directional-comparison method, power flow could continue through the faulted phase, and the directional relays at one end would not close. This is of no particular consequence on a transmission line, because, with line-to-line faults, there is little probability of sufficient through power transmission on the faulted phases to cause the relays at one end to fail to close. With a line-to-ground fault there is more of a tendency for through power transmission, but this can be readily cared for by using ground relays. It should be remembered that trouble may develop, if too much stress is placed on securing an extreme sensitivity of fault detection in any differential or pilot-wire scheme. Engineers and operators are frequently prone to overemphasize the necessity for detecting low-current faults on transmission circuits. Fault currents of low value, in general, develop rapidly into higher values or are

extinguished due to deionization of the arc as it lengthens, and no relay operations result.

It is necessary that the directional relays at each end of the line function properly for any and all fault conditions. It is possible to design the relay system to meet all conditions, even when the line is tapped or when there is a power source only at one end of the line. When the line is tapped, a set of directional relays can be installed on the tap and connected into the communication system used for the main-line protection. This is required only when there is a source of power at the end of the tap. If the tap with its associated receiving-end apparatus can be considered a part of the main circuit, then this set of relays is not required. If, however, for example, a breaker and transformer are located at the end of the tap (no breaker at junction of tap and main line), and a source of power is present, it is necessary to use directional relays. If a source of power is not present at one end of the line or the end of the tap, an instantaneous circuit-opening overcurrent relay and an instantaneous circuit-closing undervoltage relay in each phase should be used instead of directional relays. The contacts of the two relays in each phase are connected in series. Normally, the contacts of the undervoltage relays are open, but in case of a fault the contacts close. If the condition is due to a through fault, the overcurrent-relay contacts open before the voltage-relay contacts can close, hence no tripping results. If the fault is on the line, the overcurrent contacts will not open and the line will be cleared. If greater sensitivity is required for ground faults, an instantaneous circuit-opening overcurrent and an instantaneous overvoltage circuit-closing relay operating on the zero-sequence current and voltage, respectively, may be used.

In case a source of power is available at one end of the line only part of the time, it is necessary to use both the combination of current and voltage relays in conjunction with directional relays. Schemes other than those described can readily be devised which will accomplish the same results.

There are several different systems of relays that can be applied at the ends of the line in the directional-comparison scheme. For line-to-line fault protection, a polyphase directional relay, or three single-phase relays may be used. It is generally advisable to use an instantaneous overcurrent relay in conjunction

with each single-phase directional element with the trip contacts for each phase in series and the groups in the three phases in parallel. This is necessary because under certain conditions an unsymmetrical through fault may cause a single-phase directional relay having approximate wattmeter characteristics to reverse and close its contacts. The phase on which this occurs will be carrying a comparatively small current so that the overcurrent relay if set above this value of current will prevent false operation. Furthermore, the directional contacts in the three phases must be isolated from each other to prevent false operation for a condition where load current is flowing from Station B toward A and a single line-to-ground fault occurs on phase *c* on the far side of Station B. The relay on phase *c* at Station A closes due to fault current, and the relays on phases *a* and *b* at Station B close due to load current. If the directional contacts on the three phases at each end are not isolated, the circuit will be tripped. The overcurrent relays also prevent false tripping due to load current.

Single-phase directional relays, when provided with voltage restraint having proper characteristics, can be used without overcurrent relays in some cases. The voltage restraint must be such that the directional contacts will not close due to the flow of maximum load current. When polyphase directional relays are used, either voltage-restraint or overcurrent relays must be used for the same reasons as given for single-phase elements. If a single line-to-ground fault occurs between Stations A and B, the closing torque due to the fault current in the faulted phase at Station A must be greater than the sum of the opening torques due to load current in the other two phases if the load current is flowing in the opposite direction, otherwise the polyphase relay will not close.

The contacts of ground relays operating on zero-sequence current are normally open, so overcurrent relays used in conjunction are not always essential; however, it is advisable to use them in order to reduce the sensitivity. When system conditions are such that relays designed for line-to-line fault protection may operate incorrectly when certain ground faults occur, it is necessary to provide ground-fault protection so interlocked with the line-to-line protection groups that the phase groups are locked out upon the occurrence of a ground fault.

When a line is provided with directional-type relays for back-up protection, the directional elements can be used to provide directional-comparison protection if the contacts of the directional elements are placed in the relay trip circuit.

The system of directional relays used for overall protection is not limited to any particular type, as various combinations can be employed with the scheme of connections designed to secure the most satisfactory protection for the particular application. The system may be designed to operate from phase or the line currents and voltages, or from sequence quantities.

Where high-speed distance or current balance relays are used, practically instantaneous operation at one end of the line

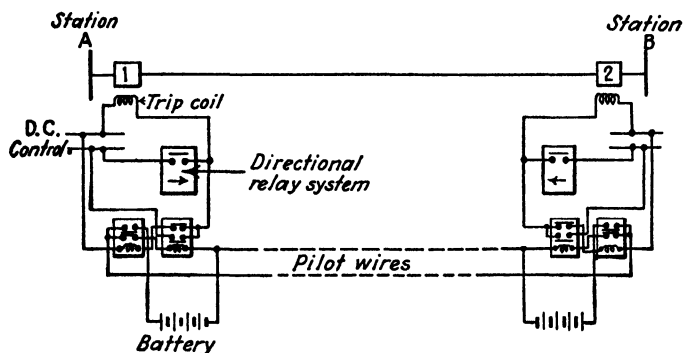


FIG. 10.—Transferred tripping through pilot wires.

or the other is obtained for any fault location. If it is necessary to trip both ends instantly, rather than wait for sequential operation, in order to maintain stability or for other considerations, a communication channel can be used by means of which the other breaker is tripped as soon as either breaker trip coil is energized. This scheme is generally known as *transferred tripping*. The elements of the system are shown in Fig. 10. The same communication or pilot-wire facilities are required as for the directional-comparison method. The selectivity of the transferred-tripping arrangement depends entirely upon the selectivity of the relay system at the ends of the lines. This scheme, although possibly more expensive and complicated, has the advantage that complete line protection is provided even when the communication system fails. Transferred tripping can be applied where selective-timing relays are already in

service more readily than the directional-comparison system. The fact should be recognized that the full benefits of transferred tripping can be realized only, if the selective-timing relays are of high-speed type.

The design of a directional-comparison pilot-wire protective scheme will generally be influenced by a variety of local conditions and particularly by the type of communication facilities available. The circuits used for pilot purposes should be properly coordinated with supervisory control, telemetering or telephone communication circuits or whatever is in service. The general types of systems that are most readily adaptable for relay communication are direct-current telegraph, low-frequency rectified alternating-current pulse telegraph, voice-frequency telegraph, carrier current over power-transmission circuits, and short-wave radio. A brief description of possible designs for each of these types for relaying will be given.

Direct-current Telegraph.—The simplest arrangement employing a two-wire metallic circuit is shown in Fig. 9. A short circuit occurring on the pilot circuit will cause the end of the circuit at which the battery is located to trip whenever the directional relays at that end close, regardless of conditions at the other end of the line. False tripping would thus result with through faults. This can be prevented by using a sensitive fault-detector relay which is normally energized. This relay operates when a fault occurs on the pilot line, to lock out the pilot circuit and give an alarm.

When a through fault occurs, such that the directional elements at the end of the line opposite the battery close, the pilot circuit will be closed through the fault-detector relay. The resistance of the detector relay must be such that the flow of current under this condition is lower than the minimum operating current for the tripping relay at the distant end. The fault-detector relay will operate under this condition; consequently, it is necessary for it to be a time-delay device so that the fault may be cleared before the pilot circuit is locked out. The continuity of the pilot circuit may be continuously checked by means of a high-resistance voltmeter or undervoltage relay at the end distant from the battery. Either of these devices must have sufficient resistance so that the maximum current is less than the minimum operating current of the fault-detector relay.

In Fig. 11 is shown an arrangement using a grounded telegraph circuit derived from a simplex telephone circuit, the latter circuit also being used for telephone communication. A single wire with ground return could be used, but it would not be so reliable. The scheme in Fig. 11 will operate with one wire open or the two wires short-circuited but not grounded. For any scheme employing ground return, the grounding points for the direct-current telegraph circuit must be made at a considerable distance from the power-station grounds, otherwise a large difference of potential can exist across the two ends of the telegraph circuit. In general, any circuit operating with ground return is less reliable than the two-wire metallic circuit.

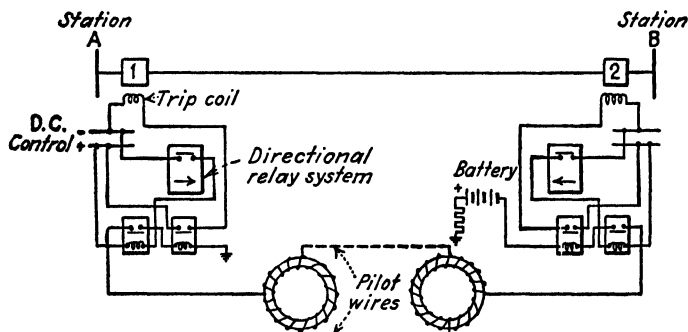


FIG. 11.—Directional-comparison scheme using grounded-pilot circuit derived from a simplex-telephone circuit.

Low-frequency Rectified Alternating-current Pulse Telegraph.

When direct-current telegraph circuits are used to provide overall protection for two or more parallel transmission lines, it is necessary to use as many pilot circuits as there are transmission circuits. It is possible to use additional batteries and polarized relays and provide protection for two transmission lines using only one pilot circuit; however, the circuit is complicated, difficult to use, and will not operate during simultaneous faults on both circuits. The rectified alternating-current pulse¹ system (Fig. 12) can be used where it is desired to use only one pilot circuit for protection of two transmission circuits in order to reduce the initial cost and operating expense. The apparatus of one circuit is operated by positive half cycles and the other

¹ J. H. NEHER, *op. cit.*, p. 595.

by negative half cycles of current and voltage from the alternating-current generator. The tripping relays must be designed to hold in on half-cycle power supply. Ordinary copper oxide rectifiers are suitable for this application. The small generators should be low frequency, say 20 cycles, to minimize the effect of line reactance. For most reliable operation the generator should be driven by a direct-current motor operating from the station battery. With the pulse system, coupling or insulating transformers cannot be used because the current pulses set up counter e.m.fs. in the circuit.

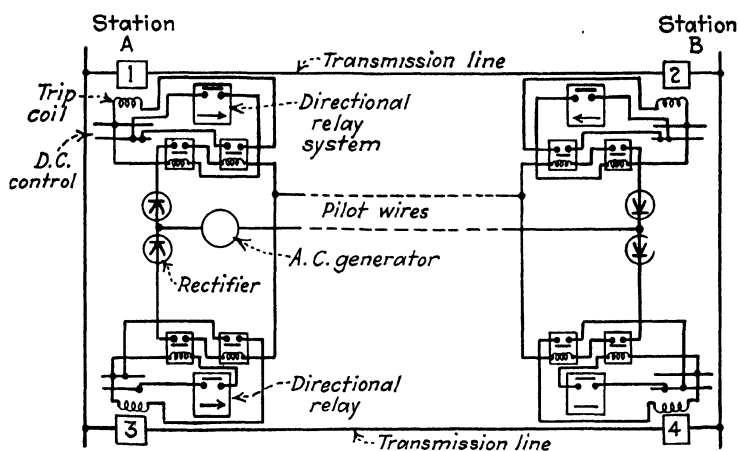


FIG. 12.—Protection of two transmission lines with one pilot circuit using low-frequency rectified alternating-current pulse system.

Voice-frequency Telegraph.—A system of communication is being used in which 12 separate messages can be sent simultaneously over a single circuit. This is accomplished by superimposing 12 different frequencies on one telephone circuit, each frequency constituting a separate channel. The various frequencies are separated at the receiving ends by means of band-pass filters, the output of the filters being applied to receiving devices.

This system can be used as the communication channel for directional-comparison relaying. An arrangement is shown (Fig. 13) in which the closing of the directional contacts at either end operates the transmitter which sends a signal of a definite frequency. The receiver at the other end of the line is

tuned only to that frequency. The breakers at both ends are tripped when a signal is received at either end at the same time that the directional element at that end is closed. To prevent the transmitter at one end from operating the receiver at the same end, two separate frequencies may be used for each circuit protected or a bridge transformer used in place of the repeating coil. The bridge transformer keeps the transmitter frequency out of the receiver at the same end so that one frequency can be used for the protection of each circuit. A single telephone circuit can then be used for the protection of 12 power circuits by the use of 12 frequencies over one pair of wires.

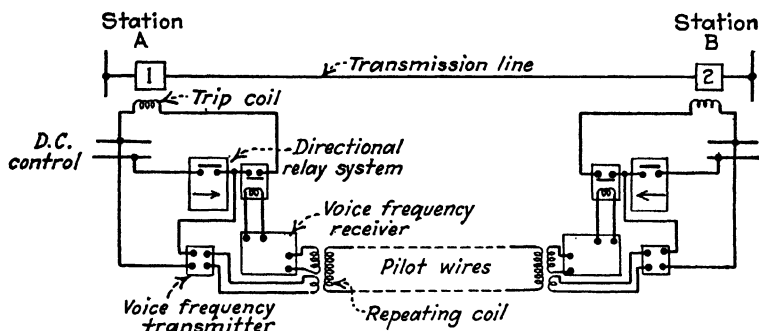


FIG. 13.—Voice-frequency relay communication for protection of 1 to 12 circuits.

Carrier Current.¹—A system of overall protection using carrier current as the communication channel has been developed and applied to important lines. The carrier-current equipment consists of a transmitter and receiver at each end of the line and connected to the line through coupling capacitors. The transmitter consists of a vacuum-tube oscillator and a power amplifier, and the receiver consists of a simple single-tube receiver with the grid biased so no plate current flows unless a signal is received. The frequency of the oscillator is usually 50 to 150 kilocycles per second. Power is supplied to the carrier equipment by a rotary inverter or motor generator operating from the station battery.

Since the carrier-current signals are superimposed upon the power-transmission conductors, a line fault might block the communication channel; therefore it is usual to arrange the equipment so that signals are used to prevent rather than

¹ The diagrams given in this section are representative of typical schemes, but refinements and new developments based on operating experience are rapidly appearing.

initiate tripping operations.¹ The scheme requires that the relays on unfaulted sections be prevented from tripping by transmitted signals, and the relays on the faulted section are permitted to trip by not receiving blocking signals. It is necessary to install traps to prevent external faults on the coupling phases from interfering with the transmission of blocking signals. The traps have a low impedance to power-frequency current and a high impedance to carrier current.

In Fig. 14 is shown a schematic elementary diagram² of an arrangement employing carrier current in which the protective-

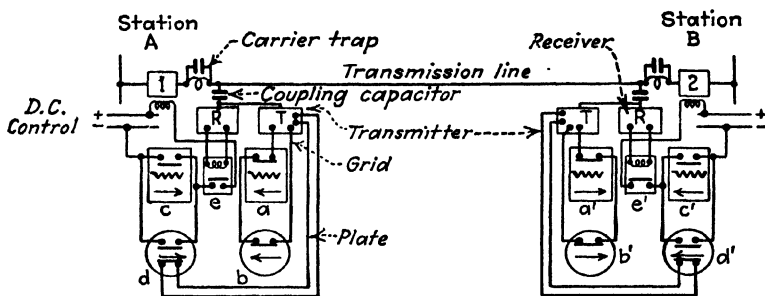


FIG. 14.—Directional comparison system of protection with carrier current. *a, a'*, Polyphase directional starting relay with voltage restraint; *b, b'*, directional ground starting relay; *c, c'*, polyphase-directional tripping relay with voltage restraint; *d, d'*, ground-directional tripping relay; *e, e'*, receiver relay.

relay system is designed for conditions where the line-to-line short-circuit current may be less than normal load current and provision is made for sensitive ground-fault protection. Such conditions arise due to the variation in transmission-system set-up and connected generating capacity. This scheme, like many others of the directional-comparison group, is susceptible to tripping during system oscillations.

In Fig. 14, relays *a, a', c*, and *c'* are of the polyphase, directional type with voltage restraint which operate with a current

¹ This is the so-called intermittent system. A continuous system has been developed in which signals are transmitted continuously and the receivers are normally energized and block the circuit-breaker trip circuits. In case of a fault, the directional relays at both ends of the faulted section close and interrupt the transmitter plate circuits, thus closing the contacts in the receivers which trip the breakers. Since the receivers at each end are energized by the transmitters at both ends, the breakers will not be tripped unless fault current is flowing in at both ends and both transmitters interrupted.

² TRAVER, O. C., JOHN AUCHINCLOSS, and E. H. BANCKER, Pilot Protection by Power Directional Relays Using Carrier Current, *Gen. Elec. Rev.*, November, 1932, p. 567.

flow as indicated by the arrows. When a fault occurs outside the section, say to the left of breaker 1, starting relay *b* operates, and its contacts open and remove a negative bias from the grid circuit, thus putting the transmitter in operation. The signal must be transmitted and the contacts of blocking relay *e'* opened before the contacts of tripping relay *c'* can close, otherwise false operation will occur. It is necessary, therefore, that the operating time of the tripping relays be slightly greater than the starting relays which initiate the blocking signal.

If a fault occurs between Stations A and B, relays *c* and *c'* close their contacts and trip the breakers. The power flow is in the wrong direction to operate relays *a* and *a'*, and hence *e* and *e'*, so no blocking signals are sent.

Relays *d* and *d'* are directional ground-tripping relays and their normally closed contacts are in series with the plate circuit of the transmitter. The ground relays have preferential control over the polyphase relays. This is necessary, because, if, for example, a single line-to-ground fault occurs between Stations A and B when heavy load current is flowing from A toward B, the voltage restraint may be so reduced that phase relays *c* and *a'* operate. Relay *a'* would start the transmitter and operate blocking relays *e* and *e'* when the internal ground fault is present. The residual current is in the proper direction to close the contacts of ground relays *d* and *d'*. The normally closed contacts on *d* and *d'* instantly open and break the transmitter plate supply at A and B and thus prevent relay *b'* from energizing the transmitter and sending blocking signals.

Another arrangement¹ of protective relays is shown in Fig. 15, which is applicable where the ground-fault current is always sufficient to operate the polyphase directional relays against the load-current torque. This scheme is less susceptible to tripping during system oscillations. Ground relays may be used if greater sensitivity is required for ground faults.

In Fig. 15, when a fault occurs, the instantaneous overcurrent relays *c* and *c'* operate and remove voltage restraint from the directional relay. If the fault is to the left of Station A, relay *a* opens its contacts and starts a blocking signal which operates *d* and *d'* thus preventing breaker 2 from tripping when the definite-time overcurrent relay *b'* closes its contacts. If the fault is in

¹ SPORN, PHILIP, Carrier-Current Relaying Proves Its Effectiveness, *Elec. World*, Sept. 10, 1932, p. 332.

the protected section, the contacts of the directional relays at each end remain closed and both breakers are tripped by the operation of relays b and b' .

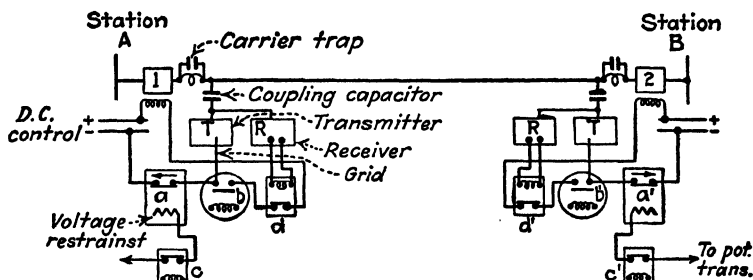


FIG. 15.—Carrier current with overcurrent relay tripping. a, a' , Polyphase directional starting relay with voltage restraint; b, b' , definite time-phase overcurrent tripping relay; c, c' , instantaneous-phase overcurrent relay; d, d' , receiver relay.

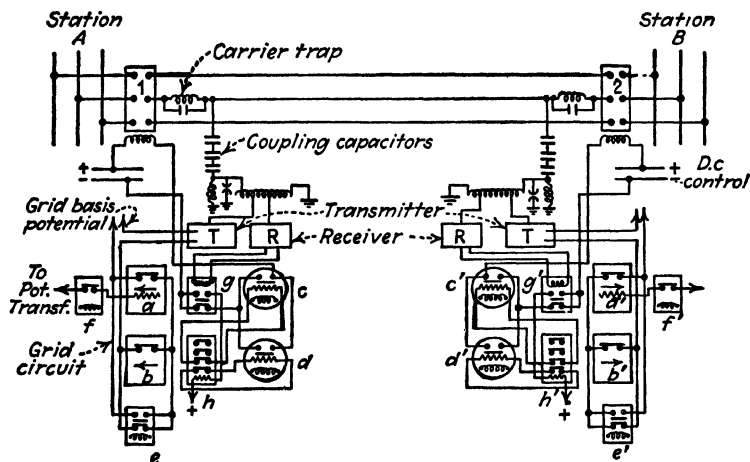


FIG. 16.— a, a' , Polyphase-directional starting relay with voltage restraint; b, b' , directional ground-starting relay; c, c' , definite-time-overcurrent phase-tripping relay with torque control; d, d' , definite-time-overcurrent ground-tripping relay with torque control; e, e' , instantaneous-overcurrent ground-current relay; f, f' , instantaneous-phase overcurrent relay; g, g' , receiver relay; h, h' , auxiliary relay.

In Fig. 16 is shown a carrier-current system fundamentally of the same type as Fig. 15, including both line and sensitive ground protection. A number of refinements are incorporated in this scheme making it less susceptible to false operation during system disturbances approaching or resulting in instability.

The overcurrent phase and ground relays are provided with torque control in which operation is prevented, even with current in the main coil, by opening the secondary circuit in the relays.

When a ground fault occurs, the instantaneous overcurrent ground relays e and e' on the sections adjacent to the fault operate, opening one set of contacts and closing another. The closing contact shunts phase-directional relays a and a' , thus giving preference in carrier control to the ground relays. If the fault is to the left of Station A, the ground-directional relay b will open its contacts and in conjunction with e , which has already operated, will interrupt the grid bias thus starting carrier transmission. Receiver relays g and g' at both stations operate and open the trip circuit by means of one set of contacts and operate auxiliary relays h and h' through another set. The operation of relays h and h' opens the torque-control circuits of the overcurrent tripping relays, allowing them to reset. This device prevents tripping in case any current disturbance occurs during the clearing of the fault which might momentarily block carrier transmission. If the fault is between Stations A and B, relays d and d' trip the breakers.

In case a phase fault not involving ground occurs to the left of Station A, instantaneous overcurrent relay f operates and removes voltage restraint from directional relay a , which in turn removes the grid bias thus actuating carrier transmission to prevent tripping. If the fault is between Stations A and B, relays c and c' trip the breakers. The tripping relays c and c' are given a higher setting than f and f' in order that the latter will not operate unless the carrier can also operate.

It will be noted in the scheme of Fig. 16 that the coupling capacitors are connected from line to ground on only one phase, and only one carrier trap is required at each end of each section protected.

The scheme of operation for the carrier-current system is really the reverse of that for other directional-comparison arrangements previously described. The problem of breaker reclosures after fault and power supply at only one end of the protected section is inherently cared for. If, for example, in Fig. 14 there is no source of power supply at Station A and an internal fault occurs, breaker 2 is tripped by c' . Relay c at Station A does not operate, and breaker 1 is not tripped since

no current flows into the fault from Station A. If the fault is external, the operation is the same, whether power supply exists at both ends or not. When a faulted section trips out and the breaker at one end is reclosed, the breaker instantly trips if the fault still exists, since no blocking signal is received from the deenergized equipment at the opposite end.

The use of starting relays with circuit-opening contacts gives a more rapid initiation of the blocking signal than circuit-closing contacts. The time required for the blocking action is approximately 1.5 cycles and tripping 3.5 cycles. The failure of a transmitter or receiver may cause false tripping on a through fault. In order to prevent this a periodic timing device at each line end momentarily energizes the transmitter. If the equipment is not at fault, the associated receiver is operated. The device is so designed that a signal is given in case the receiver fails to operate.

The same arrangement of protective relays¹ as used with the carrier-current equipment can be used with battery control and a metallic circuit for the communication channel. In case the pilot wires parallel the power line and are subject to high induced voltages or difference of ground potential between stations, it is necessary to provide sufficient insulation for the relay equipment and battery. These difficulties can be overcome by using alternating-current generators in place of batteries and inserting insulating transformers and rectifier units. The latter in conjunction with alternating-current generators of frequencies different from the power circuit will eliminate false operation from inductive interference.

Short-wave Radio Pilot Channel.—In a scheme employing short-wave radio as the communication channel,² the method of using the transmitted signal to initiate the tripping action is used. The elements of the scheme are shown in Fig. 17. The directional relays for line-to-line faults are provided with voltage restraint. The transmitter and receiver at each end of the line are normally blocked by the contacts of the directional relays. When an internal fault occurs, the directional relays at both ends of the line close. The transmitter at Station A sends a carrier of a certain frequency which is modulated by a definite-frequency oscillator, and this wave is received by the receiver

¹ TRAVER, O. C., JOHN AUCHINCLOSS, and E. H. BANCKER, *op. cit.*, p. 567.

² This scheme is in the experimental state.

at Station B tuned for the particular carrier frequency. If the directional relay at Station B is not closed, there is no reception of the signal since the receiver is blocked. The fault being internal, the signal is received and the output of the receiver is applied to a band-pass filter designed to pass the definite frequency. A relay connected to the output of the filter is operated by the signal and trips breaker 2. An exactly similar operation takes place in the opposite direction. The transmitter at Station B sends out a carrier of a different frequency from that at A but modulated at the same frequency.

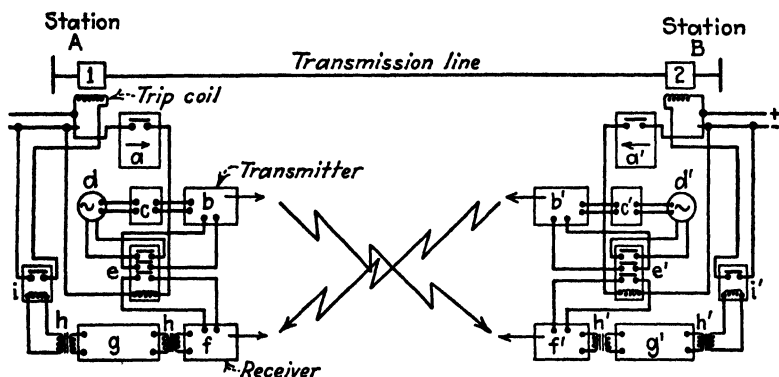


FIG. 17.—Fundamental scheme for directional-comparison protection of circuits using short-wave radio pilot channel. *a, a'*, Directional-relay system; *b, b'*, short-wave radio transmitter; *c, c'*, modulator; *d, d'*, vacuum-tube audio-frequency oscillator; *e, e'*, high-speed auxiliary relay; *f, f'*, short-wave super-regenerative receiver; *g, g'*, band-pass filter; *h, h'*, impedance-matching transformers; *i, i'*, high-speed tripping relays.

This signal is received at Station A and breaker 1 is tripped. If the fault is external and to the left of Station A, a signal may be sent by B, but no tripping action results since it cannot be received at Station A because the directional relays are open.

The transmitters are designed for short-wave operation because much less power is required and the apparatus, tubes, etc., are physically smaller and less expensive. Furthermore, it is out of the question to operate in the broadcast bands. Where distances are not too great, the use of ultra-short waves, 5 m. or less, is desirable for several reasons. First, the power requirements are insignificant, only a few watts being necessary, and the physical size of the apparatus is extremely small; second, the high-frequency carrier is not appreciably affected by external

disturbances such as lightning or static, for example; third, the carrier is definitely directional, and, by proper design of a simple reflector system, the transmission takes on the characteristic of beam transmission, thus further eliminating the influences of outside effects. It is necessary that the transmitters at opposite ends of the protected section operate at different frequencies, otherwise the transmitter at one station would operate its own receiver or smother reception from the distant station.

The receiver is of the super-regenerative type designed and tuned to receive signals from the transmitter at the opposite end of the line.

Any number of parallel circuits between two stations can be protected with only one transmitter and receiver at each station. The tripping of the two breakers on each circuit is controlled by a definite-frequency oscillator which is actuated by the directional relays. The oscillator operates through a modulator to modulate the carrier wave. A different oscillator frequency is used for each circuit, hence only an oscillator and band-pass filter are required for each breaker and a common modulator, transmitter, and receiver for each station. In case of simultaneous faults on two or more circuits, the carriers are simply simultaneously modulated with several frequencies and these frequencies are separated at the opposite station by means of the band-pass filters, and the proper breakers tripped. Two carrier frequencies can be used for the protection of any number of sections as well as any number of circuits per section.

This method of relay protection has several advantages over all other schemes described, the principal one being that the cost is only a fraction of any other where communication facilities are not privately owned. There is little possibility of trouble due to induction from power circuits. However, the radio channel may be affected by other external influences, but the transmitters are only on the air for a few cycles during a fault and any interfering frequencies would have to appear at exactly that time and contain one of the definite tripping frequencies of sufficient amplitude to operate the tripping relays. Such a group of circumstances is rather remote; in fact, there is no reason to believe the radio system is any more susceptible to false operation than any other of the schemes. Because of the short and

infrequent periods of signal transmission, no adverse effects should be imposed upon other radio transmission and reception.

General.—The directional-comparison method of relaying does not give the same protection as differential schemes in all respects. The differential system is the only one thus far devised which is not susceptible to system oscillations approaching a condition of instability. By careful design, the protective relays in a directional-comparison system may be made sufficiently free from oscillation effects to be satisfactory for practically all applications. When severe oscillations occur, voltage and current conditions approach those existing during balanced faults except that all quantities are oscillatory.

In some cases it may be advantageous to use an arrangement to lock out certain relays during through faults. For example, with power flow from Station B left toward Station A and with a fault to the right of B, if a source of power is present at Station B such as a synchronous condenser or generator, the opening of the breaker at B on the faulted circuit may produce a transient which will cause the directional element of the relay on the left of B to close. An interlock circuit can be used so that when the relays on the right of Station B close, the relays on the left are blocked from operating. The necessity for such lock-out features as this depends upon the particular application and preferably should not be used where avoidable because of the added complication.

In any of the directional-comparison schemes of the initiative type, instantaneous overall protection is provided only when both breakers are closed. Some provision must be made for making the protective scheme operative in case one end of a faulted line is reclosed after both ends have tripped. The simplest method of accomplishing this consists of introducing an additional connection between the contacts of the directional relays and the trip circuit through a contact on the breaker-closing control switch. When the control switch is in the closing position, instantaneous operation is provided.

The use of any pilot-wire relaying system as the only form of protection is unsound practice because the pilot circuit and associated equipment are rarely as reliable as the power-transmission circuits. From both an electrical and mechanical

standpoint, the power circuits are generally far superior to the communication channel. The induction-type time-delay protective relays have definitely proved their reliability superior to that of any other form of protection that has been used. In view of these considerations, it is necessary to provide additional protection in the form of back-up relays so that in case the high-speed facilities fail for any reason, the back-up protective equipment will operate to isolate the fault. In case a pilot-wire system is applied to lines in service, the protective system originally installed for protection will afford back-up protection. On new installations the back-up protection should be coordinated with all other relays.

When comparing the schemes of operation it should be noted that with carrier current the transmitted signal is used to prevent tripping or the signal is interrupted by the operation of the protective relays. With metallic circuits or the short-wave radio channel, the transmitted impulse may be used either to initiate or prevent the tripping operation. The selection of the method of operation is a matter which must be studied for each different application. With the carrier-current system, a failure of the carrier equipment does not interfere with tripping the faulted section, but false operations may result from through faults. When the transmitted impulse is used to initiate tripping, the failure of the communication channel will make the equipment inoperative, and faults will be cleared by the back-up relays. Generally, it is more desirable that the failure of communication facilities render the pilot protection inoperative and rely upon back-up protection, rather than make the power system susceptible to inselective operation. However, if the pilot protection is installed because proper selectivity cannot be secured in any other way, then the other method might be preferable, since the trouble resulting from a sustained fault could be far more serious than tripping on through faults.

The problem of selecting the type of a relay-communication system to use for a particular application is purely one of economics. The type of pilot system should be coordinated with the telephone system. If a carrier-current telephone system is in use, then this system is the logical one to use for relaying, since the same line-coupling devices can be used. If privately

owned or leased telephone circuits are used for telephone communication, then these facilities will probably prove the most economical.

The metallic circuit is the simplest type for relaying, but a separate circuit is required for each transmission line protected. If the circuits are leased, the cost may be prohibitive since this is a direct operating expense. It might prove more economical to invest in private facilities of some form. The low-frequency rectified alternating-current pulse system provides protection for two transmission lines using only one telegraph circuit. This system is limited to comparatively short distances. When stations are directly connected by telephone lines either direct-current or rectified pulse can be superimposed upon such circuits by simplexing or compositing. When circuits are available for telemetering or supervisory control, it may be possible to use the same circuits for relaying.

No specific outline can be given for the design and operation of pilot systems because such a wide variety of conditions and requirements exists. With any pilot system a considerable amount of experimental work is necessary. The schemes described in this chapter are of the more important fundamental types. Many variations of these methods can be devised to meet specific conditions.

CHAPTER XIII

HIGH-SPEED RELAY PROTECTION¹

During the last few years, many system-stability studies have been made with the view of determining the conditions under which larger amounts of power can be transmitted without danger of the generating stations falling out of step under fault conditions. As a result of these studies a number of ways of preventing loss of synchronism have been found, the most effective being high-speed isolation of the faulty section of line. The allowable time for clearing faults to prevent two generating systems or units from losing synchronism is fixed largely by the inherent stability of the system, the severity of fault, and the type of fault.

It is convenient to place systems in two general classifications in studying stability. In the first class, there are metropolitan systems which normally have a higher degree of stability, due to the large number of generating units which are interconnected by ties of comparatively low impedance; and, in the second class, those systems or generating plants which are connected by long high-voltage high-reactance transmission lines and which have comparatively a lower degree of stability. The sensitivity of long high-voltage transmission circuits to faults is considerably greater than in metropolitan systems, the high reactance of the high-voltage ties causing a much larger initial angle between the two generating systems, and hence the time required to clear a fault to maintain a given amount of load transfer over the remaining circuits is greatly reduced.

The application of relaying systems to transmission lines requiring high-speed fault isolation in order to prevent instability

¹ A part of general data in this chapter has been taken from the following sources:

SLEEPER, H. P., Relaying of High-Voltage Interconnection Transmission Lines, *Trans. A.I.E.E.*, vol. 52, pp. 808-815, 1933.

BRALEY, H. D., Assuring System Stability and Service Continuity by Relays, *Elec. World*, Feb. 13, 1932, pp. 310-314.

involves problems which are not ordinarily present in the relaying of the shorter, lower reactance ties within a system.

The requirements of an ideal high-speed relay system are as follows:

1. The relays shall operate instantaneously (2 cycles or less).
2. The relay scheme shall be inherently selective.
3. The relays at both ends of the line section shall operate simultaneously.
4. The relays shall not respond to surges between generating sources as long as the generators do not fall out of step, in which case the relays should operate.
5. Protection should be provided for phase and ground faults.

Not only must the relays isolate the faults quickly and selectively, but they must differentiate between the conditions of power flow caused by faults on the line section and the power flow caused by hunting between the generating sources. In the latter case, it is important that the interconnection line between sources of generation be kept intact so as to deliver emergency power to the part of the system which may have lost one of its generating-station ties.

There are few schemes of relay protection which meet all of these requirements. The distance type of phase-relay protection is in most common use in high-speed relay systems and will be considered first.

DISTANCE RELAYS

Distance relays consist of two general types, impedance relays and reactance relays. The impedance relay compares the magnitude of the voltage with that of the current, the ratio being the indicated impedance. The reactance relay compares the magnitude of that component of the voltage 90° out of phase with the current and the magnitude of the current, the ratio being the indicated reactance.

The Impedance Relay.¹—The high-speed impedance relay is of the *balance-point* type and has three impedance elements operating with three different impedance settings which correspond to faults occurring within three distance zones from the relay. The first zone covers 80 to 90 per cent of the first transmission-line section (Fig. 1) and is protected by an instantaneous

¹ Data of this section largely taken from published data of the Westinghouse Electric and Manufacturing Company.

balanced-beam impedance element, operating in conjunction with a high-speed directional element. The second zone includes the last 10 or 20 per cent of the first section and extends into the second section a distance of approximately 50 per cent of that line section. The third impedance element gives back-up protection and is usually set for a balance point about 25 per cent into the third line section. This system of time-zone classification is necessary because the relaying indications are not an absolutely accurate measure of the distance to the fault, so that the relay is unable to determine whether the fault is just within or just beyond the end of the section.

The instantaneous element consists of a balanced beam of Hipernick iron, which is pulled downward on the forward end by a current coil. The pull of the current coil is opposed by two voltage coils acting on the other end of the beam. The

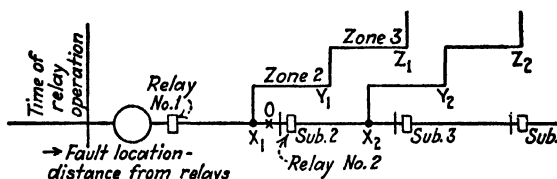


FIG. 1.—Distance-time characteristic of balance-point distance relays.

fluxes of these two potential coils are shifted 90° out of phase with respect to each other, in order to produce a steady pull, so that a practically constant balance is obtained, regardless of the phase angle between the potential flux and the current flux.

The contacts of the instantaneous element are in series with the contacts of the directional element, so that the breaker can be tripped only in case the power flow is in the proper direction (Fig. 2). In order to secure high speed, a new design of directional element has been developed. A small voltage transformer causes a large current to flow in a one-turn movable secondary, which current is substantially in phase with the voltage. The aluminum one-turn secondary loop is placed in a field produced by the current, and a torque is produced which rotates the loop in one direction or another, depending upon the direction of the power flow. This element has true wattmeter characteristics and is extremely fast.

The synchronous timer consists of a small synchronous motor which operates from the current circuit through a small trans-

former. The synchronous motor drives a moving contact through a set of gears, the moving contact completing the circuit by bridging the contacts on the adjustable contact blocks. The synchronous motor has a floating rotor and is arranged so that it is in gear only when energized. The rotor instantly falls out of gear when the motor is deenergized, allowing the spring

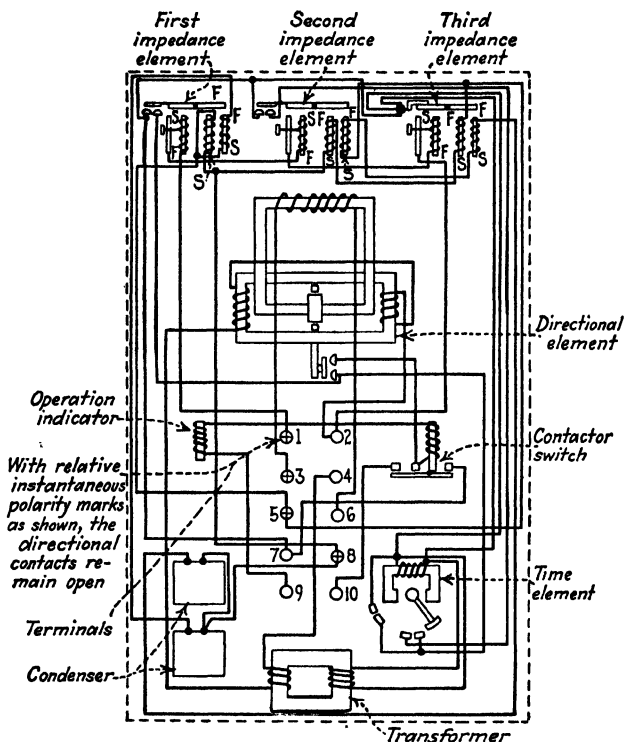


FIG. 2.—Internal diagram of high-speed impedance relay.

to stop the motion of the contact, giving quick reset. The timer includes two sets of adjustable contacts, the first in series with the second-impedance-element contacts and the second in series with the third-impedance-element contacts. The maximum time setting of the synchronous timer is 3 sec.

By means of a core screw and taps on the current coil of the first impedance element, the pull of this coil can be adjusted so

that it just balances the pull of the voltage when a fault occurs at point X_1 , Fig. 1. Under this balanced condition the contacts will not close. If a short circuit occurs to the left of X_1 , the current pull overcomes the voltage pull and closes the impedance-element contacts of the first impedance element. If the fault occurs at point O , the first impedance element cannot close contacts in that the ratio of E to I is too high. However, the second impedance element is adjusted to balance a short circuit at Y_1 and therefore operates for a fault at O . The third element is adjusted to balance at Z_1 . Settings for relay 2 are shown in Fig. 1 as X_2 , Y_2 , and Z_2 .

For line-to-line fault conditions on an individual single-phase line, the impedance measured by the relay is equal to the actual impedance of the circuit from the relay to the fault, plus the effect of any fault impedance which is present. On polyphase systems the distance to the fault may not necessarily be correctly measured by the impedance relay and may be affected in various ways by the type of fault, the system load, and other conditions external to the section being protected, as well as by fault impedance. This is covered later in the chapter under the heading Fundamental Basis for Distance Relaying of Line Faults, page 468.

A simplified form of high-speed impedance relaying is available, which consists of one impedance element and one directional element. The second and third impedance elements and the synchronous timer are eliminated. This relay can be added as supplementary protection to existing induction-impedance relaying or overcurrent directional protection. This single-element impedance relay will operate for faults occurring on 80 to 90 per cent of its own line section. The existing induction-impedance relays will operate for faults on the remaining 10 or 20 per cent of the line and operate selectively for faults on adjacent line sections.

Another type of relay is available which combines the standard induction-impedance relay element and the single high-speed impedance element. The high-speed impedance element operates for faults on 80 to 90 per cent of the line section and the induction-impedance element operates for the remaining 10 or 20 per cent of its own line section, and will act selectively for faults on adjacent line sections.

Reactance Relays.¹—The reactance relay compares the magnitude of that component of the voltage 90° out of phase with the current and the magnitude of the current, the ratio being the indicated reactance. The relay as shown in Fig. 3a consists of three units—a starting unit (lower), an ohm unit (middle), and a timing unit (upper).

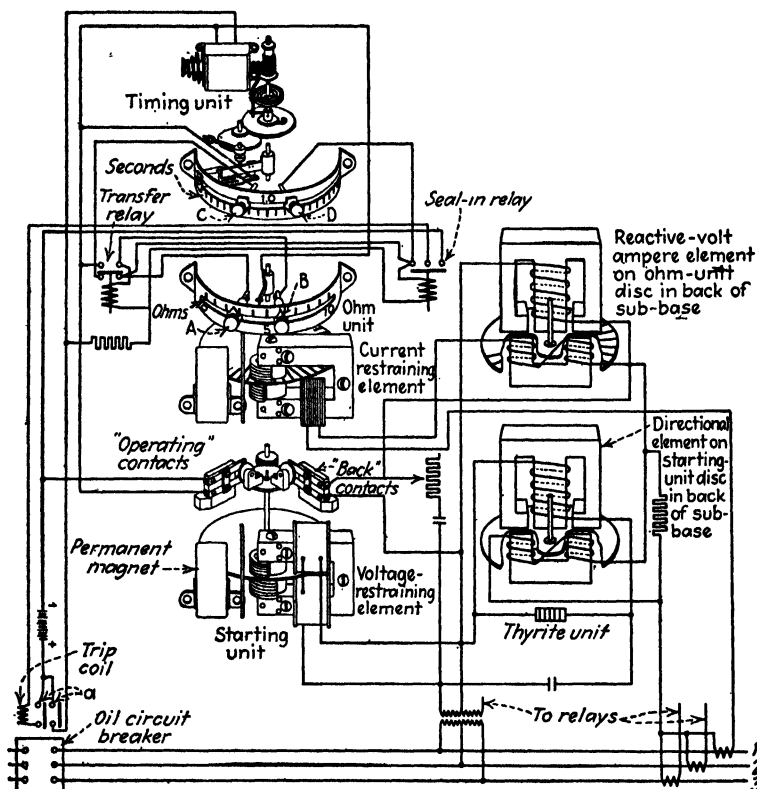


FIG. 3a.—Diagram showing relation of parts in reactance relay.

The starting unit is a directional impedance relay which operates only on faults which are on the outgoing side of the bus, and are within the total ohmic distance over which it is desired to give intermediate or long-time protection. It consists of an induction disk subjected to the torques of a directional element

¹ Data taken from published data of General Electric Company.

and a voltage element in opposition. The directional element tends to close the operating contacts only when power flows away from the bus. The directional element has maximum torque with current lagging 75° behind the voltage. The voltage element normally overcomes the torque of the directional element and prevents the relay from operating until the volts, amperes, and power factor indicate a fault within the zone over which the reactance relay is set to operate. When the starting unit operates, it first opens one set of contacts (which normally short-circuit the ohm-unit potential coil) and then closes another set

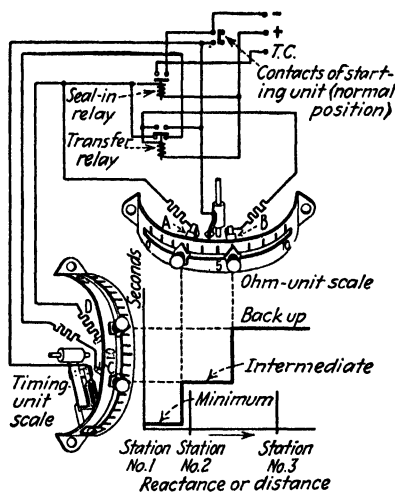


FIG. 3b.—Diagram of reactance-relay contact scheme.

of contacts which starts the timing unit and puts negative potential on the tripping contacts of the relay.

The ohm unit measures the secondary reactance from the relay to the fault and determines in which of three ohmic ranges the fault is located. It is similar to the starting unit except that the voltage-restraining element is replaced by a current element and the disk is slotted so that it tends to take up a position of equilibrium along its travel with its moving contact arm at a position on a graduated scale proportional to the reactance up to the point of fault. The travel of the contact arm is limited by two stationary contacts on the ohm-unit scale as shown in Fig. 3b. The left-hand contact is set to correspond to a point near the end of the protected section, and the right-hand contact is set to

about the middle of the next section. The contact arm of the ohm unit is normally held against the left contact by the torque of the current element (the potential coil of the ohm unit being short-circuited by the normally closed contact of the starting unit.) If, when a fault occurs, the reactance is greater than the setting of this left-hand contact of the ohm unit, the contact arm will immediately move away from it. As the arm moves up the scale, the disk slots under the *EI* element become deeper and weaken its torque until it balances that of the current magnet. If the ohm unit indicates reactance below the setting of the left-hand contact, the circuit breaker will be tripped immediately after the starting-unit operating contacts close. The minimum operating time of the relay is therefore the operating time of the starting unit plus that of the seal-in relay. This minimum time is two to three cycles. If the ohm unit takes up a position between the two stationary contacts, tripping will occur in a longer (intermediate) time. If it reaches the right-hand contact (indicating a fault beyond the middle of the next line section), the breaker will be tripped in a still longer (back-up) time, but only if the breaker in the adjoining faulty section has not cleared the trouble.

The timing or upper unit of the reactance relay measures out the intermediate and back-up times referred to above. It consists of a clockwork mechanism controlled by an escapement which is self-wound by means of a direct-current solenoid. The time arm sweeps the scale at a uniform rate and makes a passing contact at the first stationary (intermediate) contact mounted on the time scale, going on to the second (back-up) stationary contact only in emergency. When the solenoid is deenergized, its plunger is reset instantaneously by a spring, taking with it the moving contact.

On polyphase systems the distance to the fault may not necessarily be correctly measured by the reactance relay. This is covered in this chapter under the heading Fundamental Basis for Distance Relaying of Line Faults, page 468.

Composite Impedance-reactance High-speed Distance Relay.¹
A composite impedance-reactance relay combines the charac-

¹ Part of data taken from published data of the Westinghouse Electric and Manufacturing Company.

GOLDSBOROUGH, S. L., and W. A. LEWIS, A New High-Speed Distance Relay, paper, meeting of A.I.E.E., New York, Jan. 25-29, 1932.

teristics of the impedance and reactance types so that the operation will be essentially independent of fault resistance, without requiring the use of an auxiliary element to prevent operation under load conditions. The first element, which protects the instantaneous zone is an impedance-measuring element as already described. The intermediate zone is protected by a second element, mechanically a duplicate of the instantaneous element. However, on the same core on which the current coil is wound is an operating potential coil which modifies the characteristics from pure impedance, as shown schematically in Fig. 4. This potential coil is in series with a condenser which leads its current by 70° . The pull of this coil alone is equal to 70 per cent of the pull of the restraining potential coils. The bridging contacts of this element are in series with the intermediate time

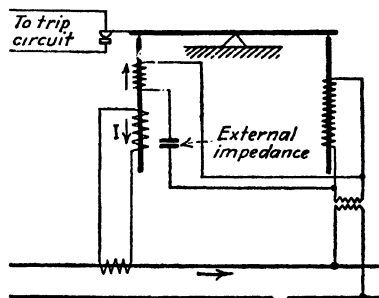


FIG. 4.—Schematic diagram of composite impedance-reactance relay.

contacts on the synchronous-motor-driven timer and also with the contacts on the directional element. The third, or back-up, element is similar to the third element of the pure-impedance relay.

This relay is intended for application on the shorter line sections where the arc resistance may reach a value comparable to the line impedance. The factor which limits the length of line to which the relay can be applied is the possibility of the element, with the composite characteristic, tripping on load currents. The longer the line the greater is such possibility, and for the particular characteristic discussed, this limit is approximately two secondary ohms. To obtain secondary ohms, the primary ohms are multiplied by the ratio-quotient of the current-transformer ratio, divided by the potential-transformer ratio. Hence, 20 ohms at 138 kv. with 400:5 current transformers 80:1 and 1200:1 potential transformers gives a secondary ohmic value of $20 \times \frac{80}{1200}$ or 1.33.

DISTANCE-RELAYING APPLICATION

Distance relaying has many inherent advantages to recommend it, such as its feature of automatic selective operation by fault

location, its simplicity, and its relative low cost as compared with other schemes such as pilot-wire protection. It operates instantaneously (two cycles or less) for a large proportion of faults on the line. It further provides back-up protection by use of timers which are set into operation by other distance-measuring units within the relay. However, it does not meet the ideal requirements of high-speed relaying since its principle of operation leaves a zone of 10 to 20 per cent of the line at the far end for a safety factor in distance differentiation. Faults in this zone are detected by the operation of time-delay devices. However, the distance relays will clear the majority of line troubles instantaneously from both ends of the section. In the minority of cases, where the trouble is near one end, that end will be cleared very fast. The reactance of the line to the point of fault, after the first breaker has opened, will, in some cases, materially increase the stability limit so that the sequential clearing of the second breaker may not cause instability.

There is one system connection wherein instantaneous protection can be provided for the full length of the line section. This is the case where a high-voltage line has transformers at each end and the line circuit breakers are connected to the low-voltage side of the transformers. The transformers can be considered as part of the line, and their lumped impedances permit the distance relay to be set so that all faults on the line and in parts of the high-voltage windings of the transformer will cause operation of the instantaneous elements of the relays. This gives instantaneous clearing of all line faults at both ends of the line section.

Instantaneous operation of high-speed directional relays requires that the directional elements operate at high speed. The voltage is usually of sufficient magnitude, unless a three-phase fault occurs too close to the bus. Where possible, consideration should be given to connecting the directional relay potential to the opposite side of the station power transformers. Any feed through these transformers will then give an impedance drop which will increase the voltage on the relay directional elements and speed up their operation.

Observations on several unstable interconnections have shown that distance relays will trip under violent conditions of system surging. The impedance relay, in general, seems to offer the

better chance of holding the systems together under such conditions, since only ratios of currents and voltages are being compared in the relays, but may give delayed operation for out-of-step operation. On the other hand, the reactance relay will be more sensitive to hunting but should operate readily for out-of-step operation.

Distance relays have in some cases been falsely accused in that distinction has not always been recognized between hunting conditions and out-of-step system operation. Relays should operate promptly for out-of-step operation. The ability to separate interconnected systems during instability at the most desirable geographical point may offer a difficult problem with any type of relaying. It is important to apply high-speed relaying to all feeders off the main lines so protected so as to prevent the condition of instability and minimize system oscillations. High-speed differential protection of buses may be justified in many instances. The application of high-speed bus protection is discussed in Chap. X.

The impedance relay is fundamentally the simplest and can be made to operate at a higher speed. However, the presence of arc resistance at the point of short circuit increases the impedance as viewed from the relay and makes the fault appear more distant than it really is. The reactance relay, on the other hand, is affected to a less extent by the presence of arc resistance but in general requires a more complicated structure. Furthermore, additional features are required to prevent the operation of this type of relay under normal load conditions.

Tests have shown that the arc resistance is generally quite small at the inception of the fault, but that it may increase to rather large values if the arc is allowed to persist. This indicates that in the first zone, in which the relay is designed to trip instantaneously, the error in distance measurement, due to the effect of arc resistance, will not seriously affect the operation of the relay. However, in the second zone of the relay, where a definite time is purposely introduced into the operation of the relay, the arc resistance may increase to a sufficient value to affect the operation of the relay. On lines of average length the fault resistance generally remains within such limits that the error with either an impedance or a reactance relay is small enough to give proper selectivity. However, on the shorter

lines, the arc resistance, after several cycles, may become as large as or perhaps even larger than the impedance of the line section, and for such a case the error in distance measurement using an impedance relay becomes considerably larger than the corresponding error using a reactance relay. Hence, for the shorter lines, the use of a relay possessing essentially a reactance characteristic is necessary.

FUNDAMENTAL BASIS FOR DISTANCE RELAYING OF LINE FAULTS¹

In order that high-speed relays employing the distance principle be on a sound basis, it is essential that, for a fault at any particular location, the distance measurement must be reasonably constant irrespective of the type of fault and number of phases involved. This is a difficult requirement to attain in that the impedance is determined by voltages and currents at the relaying

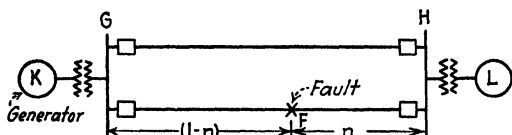


Fig. 5.—Assumed system for analyzing relay-impedance indications.

point, and these quantities are affected by fault impedance, load conditions, phase angle between generating sources, and mutual coupling between lines.

An analysis will be developed for the impedance measured at the relaying point for line-to-line, double-line-to-ground, and three-phase faults. The general method employed has been completely outlined in preceding chapters. The analysis will be applied to the two-circuit transmission lines with a generating source at each end, as shown in Fig. 5. It is assumed that the two circuits are closely spaced and the zero-sequence mutual coupling is considered, but shunt loads, capacitance, and mutual coupling are neglected in the positive- and negative-sequence networks and capacitance is neglected in the zero-sequence network. It is not permissible to consider the internal voltages

¹ LEWIS, W. A., and L. S. TIPPETT, Fundamental Basis for Distance Relaying on Three-Phase Systems, an A.I.E.E. paper, Abstract, *Elec. Eng.*, June, 1931, p. 420. Acknowledgment is given to the above mentioned authors for data appearing in their paper, used in this section.

of the two generating sources to be in phase since the phase relations have an important influence upon the relay operation. The problem is to determine the impedance or ratio of current and voltage, measured at the relaying point for faults involving more than one phase at some particular point. The proper combination of current and voltage that will give the most nearly constant indication of impedance, irrespective of type of fault, should determine the proper connections for distance relays.

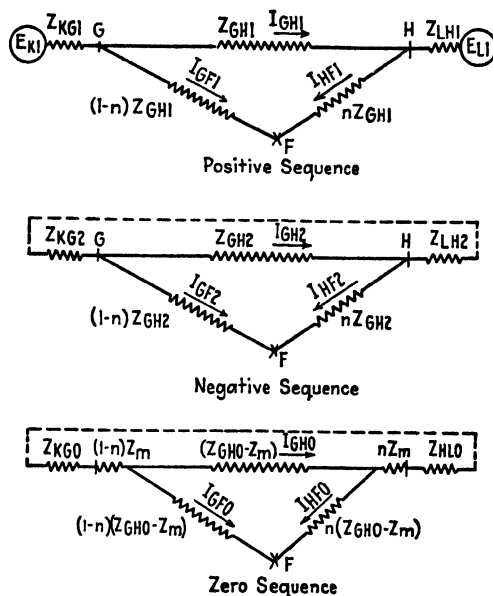


FIG. 6.—Sequence networks corresponding to Fig. 5.

In Fig. 6 are shown the positive-, negative-, and zero-sequence networks corresponding to the system of Fig. 5. The two transmission lines are assumed to have identical electrical characteristics. The fault location is at F . The subscripts on the impedance symbol Z indicate the section of the network included, thus Z_{GH1} is the positive-sequence impedance of one circuit between Stations G and H . The voltages and currents at Station G will be calculated in terms of the constants of the system for faults at point F .

Line-to-line Fault.—Assume a line-to-line fault between phases b and c and a fault resistance R_f between these conductors.

Tests have shown that for practical purpose the impedance of an arc is pure resistance. For this analysis it is convenient to consider a resistance $R_F/2$ connected to each line conductor at F and the terminals of the resistors on b and c closed at W , as shown in Fig. 7. This makes the system symmetrical to the point W . A little thought will show that this will give the same results as the analysis for line-to-line fault with impedance given in Chap. III.

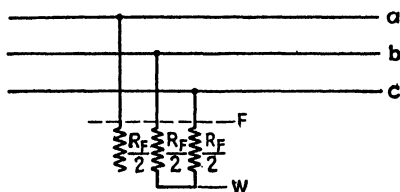


FIG. 7.—Line-to-line fault representation.

The sequence voltages at G can be determined by adding the sequence-voltage drops between W and G to the sequence voltages existing at W :

$$E_{G1} = I_{GF1} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{HF1} \frac{R_F}{2} + E_{W1} \quad (1)$$

$$E_{G2} = I_{GF2} \left[(1 - n)Z_{GH2} + \frac{R_F}{2} \right] + I_{HF2} \frac{R_F}{2} + E_{W2} \quad (2)$$

$$E_{G0} = 0 \quad (3)$$

The same phase rotation and relations between phase and line voltages and currents will be used as given in Chap. II. Resolving the sequence voltages at G into terms of phase voltages by the use of the equations developed in Chap. II and remembering that $Z_{GH1} = Z_{GH2}$, $E_{W1} = E_{W2}$, and $I_{FW1} = -I_{FW2}$,

$$E_{aG} = I_{aGF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{aHF} \frac{R_F}{2} + 2E_{W1} \quad (4)$$

$$E_{bG} = I_{bGF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{bHF} \frac{R_F}{2} - E_{W1} \quad (5)$$

$$E_{cG} = I_{cGF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{cHF} \frac{R_F}{2} - E_{W1} \quad (6)$$

A mathematical investigation has shown that special forms of relay and current-transformer connections are necessary to

secure the most constant impedance indication at the relaying point. One such circuit is that shown in Fig. 8 in which the current transformers are connected delta and the relays, star. All instrument transformers are assumed to be 1:1 ratio. The

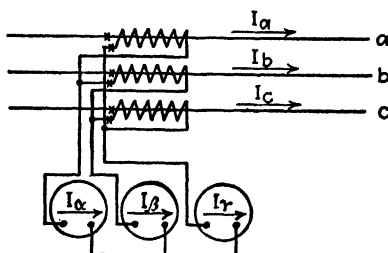


FIG. 8.—Delta-connected current transformers. Y-connected relays.

currents I_α , I_β , I_γ in the relays have the following values in terms of the phase currents:

$$I_\alpha = I_c - I_b = j\sqrt{3}(I_1 - I_2) \quad (7)$$

$$I_\beta = I_a - I_c = j\sqrt{3}(a^2 I_1 - a I_2) \quad (8)$$

$$I_\gamma = I_b - I_a = j\sqrt{3}(a I_1 - a^2 I_2) \quad (9)$$

Making use of these relations the line voltages at G are:

$$E_{AG} = E_{cG} - E_{bG} = I_{\alpha GF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{\alpha HF} \frac{R_F}{2} \quad (10)$$

$$E_{BG} = E_{aG} - E_{cG} = I_{\beta GF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{\beta HF} \frac{R_F}{2} + 3E_{W1} \quad (11)$$

$$E_{CG} = E_{bG} - E_{aG} = I_{\gamma GF} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{\gamma HF} \frac{R_F}{2} - 3E_{W1} \quad (12)$$

It was formerly the practice to use line voltage and current for operating the relay in which the phase relations were such that the current in each relay lagged the voltage by 30° at unity power factor. In order to analyze this connection, it is convenient to change the form of the above equations. Since $I_0 = 0$, we may write

$$(I_{\alpha GF} + I_{\beta GF} + I_{\gamma GF}) \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + (I_{\alpha HF} + I_{\beta HF} + I_{\gamma HF}) \left(\frac{R_F}{2} \right) = 0$$

Subtracting this expression from the above equations gives

$$E_{AG} = -2I_{bGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - 2I_{bHF} \frac{R_F}{2} - I_{aGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - I_{aHF} \frac{R_F}{2} \quad (13)$$

$$E_{BG} = -2I_{cGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - 2I_{cHF} \frac{R_F}{2} - I_{bGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - I_{aHF} \frac{R_F}{2} + 3E_{W1} \quad (14)$$

$$E_{CG} = 2I_{aGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - 2I_{aHF} \frac{R_F}{2} - I_{cGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - I_{cHF} \frac{R_F}{2} - 3E_{W1} \quad (15)$$

Since $I_{aGF} + I_{aHF} = I_{aFW} = 0$, Eq. (13) simplifies to

$$E_{AG} = -2I_{bGF} \left[(1-n)Z_{GH1} + \frac{R_F}{2} \right] - 2I_{bHF} \frac{R_F}{2} - I_{aGF}(1-n)Z_{GH1} \quad (16)$$

Double-line-to-ground Fault.—The conditions for a double-line-to-ground fault are shown in Fig. 9 which is similar to the

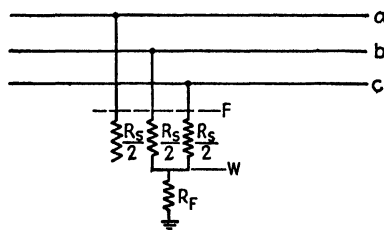


FIG. 9.—Double-line-to-ground fault representation.

line-to-line case except that a resistance R_F is connected at the midpoint between phase b and c and ground. The system is symmetrical up to W . The sequence voltages at G are the summation of the voltage drops from the fault to the point G .

$$E_{G1} = I_{GF1} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + I_{HF1} \frac{R_s}{2} + E_{W1} \quad (17)$$

$$E_{G2} = I_{GF2} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + I_{HF2} \frac{R_s}{2} + E_{W1} \quad (18)$$

$$E_{G0} = I_{GF0} \left[(1-n)Z_{GH0} + \frac{R_s}{2} + 3R_F \right] + I_{HF0} \left(\frac{R_s}{2} + 3R_F \right) + I_{GH0}(1-n)Z_m + E_{W1} \quad (19)$$

The line-to-neutral and line-to-line voltages can be found from these equations:

$$E_{bG} = I_{bGF} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + I_{bHF} \frac{R_s}{2} + I_{GF0}[(1-n)Z_{GH0} - (1-n)Z_{GH1} + 3R_F] + 3I_{HF0}R_F + I_{GH0}(1-n)Z_m \quad (20)$$

$$E_{cG} = I_{cGF} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + I_{cHF} \frac{R_s}{2} + I_{GF0}[(1-n)Z_{GH0} - (1-n)Z_{GH1} + 3R_F] + 3I_{HF0}R_F + I_{GH0}(1-n)Z_m \quad (21)$$

$$E_{AG} = E_{cG} - E_{bG} = I_{aGF} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + I_{aHF} \frac{R_s}{2} \quad (22)$$

Equation (22) can be put in a form similar to Eq. (13), since

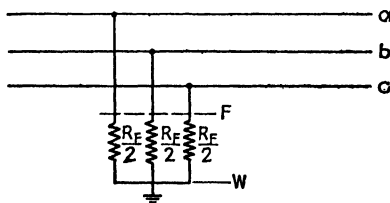


FIG. 10.—Three-phase fault representation.

$I_{aGF} + I_{bGF} + I_{cGF} = 3I_{GF0}$ and $I_{aHF} + I_{bHF} + I_{cHF} = 3I_{HF0}$ and the following arbitrary expression can be written:

$$I_{aGF} + I_{bGF} + I_{cGF} - 3I_{GF0} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + [I_{aHF} + I_{bHF} + I_{cHF} - 3I_{HF0}] \frac{R_s}{2} = 0$$

Subtracting this expression from Eq. (22) and simplifying by use of the relation $I_{aGF} = -I_{aHF}$ gives

$$E_{AG} = -2I_{bGF} \left[(1-n)Z_{GH1} - \frac{R_s}{2} \right] - 2I_{bHF} \frac{R_s}{2} + 3I_{GF0} \left[(1-n)Z_{GH1} + \frac{R_s}{2} \right] + 3I_{HF0} \frac{R_s}{2} - I_{aGF}(1-n)Z_{GH1} \quad (23)$$

Three-phase Faults.—The three-phase fault condition is shown in Fig. 10 in which a fault resistance $R_F/2$ is connected between each phase and ground. For this case $E_{G2} = E_{G0} = 0$.

$$E_{bg} = I_{bgf} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{bhf} \frac{R_F}{2} \quad (24)$$

$$E_{og} = I_{ogf} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{chf} \frac{R_F}{2} \quad (25)$$

$$E_{Ag} = I_{agf} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] + I_{ahf} \frac{R_F}{2} \quad (26)$$

We may write $(I_{agf} + I_{bgf} + I_{ogf}) \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] = 0$ and $(I_{ahf} + I_{bhf} + I_{chf}) \left(\frac{R_F}{2} \right) = 0$. Subtracting these identities from the above gives

$$E_{Ag} = -2I_{bgf} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] - 2I_{bhf} \frac{R_F}{2} - I_{agf} \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] - I_{ahf} \frac{R_F}{2} \quad (27)$$

Measured Impedance with 30° Connection.—In comparing the impedance measured with the 30° connection for the three different types of faults, consider first the conditions when the fault impedance is zero. The impedance measured by the relay is simply $Z_R = E_{Ag} / -I_{bgf}$. The impedance relay measures the magnitude and the reactance relay the reactive component of the resulting expression. Dividing Eqs. (13), (23), and (27) by $-I_{bgf}$ and setting the fault impedance at zero gives

$$Z_R = 2(1 - n)Z_{GH1} + \left(\frac{I_{agf}}{I_{bgf}} \right) (1 - n)Z_{GH1} \text{ (line to line)} \quad (28)$$

$$Z_R = 2(1 - n)Z_{GH1} + \left(\frac{I_{agf}}{I_{bgf}} \right) (1 - n)Z_{GH1} - \left(\frac{3I_{gfo}}{I_{bgf}} \right) (1 - n)Z_{GH1} \text{ (double line to ground)} \quad (29)$$

$$Z_R = 2(1 - n)Z_{GH1} + \left(\frac{I_{agf}}{I_{bgf}} \right) (1 - n)Z_{GH1} \text{ (three phase)} \quad (30)$$

The first term of each of these equations is similar and proportional to the distance from the relay to the fault, but the second terms involve not only line constants but currents in other conductors. Equation (29) also includes an additional term containing zero-sequence current. The current I_{agf} in the

unfaulted conductor for the line-to-line and double-line-to-ground faults is not controlled by fault conditions and may vary over a wide range. This current is a function of load conditions, flow of synchronizing power, and system-operating set-up. The third term of Eq. (29) also causes variation of impedance. These variable factors produce errors in both impedance and reactance relays.

After the breaker on the faulted circuit at H opens, the value of the impedance changes. With this condition $I_{bGF} = I_{aGF} = 0$ for line-to-line fault, $I_{AGF} = I_{bHF} = I_{HF0} = 0$ for double-line-to-ground fault, and $I_{aHF} = I_{bHF} = 0$ for three-phase fault. Dividing Eqs. (13), (23), and (27) by $-I_{bGF}$ and substituting the above relations gives

$$Z_R = 2 \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] \text{ (line to line)} \quad (31)$$

$$Z_R = 2 \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] - \left(\frac{3I_{GF0}}{I_{bGF}} \right) \left[(1 - n) \left(Z_{GH1} + \frac{R_F}{2} \right) \right] \text{ (double line to ground)} \quad (32)$$

$$Z_R = 1.73 \left[(1 - n)Z_{GH1} + \frac{R_F}{2} \right] e^{j30^\circ} \text{ (three phase)} \quad (33)$$

If the fault resistance is zero, this reduces to

$$Z_R = 2(1 - n)Z_{GH1} \text{ (line to line)} \quad (34)$$

$$Z_R = 2(1 - n)Z_{GH1} - \left(\frac{3I_{GF0}}{I_{bGF}} \right) (1 - n)Z_{GH1} \text{ (double line to ground)} \quad (35)$$

$$Z_R = 1.73[(1 - n)Z_{GH1}]e^{j30^\circ} \text{ (three phase)} \quad (36)$$

All variable terms are eliminated from Eqs. (34) and (36), but the measured impedance is approximately 15 per cent less for a three-phase than for a line-to-line fault. The magnitude of the impedance is not affected by the 30° rotation of Eq. (36), but the reactive component, as measured by a reactance relay, is affected by phase rotation, and the measured reactance depends upon the phase angle of Z_{GH1} . The reduction may be greater or less than 15 per cent depending upon whether the phase angle is greater or less than 75° .

The effect of fault resistance is to cause a variation of the measured impedance for the different types of faults. There is

always one term involving pure resistance, but other terms of the form $\frac{I}{I_{bgf}} \left(\frac{R_f}{2} \right)$ that represent pure resistance only when the currents are in phase. If the currents are not in phase, the term will contain resistance and apparent reactance component. The reactance component is proportional to the sine of the angle between the currents and may be positive or negative depending upon the phase relations of the two currents. The fault-impedance term causes the balance point for an impedance relay to move closer to the relay, but for the reactance relay the apparent reactance component of the fault-resistance term may cause the balance point to move in either direction.

Measured Impedance with Delta Connection.—An examination of Eqs. (10), (22), and (26) will show that if the current I_{agf} is used for relaying instead of $-I_{bgf}$, the measured impedance is identical for line-to-line, double-line-to-ground, and three-phase faults. If the breaker at H is open, the measured impedance for all three types of faults reduces to

$$Z_R = (1 - n)Z_{GH1} + \frac{R_f}{2} \quad (37)$$

For this condition a reactance relay will have a constant indication, and an impedance relay will receive an indication which will vary with the fault resistance.

The measured impedance with all breakers closed, using Eqs. (10), (22), and (26) for all three types of faults, reduces to

$$Z_R = (1 - n)Z_{GH1} + \frac{R_f}{2} + \left(\frac{I_{ahf}}{I_{agf}} \right) \frac{R_f}{2} \quad (38)$$

For this case the reactance relay is also affected by the fault resistance if the two currents are not in phase, but all other errors are eliminated. The delta connection is therefore the proper one for use with distance relays.

The delta relaying currents may be obtained as shown in Fig. 8 or by connecting the current transformers in Y and the relays in delta, and using a filter to remove zero-sequence currents from the relays. The indicated impedance is three times as great with this connection as the one in Fig. 8, since the relay currents are one-third those with the delta-connected current-transformer scheme.

HIGH-SPEED DISTANCE RELAYING FOR LINE-TO-GROUND FAULTS

It is generally recognized that the high value of resistance which may be present in ground faults is sufficient to eliminate the ordinary line type of impedance relay or reactance relay from being used for line-to-ground fault protection. Even though the resistance component is effectively eliminated, there are still other factors which cause error in the relay measurement of distance. Assuming line-to-ground potential and line current for energizing the relay on a line-to-ground fault, it is found that the impedance measured by the relay is affected by the fault resistance, the residual current in the line being protected, and by the residual current in the parallel line. Where two lines parallel each other at close spacings, the zero-sequence mutual impedance between the lines becomes quite appreciable in comparison with the self-impedance, particularly when a fault occurs near the remote end of the section. If the parallel line is not in service, a considerable variation in the indicated impedance is possible. When the fault current divides and comes from two or more points to the fault, the proportion of the current in the conductor which is residual or zero-sequence depends upon the relative impedance to phase and ground currents in that branch, and, due to the fact that the impedance of the transmission line is much higher to ground current than to phase current, a relatively greater portion of the residual current will usually come from the end of the line nearer the fault. These variations in indicated vector impedance affect the indicated reactance in much the same proportions. From the standpoint of these errors alone, the reactance relay presents no advantages, but the effect of the resistance terms in the distance-relation equations is reduced. The use of residual current instead of line current involves similar errors in distance measurement since the voltage to neutral of the faulted phase is not directly proportional to the residual current, even when fault resistance is neglected.

The use of distance relays for ground protection is a very recent development, and they have been installed to a limited extent in practice. Special current or voltage compensation is used, and an additional set of three relays to those used for phase protection is required. Even though there are not, to date, any

comprehensive operating data in connection with specially compensated distance relays for line-to-ground faults, calculations indicate that it is possible that suitable ground protection can be obtained by this means provided the impedance of the section being protected is sufficiently great.

Sufficient data are not yet available to state definitely the characteristics of fault impedance, but undoubtedly further information will be obtained from future investigations. Until more accurate data are available, certain margins determined by experiment must be allowed in relay settings to allow for deviations from the assumptions.

The relations between arc current, voltage, and length of arcs on high-voltage circuits have been found to be given approximately by the expression¹

$$V = \frac{8000L}{2.5\sqrt{I}}$$

where V is the voltage across the arc and L the length of arc in feet. This is a rough approximation derived from field tests in which the arc was initiated by small copper wire. The arc was undoubtedly influenced somewhat by copper vapors, which would tend to give a lower voltage. Above 800 amp., the arc voltage seems to become practically independent of the current. The arc tends to stretch, due to the effect of air currents produced by the heat of the arc and by wind. The arc has negligible mass, hence will stretch at essentially the same velocity as that of the wind. If the current in the arc decreases to some critical value, the arc may extinguish without circuit interruption. Exact data on arc characteristics in air will not be obtained until tests are made in which the arc is initiated by a lightning generator in order to eliminate the metallic vapor effect.

Considerable study and investigation are yet required in order to perfect reactance ground relaying. Fortunately, line-to-ground faults do not have to be cleared so quickly as phase faults in order to maintain stability, and therefore other forms of ground relaying such as current balance, directional overcurrent, and inverse-time overcurrent, can in many cases be used to effect the clearing of ground faults in sufficient time to prevent instability. A complete analysis of the present development in

¹ From published data of the General Electric Company.

reactance-ground-relaying principles is therefore not included in this chapter.

HIGH-SPEED BALANCED RELAYING OF PARALLEL LINES¹

For parallel circuits a current balance relay can be used to advantage in quickly isolating the line on which a fault develops, leaving the second line intact. It is a simple form of relaying and can be applied at low cost. It should not be overlooked, however, that single-line relaying must be provided also, if high-speed relaying is desired under all conditions of operation. For

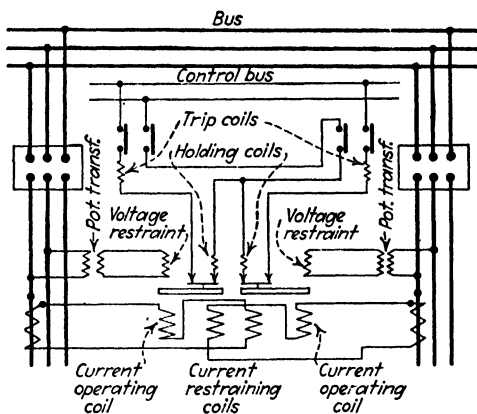


FIG. 11.—Voltage-restraint high-speed current balance relay connections.

this reason, distance relaying on each of the lines may prove to be the better scheme for a particular application regardless of the greater initial cost. The choice depends upon many factors and requires careful analysis of system-stability limitations. The current balance relay has many important engineering advantages to recommend it for high-speed protection.

Voltage-restraint Type of Current Balance Relay.—In Fig. 11 is shown the voltage-restraint type of current balance relay. This relay comprises two balanced-beam units each consisting of an armature, two contacts, and four coils. The coils consist of one operating coil, one current-restraining coil, one voltage-restraining coil, and one direct-current holding coil. The

¹ General description of relays and their characteristics are obtained from published data of the Westinghouse Electric and Manufacturing Company and the General Electric Company.

operating and current-restraining coils are located below the armature, while the voltage-restraining and holding coils are above the armature.

Under normal operation, the pull on the armature, produced by the current-restraining coil and the voltage-restraining coil, tends to hold the contacts open. The pull from the operating coil operates against the total pull of the restraining coils and

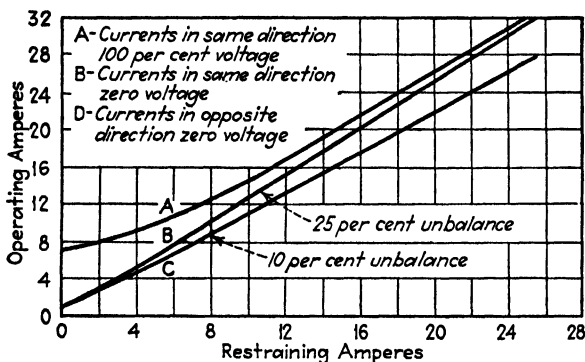


FIG. 12.—Operating characteristics of current balance relay shown in Fig. 11.

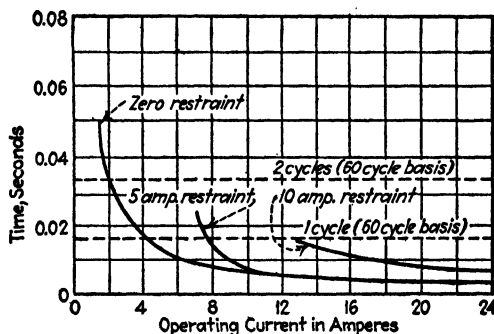


FIG. 13.—Time-current characteristics of current balance relay shown in Fig. 11.

tends to close the contacts. In case of short circuit on one of the protected lines, the current in the operating winding will be in excess of that in the current-restraining winding. When this unbalance exceeds 25 per cent, and the voltage is low, the pull of the operating coil will be greater than that of the restraining coil and close the contacts. If, however, full voltage is maintained, current through the operating winding in excess of 125 per cent of the restraining current will be required. The operating and

restraining characteristics of the relay, with and without voltage restraint, are shown in Fig. 12. For practically all short-circuit conditions a time of less than one cycle will be obtained. Time-current characteristics for various values of current restraint are shown in Fig. 13.

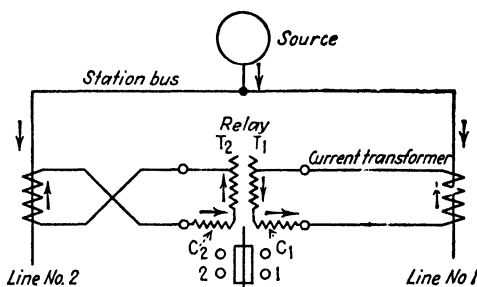


FIG. 14.—Schematic connections of induction-loop type of current balance high-speed relay.

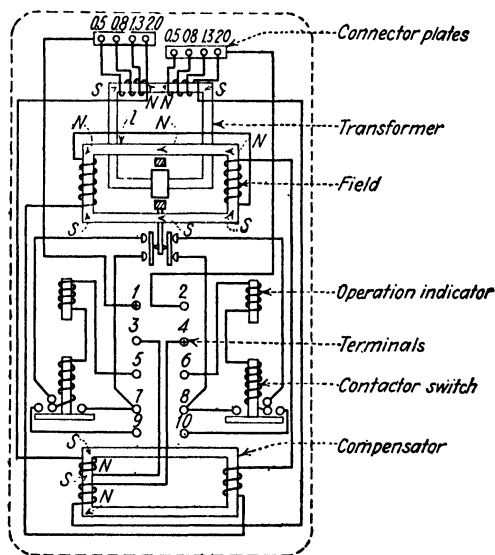


FIG. 15.—Internal diagram of current balance relay shown in Fig. 14.

Induction-loop Type of Current Balance Relay.—A schematic connection of the induction-loop type of current balance relay is shown in Fig. 14 and a more complete internal scheme of connections in Fig. 15. The windings T_1 and T_2 are on the transformer which excite the aluminum loop. This rectangular loop

of aluminum is pivoted on the two ends, and forms a short-circuited secondary of a small transformer. A current is induced in the loop which is proportional to the difference of the two line currents. The loop is located in a magnetic field produced by a current winding which is fed from a small compensator excited by windings C_1 and C_2 and is rotated one way or the other,

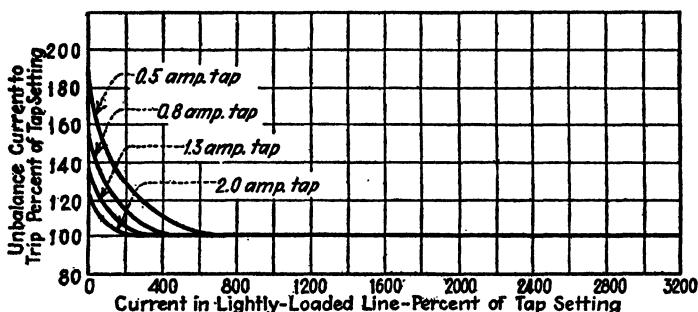


FIG. 16.—Tripping currents of relay shown in Figs. 14 and 15 with currents in current-transformer windings in opposite directions.

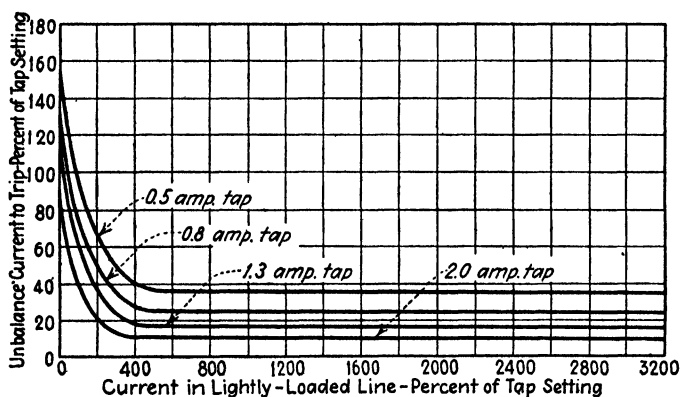


FIG. 17.—Tripping currents of relay shown in Figs. 14 and 15 with currents in current-transformer windings in opposite directions.

depending upon the relative direction of the current in the loop and the flux in the field which cuts the loop. This relay operates in one cycle or less. Operating characteristics are shown in Figs. 16 and 17. Current balance relays for phase protection cannot be made as sensitive as the ground relay, the reason being that the relay must carry the load current, and the load current on one line with the other one out of service will trip the

remaining line. This has been overcome by making the pick-up current high enough so that the load current, even if all on one line, will not be sufficient to trip the relay. The pick-up value automatically increases two to three times when one line goes out of service.

Current Balance Ground Protection.—The high-speed current balance relay can be applied for the protection of line-to-ground faults. Owing to the difficulties involved in applying distance relays for ground protection, the current-balance scheme offers a very satisfactory solution for the protection of parallel lines.

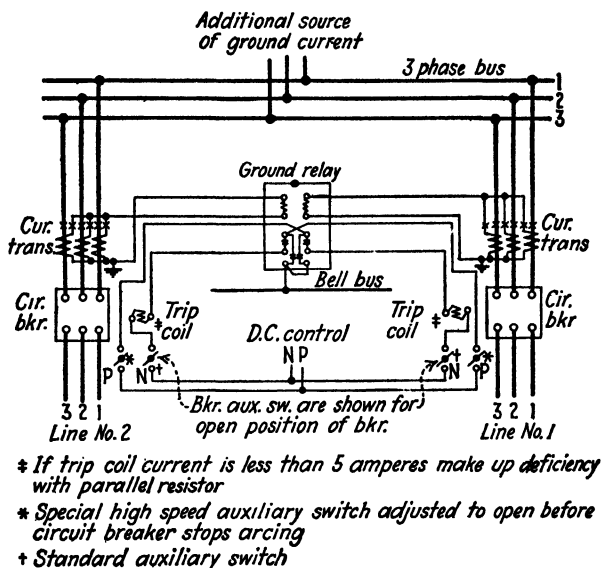


FIG. 18.—Diagram of connections for high-speed current balance ground protection.

The only limit to the maximum sensitivity of the relay is the accuracy of the current transformers. If the current transformers balance with very small ratio and phase-angle errors, the residual line current is very small, and a very sensitive setting of the ground-current balance relay may be utilized. (See Fig. 18 for a diagram of connections.)

Comparison of Current Balance and Distance Relaying.—The high-speed current balance relay gives instantaneous sequential operation of the relays on the two ends of a line section protected by balanced relays, as compared with the instantaneous and first-

time-zone operation of distance relays for all faults occurring in the end zones of such a paired line. By end zone is meant the 10 or 20 per cent of the line beyond the balance point of the instantaneous element of the distance relay. For the sake of simplicity, a 15 per cent end zone will be assumed. The total maximum time of clearing an end-zone fault with distance relays is the instantaneous relay operation plus the first time element of the relay at the other end of the section plus the operating time of two circuit breakers. In many cases the maximum time will be that of the first time element plus its breaker time, assuming the relays at both ends start operating immediately at the inception of the fault. With the balanced relay system the total time for clearing a short-circuit close to one bus is that of one instantaneous relay plus one circuit breaker on one end, plus one instantaneous relay and its circuit breaker on the other end. After the first end clears, all of the short-circuit current is available to unbalance the instantaneous current balance relay at the other end.

Faults in the 70 per cent (two 15 per cent end zones subtracted from 100 per cent line length) instantaneous operating zone of the line will of course be cleared by the instantaneous simultaneous operation of distance relays or the current balance relays at both ends. It is obvious that the balanced system of relaying has certain advantages, but there are also certain disadvantages as follows:

1. Cross-connection between current transformers.
2. Necessity for interlocking the relays with the circuit breakers to prevent the tripping of the good line while the faulted line is clearing the second end.
3. Necessity of using special high-speed auxiliary switches adjusted to open before circuit breaker stops arcing.
4. Hazard of tripping a loaded line while switching in the other line of the pair.
5. Necessity for providing single-line protection.

It is, of course, possible to combine the two schemes by using balanced relays for parallel-line operation and distance relays for single-line operation.

Simple Overcurrent High-speed Relaying.—Simple overcurrent relays can in some cases be used to operate instantaneously for certain locations of faults, but their zone of safe

selective operation is necessarily limited because of the variations in the magnitudes of ground-fault currents under various system-operating conditions. In some instances the relays can be set to operate instantaneously to disconnect the line-to-ground faults over a very maximum of 75 per cent of its length from each end, giving simultaneous and instantaneous operation at both ends for faults occurring over 50 per cent of the line. However, a large percentage of the faults outside the 50 per cent zone will cause sequential operation of the instantaneous ground relays on the two ends of the line, due to the resulting increase in current after the first breaker has opened. The fast operation, for faults close to the ends of a line section, of the breaker near the fault usually permits stability to be maintained on the near system, and the reduced power from the far system frequently permits that end to be cleared quickly enough to prevent a serious disturbance. It should be kept in mind that a longer time is allowable for clearing ground faults before the stability limit is reached than is the case for phase faults.

This type of relay consists merely of a solenoid coil and plunger to which contacts are fastened. The speed of operation is one to two cycles above approximately 125 per cent pick-up. The operating value is adjustable over a range of 3 to 1, and many different coil ratings are available. The drop-out values range from 90 to 98 per cent of the pick-up value.

High-speed Directional Overcurrent Relays.—The application of the conventional-type slower speed overcurrent directional relays is covered in detail in Chap. XI. The principles of application of the high-speed type are similar, the main difference being that the relays operate at higher speed. However, the construction of the relays differs materially.

The type of overcurrent directional relay giving the lowest operating time is of the solenoid current element and aluminum-vane directional type. The current element is similar to the instantaneous overcurrent relay and the directional element similar to that used in the impedance relay. The directional element can be polarized by use of a potential coil connected to the secondary of potential transformers, or it can be supplied with a coil which can be polarized from a current transformer in the neutral of the transformer bank. This is illustrated in Figs. 19 and 20. This relay can be applied to the cross-connected

system of relaying as discussed in Chap. XI. It is particularly applicable for line-to-ground fault protection of parallel lines at a substation which does not have a separate source of power connected to the bus. For parallel-line protection the overcurrent winding is connected in the differential circuit of cross-connected current transformers and receives but little current normally or in case of through faults. During faults within the relay's own line section, a large differential current flows through

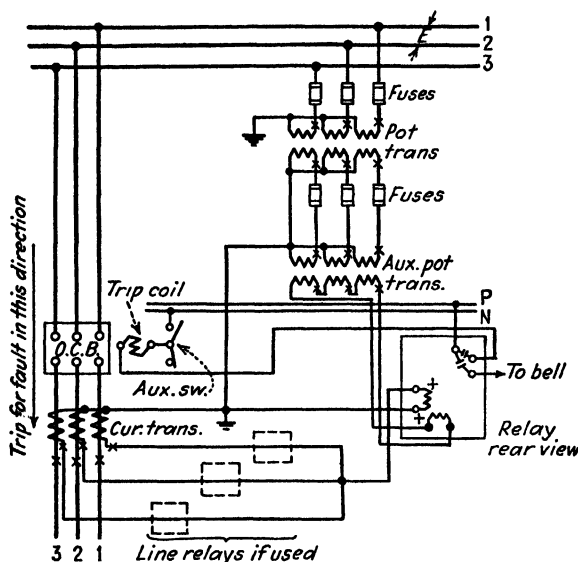


FIG. 19.—Overcurrent directional high-speed ground relay polarized by potential transformers.

the overcurrent element, and the faulty line is selected by means of the directional element. The relays are unidirectional, three relays being required per line for phase protection and one relay per line for ground protection. For protection during single-line operation, additional relays with appropriate time characteristics are required. For further details concerning parallel-line relay systems refer to Chap. XI. In many instances it is possible to apply one ground relay to each of the parallel lines, eliminating cross-connections. This then gives parallel- and single-line operation with the same relays.

A second type of high-speed directional relay has three complete power elements, taking current and voltage from all three

phases of the protected circuit, which exercise torques on the rear sectors of three aluminum disks. In addition, the front portion of the lower disk rotates in the field created by two supplementary potential circuits connected in open delta, which restrain the action of the relay in proportion to the area of the voltage triangle existing at the relay location. This restraint prevents operation of the relay during power reversals incidental to ordinary operation which do not result from the presence of a fault. As in other relays of this type the direction of torque is controlled

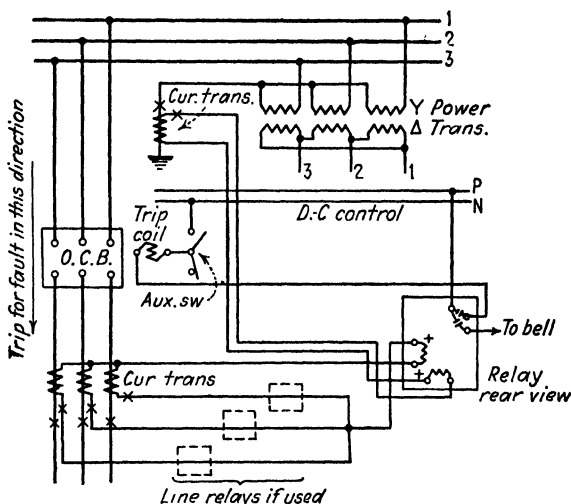


FIG. 20.—Overcurrent directional high-speed ground relay polarized by neutral current transformer.

by the direction of power flow at the point of application. However, this relay is much faster in its operation than the conventional induction-type directional relay. The resonant character of its potential circuit gives it a faster operating time at low voltages than is the case with conventional normal-speed relays. The potential coil of the directional element is in series with a capacitor which tunes it at the system frequency. A special resistor is connected in parallel with the coil. At low voltage the special resistor has the inherent characteristic of a high-resistance value, and hence practically no current passes through it. The coil and capacitor are, therefore, in series resonance, and the voltage across the coil is several times the line voltage so that the torque of the element is maintained even at a low

voltage. Special lagging of the potential flux and the use of the quadrature connections further aid in obtaining high speed. Time curves are shown in Fig. 21. High-speed overcurrent relays are required in the equipment for power-directional protection when using this high-speed polyphase directional relay, generally for the purpose of preventing the tripping of a line until certain predetermined current values are reached. In the

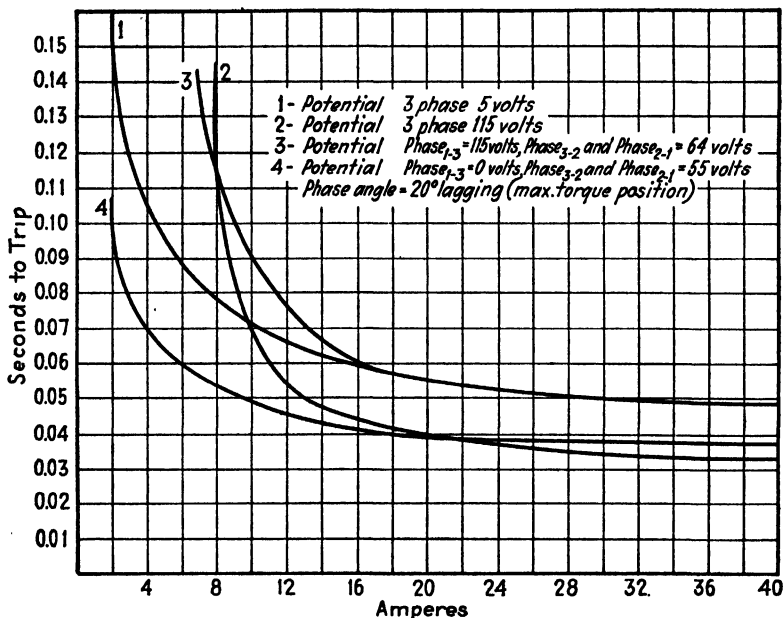


FIG. 21.—Time-current characteristics of special high-speed induction-type power-directional relay.

case of balanced lines, they insure that neither line will be tripped unless the vectorial difference of current in the two lines is in excess of their settings.

APPLICATION OF HIGH-SPEED RELAYS TO A COMPLETE TRANSMISSION SYSTEM

In Fig. 22¹ is shown a single-line diagram of a large power system on which complete high-speed relaying is required if stability limits are to be maintained. Conventional normal-

¹ A paper was presented by G. W. Gerell at A.I.E.E. meeting, Cleveland, Ohio, June 20-24, 1932—"Application of High Speed Relays"—which analyzes a Protection System of this same general system-circuit arrangement.

speed relays and breakers would cause Station A to pull out of step with Station C as a result of faults occurring on the 132- or 66-kv. lines.

In applying high-speed relays to a system of this kind, it is not only necessary to make a complete short-circuit study, but it is also essential that a thorough analysis of stability limits be

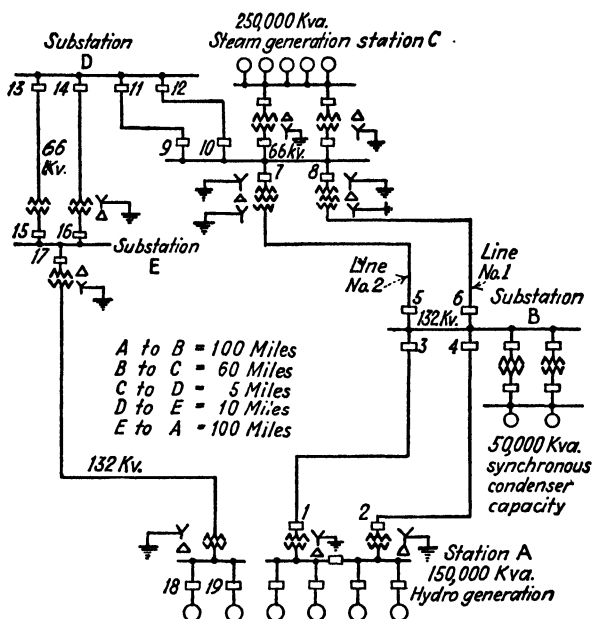


FIG. 22.—Typical high-voltage transmission system which requires high-speed relay protection.

made in order to determine in what time line-to-ground, two-line-to-ground, and phase faults must be cleared in order to prevent instability. Such studies are usually made on an alternating-current calculating board in order to eliminate laborious analytical calculations.

Parallel-line sections connect Station A to Substation B, Substation B to Station C, Station C to Substation D, and Substation D to Substation E. A single-line section connects Substation E to Station A.

Relaying of Line Section, Station A to Substation B.—The parallel-line section immediately suggests the use of current balance relays at both ends of the section. However, single-line

protection must be provided and this suggests the use of distance relays. The arrangement of Station A is such that the station may be operated with the bus-tie breaker open; in fact such operation may work out to advantage for certain system conditions. The output of a hydroelectric station varies within wide limits for different seasons of the year, and therefore the relaying and switching equipments should be arranged to give as flexible an operating schedule as possible.

For parallel-line operation the high-speed current-balance system of operation at Station A will give effective phase protection and ground protection. Three-line type current balance relays and one current balance ground relay will be required at Station A for parallel-line operation. At Substation B, distance relays for phase protection and high-speed directional over-current relays for ground protection will give complete protection for phase and ground faults under conditions of parallel- or single-line operation.

For parallel-line operation, faults on 85 per cent of a line section out of Station A will give simultaneous relay operation at both ends. Faults close to Substation B will not give sufficient unbalance of current in the current balance relays at Station A to operate the relays. However, as soon as the breaker at Substation B on the faulted line has opened, there will be sufficient unbalance at Station A to give operation. The distance phase relays and directional ground relays at Substation B will operate instantaneously for faults on the complete line section from B to A. The distance relays for breakers 3 and 4 can be set to operate in the instantaneous zone for the complete line impedance plus part of the transformer bank. Faults on the low side of the transformer bank will throw the impedance measurement into the second or timed zone of the relay. This is important because the timed zone of the distance relay will allow bus differential to clear a faulted bus section, or differential protection of the transformer to trip off the transformer, and yet not open up the line connected to the good bus section. Faults on 85 per cent of the line section for parallel operation between Stations A and B will operate the relays simultaneously in 1 cycle. This assumes that impedance relays are used for breakers 3 and 4; if reactance relays are used the time for this end will be 2 to 3 cycles. In giving the time of operation at points where

either impedance or reactance relays are applicable the lower time will be given, which is one cycle. Assuming a 6-cycle breaker, this gives a total clearing time of 7 cycles. For 15 per cent of the line close to Substation B the total clearing time for parallel-line operation will be on a sequential basis, or 1 cycle plus 6 cycles for distance relay and breaker at Substation B, plus 1 cycle and 6 cycles for current balance relay and breaker at Station A, giving a total of 14 cycles.

For single-line operation, distance relays of the reactance or impedance type can be applied to Station A, breakers 1 and 2, to give phase-fault protection. The line is 100 miles in length so that the arc resistance will not materially affect the distance as measured by an impedance relay at Station A. The distance relay will operate in its instantaneous zone for faults on 85 per cent of the line section from Station A. This means that the distance relays at both ends will give simultaneous operation for faults within 85 per cent of the line length out of Station A. This gives a total clearing time of 7 cycles. For faults within 15 per cent of the line length out of Substation B, the distance relays at Station A will operate in the intermediate-timed zone of 20 cycles. The clearing time for the breaker at Substation B will then be 1 cycle plus 6 cycles or a total of 7 cycles. The clearing time for breaker at Station A will be 20 cycles for relay plus 6 cycles for breaker, giving a total of 26 cycles. This is not serious in this particular case since the opening of the breaker at Substation B in 7 cycles removes the fault from the rest of the system so that the long time for opening the breaker at Station A will not affect the stability limits of the complete system. (The case just described assumes, of course, single-line operation, that is, only one line section operating between Stations A and B, or two line sections operating with tie breaker open at Station A.) The necessity for a time as long as 20 cycles for the intermediate zone of the distance relay will be obvious when the relaying of the next line section is discussed.

Now let it be assumed that only distance relays are used at Station A (the line-type current balance relays being eliminated). In other words, distance relays are used for both single- and parallel-line operation. Assuming parallel-line operation, a fault on one line within 15 per cent of the line out of Station B will require a total clearing time of 20 cycles for relay and 6 cycles

for the breaker at Station A, or a total of 26 cycles. It is true that, should the fault occur near breaker 4, this breaker will open in 7 cycles, but the fault is held on the system until breaker 2 opens. Distance relay for breaker 3 will measure a distance such that it will operate in the back-up time zone, thus giving selective action. If current balance relays are used at Station A for parallel-line operation, the fault will be cleared in 14 cycles, giving faster time by a margin of 12 cycles.

For protection of ground faults during parallel operation, it has already been stated that a high-speed current balance ground relay is used at Station A for breakers 1 and 2, and instantaneous directional overcurrent ground relays at breakers 3 and 4. Ground faults on 85 per cent of the line length will give an overall clearing time of 7 cycles; the remaining 15 per cent of the line giving 14-cycle operation. However, for single-line operation another condition exists. The ground relays for breakers 1 and 2 during single-line operation must be timed so as to act selectively with breakers 5 and 6. This again does not present a serious problem in that, after the breaker opens at B on section A-B, during single-line operation, the rest of the system is relieved of the fault.

Relaying of Line Section; Substation B to Substation C.—This section is very similar to the line section just described. It is 60 miles in length, somewhat shorter than the 100-mile section from A to B, but still long enough so that distance relays of the impedance type as well as of the reactance type are applicable. At Substation C the two lines from Substation B terminate in transformers, which can therefore be considered as part of the line.

At Substation B for breakers 5 and 6, high-speed line-type current balance relays for phase-fault protection will give the fastest overall clearing time, as described in detail for the line section between Station A and Substation B. For single-line operation the distance relay is applicable. Effective ground protection can be provided by the use of a high-speed directional overcurrent-relay for each of the breakers 5 and 6.

It should be noted particularly that instantaneous directional ground relays are applicable at breakers 3, 4, 5, and 6, because these breakers represent the first opening point in selective clearing of faults with breakers 1, 2, 7, and 8. Breaker 6 must

be selective with breaker 2, and breaker 5 with breaker 1. In the opposite direction, breaker 4 must be made selective with breaker 8, and breaker 3 with breaker 7. Also, a study of the transformer connections will make it obvious that the two line sections C to B to A represent a system in itself as far as ground currents are concerned. Directional ground relays can therefore be made to operate instantaneously for breakers 5 and 6 in that selective operation is not required for ground faults beyond Station C, on line sections C to D and D to E.

At Station C, current balance line and ground relays are applied for parallel-line protection and distance relays for single-line operation. For ground-fault protection during single-line operation, it is necessary to provide time-delay relays for breakers 7 and 8. It is recognized, however, that it is desirable to open these breakers as quickly as possible, particularly for a fault within 50 per cent of the line section out of Station C. An induction-type inverse time-delay relay is available, having two current coils and developing a torque equal to the square of the current. This relay can be designed to give 4- to 5-cycle operation with faults close to Substation C, yet give sufficient time delay for selective operation with faults in the line section B-A. This relay is more fully described in Chap. XI.

Relaying of Line Section; Station C to Substation D.—This parallel-line section is only 5 miles in length. High-speed current balance relays can be provided at both ends for line and ground faults during parallel-line operation. Reactance-type distance relays can be applied for phase faults during single-line operation. For ground protection during single-line operation, time-delay ground relays are required for breakers 9 and 10. The line section is so short that it is not possible to discriminate between ground faults close to Station C as against those occurring close to Substation D. A ground relay having a highly inverse operating characteristic is therefore not applicable. This time delay for ground faults close to Station C during single-line operation is very objectionable. Instantaneous overcurrent directional ground relays can be applied to breakers 11 and 12.

The section between Station C and Substation D is difficult to relay satisfactorily at high speed. For such a section, pilot-wire relaying may be the only satisfactory solution. With pilot-wire relaying, line and ground faults can be cleared in a

total of 7 cycles. This form of relaying has been developed to a point where it should be given serious consideration and is very likely to be the best choice for an important short section out of the main generating station.

Relaying of Line Section; Substation D to Substation E.—This section is approximately 10 miles in length and in addition has a transformer bank in each line at Substation E. High-speed current balance relays and reactance relays will give better protection than can be obtained by this same type of relaying when applied to section C-D. Instantaneous directional overcurrent ground relays can be applied to breakers 13 and 14, but time-delay ground relays are required for breakers 15 and 16.

Pilot-wire relaying would give fast and effective protection to this section and should be given serious study as a substitute to the above relaying.

Relaying of Line Section; Substation E to Station H.—This section gives an ideal set-up for simple effective relaying by use of distance relays for phase faults and instantaneous overcurrent relays for ground faults. A transformer bank is connected to each end of the line. The distance relays can therefore be set for instantaneous operation for the complete line length. A simple instantaneous overcurrent relay at each end will clear all ground faults instantaneously. This gives 7-cycle clearing of phase and ground faults for the full section.

SUMMARY—HIGH-SPEED RELAYING

The relaying described for system connections (Fig. 22) is not given as an example of relaying which has reached the ideal form of protection. An attempt has been made to indicate how difficult it is to clear all faults on a system at high speed. No relay system yet devised can be expected to give perfect operation. If a scheme fulfills the theoretical ideal, there are practical disadvantages such as initial cost and maintenance expense. The question naturally presents itself as to how far the complications of relaying should be considered in order to attempt to get high-speed operation for all types of faults. This question cannot be answered in a general way because each system has individual characteristics, and obviously the economic factors must be given due weight. There are interconnections now in operation which by calculation cannot maintain stability, if a three-phase

short circuit exists longer than 3 or 4 cycles. No protective scheme can accomplish this, if the circuit breakers require 6 cycles or more to clear the arc. It is true that very recent developments indicate that a breaker-opening time of 3 cycles is possible for certain classes of voltage and interrupting capacity, but these breakers are not only very special but highly expensive.

In the early part of this chapter, it is explained that the ideal high-speed relay system should be unaffected by swinging conditions between sources of generation, as long as the systems do not get out of step, in which case the relays should operate. The system of relaying described for the system connections shown in Fig. 22 does not fulfill this ideal for all conditions but does give the best practical solution available at present.

In Chap. X, on Differential Protection, it is explained how important it is not to overlook high-speed bus protection and apparatus protection, if the transmission-line protection is set up on a high-speed basis. Here again these questions present themselves: How much is such protection worth? How often do such faults occur? On many old substations, that have been later tied into a high-speed transmission system, such protection is imperative. In installations where thought has been given to the recent developments in insulation coordination and lightning protection, differential bus protection may not be necessary.

A complete chapter has been devoted to the principles involved in pilot-wire relaying. The relay systems which require pilot lines give protection which is in the high-speed class. Carrier-current pilot-relay schemes are not in the instantaneous zone of high-speed relaying in that approximately 4 cycles is required for the relay time. However, its average time for all locations of faults is still 4 cycles.¹ In distance-relay systems there may be a zone of fault location which requires 20 cycles for relay time in order to provide proper selectivity, and averaging this with the instantaneous-zone operation may give a fictitious average time higher than that for carrier-current relaying. It is not entirely logical to use a fictitious average time of distance schemes for purposes of comparison. The high-speed clearing of one end often prevents instability, so that, even though the opening of the second breaker is given a time delay, the distance-relay

¹ Refinements based on operating experience are rapidly being made in carrier-current systems. At present two systems are available, one having an operating time of four cycles and the other two cycles. It is probable that this time will be further reduced as new developments are brought out.

system should be considered a high-speed relay system if it prevents instability. The slower opening of the first breaker in a carrier-current relay system may partially offset the fact that its fictitious overall clearing average may be less. The distance-relay system has practical operating data on which to base its application. The carrier-current scheme is a new development and shows great promise for affording high-speed protection of important circuits.

AUTOMATIC-OSCILLOGRAPH DATA; HIGH-SPEED RELAYING

To analyze properly high-speed-relay and circuit-breaker operation during system faults it is necessary to have oscillograph records available. Automatic oscillographs that start operating at the instant a fault occurs will provide these data. The line sections clear so quickly that it is impossible to study

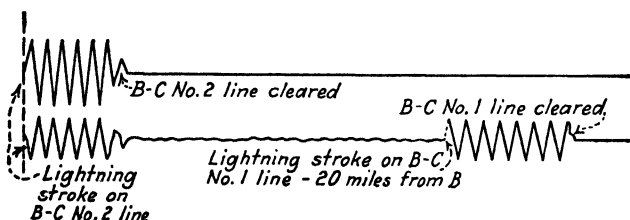


FIG. 23.—Tracing of actual oscillograph recorded on 132-kv. parallel line section (line section B-C, Fig. 22).

intelligently and improve the relay system without knowing exactly the sequence and time of opening of the circuit breakers. Automatic oscillographs should be installed at all important stations and substations in order to obtain complete data.

In Fig. 23 is shown a typical oscillograph record obtained on a line section similar to B-C, Fig. 22. Breakers 5, 6, 7, and 8 tripped. An intelligent analysis of what occurred would have been impossible without such an oscillogram available.

The oscillogram indicates that a line-to-ground fault occurred on line 2 and cleared in 6 to 7 cycles. A second line-to-ground fault occurred on line 1, approximately 20 cycles after the fault had cleared on line 2. The second fault cleared in approximately 7 cycles. Without the oscillogram it would have been assumed that a simultaneous stroke occurred on both lines, but investigation of the lines substantiated the oscillogram and it was found that the faults occurred approximately 20 miles apart.

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CHAPTER XIV

WAVE-FORM PHENOMENA AND THEIR EFFECTS IN RELAY CIRCUITS

The assumptions usually made in the study and application of protective relay schemes are:

1. The wave shape of the fault currents and voltages are sinusoidal.
2. Transient components of fault currents can be neglected.
3. The wave shape and magnitude of currents and voltages in the low-voltage circuits of instrument transformers are exactly equivalent to those quantities in the high-voltage circuits.
4. The speed of operation of relays under test and under actual system faults for similar conditions are the same.

In recent years the complexity of relay problems has increased immeasurably with the expansion and interconnection of power systems. Many large companies have found it necessary to revise completely their major relay systems. To meet the more involved operating conditions, relays must be fundamentally more accurate; applications by the method of complete short-circuit studies for steady state and, in special cases, transient conditions must be made. Field tests must be carefully made, and attention must be given to the characteristics of auxiliary devices, current transformers, potential devices, etc. In brief, all errors entering into the operation of relays should be thoroughly understood and corrected, or allowance made where complete correction is not practical.

When testing and calibrating relays it is important to use a test circuit which will provide a good wave form.¹ With the same current, the time of operation of protective relays should be the same during faults or tests. In general, the wave form of short-circuit currents approaches a pure sine form, if an appreciable amount of impedance is present in the circuit. Harmonics of considerable magnitude may exist in the fault current for terminal short circuits on rotating machines, but for the more usual

¹ ROBINSON, P. H., and I. T. MONSETH, Theory and Application of Relay Systems, *Elec. Jour.*, December, 1931, p. 685.

line faults the impedance of transformers, lines, etc., is so high that harmonics of large magnitude cannot flow.

If relays are tested, using current of other than sine wave form, the time of operation under test and actual fault conditions will be different. In extreme cases this may lead to inselective relay operation. Such a case of trouble would probably never be isolated without the use of an oscillograph. The proper calibration of relays is, of course, most important on complex systems, where various line sections must be cleared selectively.

The usual effect of a distorted wave form is to cause a delay in the operation of the relay. Therefore, if relays are tested with a distorted wave, the actual operating time under fault will be shorter than that indicated by test.

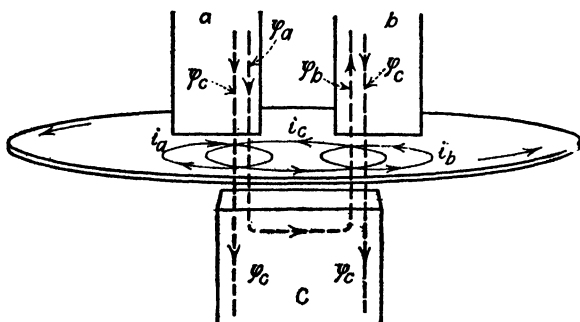


FIG. 1.—Fluxes and currents in induction-type relay.

The majority of relays in use today operate on the induction principle, including simple overcurrent, distance, current balance, phase-balance, differential, volt-ampere or residual relays, voltage, and reactance relays. The advent of high-speed relays has opened a new field of development but even here the induction principle is employed in certain types.

There are two general ways of obtaining a torque in induction-type relays. One is by arranging the physical position of the flux-producing poles so that the flux of the pole reacts with the portion of the eddy currents induced by a second pole which lies in the field of the first. Thus, in Fig. 1, the flux of pole *a* reacts with that part of the eddy currents induced in the disk by the flux of pole *c* which is within the field of *a*. Similarly, the flux of pole *b* reacts with the portion of the eddy currents induced by the flux of pole *c* that is in its field; and the flux of pole *c* reacts

with the eddy currents of poles *a* and *b* in its field. Application of Lenz's law for the direction of the induced currents, and the law for the direction of force exerted by a magnetic field on a current flowing at right angles to the field, will show that the torque produced by all reactions will be in the same direction. Therefore, the resultant torque at any instant will be the sum of these. The relation of the three fluxes is shown vectorially in Fig. 2. The flux ϕ_c is shown lagging ϕ_a by 90° and, therefore,

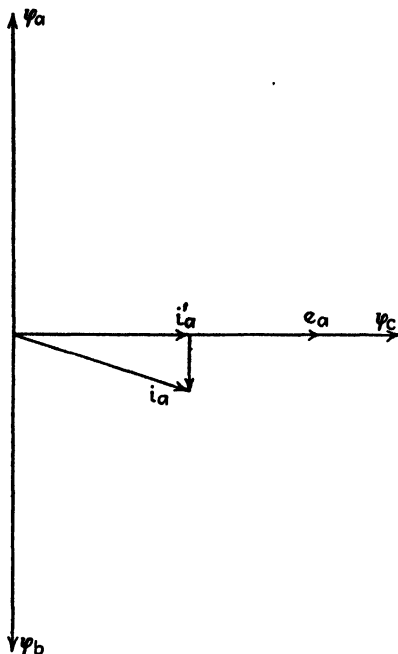


FIG. 2.—Vector relations of fluxes and currents in induction-type relay of Fig. 1.

in phase with the voltage e_a induced by the flux ϕ_a . This voltage e_a produces the current i_a lagging at an angle depending upon the impedance of the eddy-current path. A torque will be produced by the reaction of flux ϕ_c with the component of i_a in phase with it. This torque will always be in the same direction, varying from zero to a maximum. For maximum torque, ϕ_c should lag ϕ_a a little more than 90° . The reaction between ϕ_c and the quadrature component of i_a will be alternately positive and negative and, therefore, will produce no resultant torque. A similar analysis may be made for the other reactions.

The second method of producing a torque is shown diagrammatically in Fig. 3. This is the well-known shaded-pole principle. A short-circuited turn around a part of the pole causes the flux under the turn to lag the flux in the other part of the pole, its rise to a maximum value and subsequent fall being retarded by the current induced in the short-circuited turn. This results in a sweeping of the flux across the air gap under the pole, which induces eddy currents in the disk. The reaction of the induced currents with the pole flux produces the torque. The relation

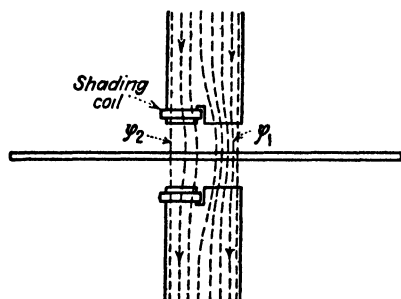


FIG. 3.—Fluxes in shaded-pole induction type relay.

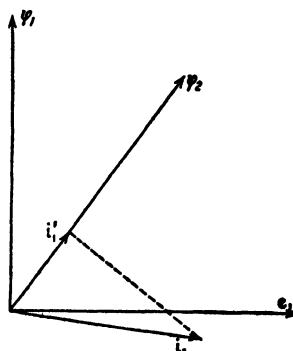


FIG. 4.—Vector relations of fluxes and currents in shaded-pole induction-type relay of Fig. 3.

of the fluxes and current is shown on Fig. 4. The flux in the unshaded portion of the pole is represented by ϕ_1 and that in the shaded portion by ϕ_2 . The flux ϕ_1 induces the voltage e_1 in quadrature with it. The voltage e_1 produces the current i_1 lagging it by a small angle. The component of i_1 in phase with ϕ_2 reacts with ϕ_2 to produce a torque. It is easily seen that the greater the angle of lag between ϕ_1 and ϕ_2 the greater will be the component of i_1 in phase with ϕ_2 and, therefore, the greater will be the torque.

EFFECT OF WAVE FORM ON RELAY TORQUE

A general equation for relay torque with a symmetrical non-sinusoidal wave may readily be developed.

Let

T_2 = average torque produced by ϕ_u and i_{a1} .

$T = T_1 + T_2$ = total torque acting on relay.

Φ and I = maximum values of flux and current. (Same subscripts as for instantaneous values used to indicate particular flux or current.)

α, β, γ , and λ = phase displacement of quantities with respect to a common reference.

$$\tau = \frac{2\pi}{\omega} = \frac{1}{f}$$

k and k' = constants.

Z_d = impedance of relay disk.

Subscripts 1, 3, . . . indicate order of harmonic.¹

Assume a flow of current in the upper coil which produces a flux of some form:

$$\phi_u = \Phi_{u1} \sin(\omega t + \alpha_1) + \Phi_{u3} \sin(3\omega t + \alpha_3) + \dots \quad (1)$$

This flux induces a voltage e_{du} in the disk:

$$e_{du} = -N \frac{d\phi_u}{dt}$$

which produces a current i_{du} .

This current will have some wave form

$$i_{du} = I_{du1} \sin(\omega t + \beta_1) + I_{du3} \sin(3\omega t + \beta_3) + \dots \quad (2)$$

different from that of the flux ϕ_u .

The instantaneous torque produced by the reaction of i_{du} and the flux ϕ_l will be:

$$\begin{aligned} t_1 &= k\phi_l i_{du} \\ &= k[\Phi_{l1} \sin(\omega t + \gamma_1) + \Phi_{l3} \sin(3\omega t + \gamma_3) + \dots] \\ &\quad [I_{du1} \sin(\omega t + \beta_1) + I_{du3} \sin(3\omega t + \beta_3) + \dots] \end{aligned} \quad (3)$$

Similarly, the instantaneous torque produced by i_{dl} and ϕ_u will be

$$t_2 = k'\phi_u i_{dl} \quad (4)$$

The average torque produced by the reaction of i_{du} and ϕ_l is

$$\begin{aligned} T_1 &= \frac{1}{\tau} \int_0^\tau t_1 = \frac{1}{\tau} \int_0^\tau k\phi_l i_{du} \\ &= \frac{k}{\tau} \int_0^\tau [\Phi_{l1} \sin(\omega t + \gamma_1) + \Phi_{l3} \sin(3\omega t + \gamma_3) + \dots] \\ &\quad [I_{du1} \sin(\omega t + \beta_1) + I_{du3} \sin(3\omega t + \beta_3) + \dots] \end{aligned}$$

¹Symbols having the subscript l , u , or d refer to the lower coil, upper coil, and disk respectively.

The expression under the integral sign involves product terms of like and unlike frequency.

Upon integration, the product terms of like frequency become

$$\frac{k \Phi_{1p} I_{du p}}{2} \cos (\gamma_p - \beta_p)$$

The product terms of unlike frequency become zero, showing that harmonics of different frequencies do not react to produce torque. Hence, the average torque due to the reaction of i_{du} and ϕ_i is

$$\begin{aligned} T_1 &= \frac{k \Phi_{11} I_{du 1}}{2} \cos (\gamma_1 - \beta_1) + \frac{k \Phi_{13} I_{du 3}}{2} \cos (\gamma_3 - \beta_3) + \cdots \\ &= K_1 \cos (\gamma_1 - \beta_1) + K_3 \cos (\gamma_3 - \beta_3) + \cdots \end{aligned} \quad (5)$$

Similarly, it can be shown that the average torque produced by the reaction of i_{dl} and ϕ_u is

$$T_2 = K'_1 \cos (\alpha_1 - \lambda_1) + K'_3 \cos (\alpha_3 - \lambda_3) + \cdots \quad (6)$$

The total torque tending to produce rotation is the sum of these average torques.

$$\begin{aligned} T &= T_1 + T_2 \\ &= K_1 \cos (\gamma_1 - \beta_1) + K_3 \cos (\gamma_3 - \beta_3) + \cdots \\ &\quad + K'_1 \cos (\alpha_1 - \lambda_1) + K'_3 \cos (\alpha_3 - \lambda_3) + \cdots \end{aligned} \quad (7)$$

This expression for the torque acting on the relay disk shows that the torque produced by the reaction of the fundamentals or corresponding harmonics of flux and disk current is directly proportional to the cosine of the angle of phase difference between the fundamentals or harmonics considered. Therefore an increase in the phase angle between corresponding components of flux and current reduces the active torque. The torque thus depends upon the magnitude and phase relations of the fundamental and harmonics of effective pole flux reacting with corresponding components of current induced in the relay disk.

Torque-producing flux and current relations in a relay such as the one shown in Fig. 5 can be determined qualitatively by placing an exploring coil on one of the upper poles and substituting a coil having only a few turns for the disk. Current is then circulated through the main coil and oscillograph records taken of the voltage induced in the exploring coil on the upper pole and

the current flowing in the coil between the upper and lower poles. Knowing the magnitude and wave form of the voltage of the exploring coil, it is possible to obtain the wave form of the flux

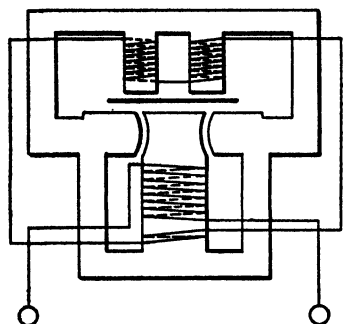


FIG. 5.—Internal connections of inverse-time characteristic relay.

from the relation $e = -N \frac{d\phi}{dt}$. The upper pole flux and the disk current are thus obtained and analyzed to find the magnitude and phase relations of the fundamental and harmonics. The torque produced is proportional to the summation of the instantaneous products of the fundamental and harmonics in the upper pole flux and the fundamental and harmonics of current induced in the disk by the lower coil.

In Fig. 6a is shown an oscillogram of the relations with a nearly sine-wave current in the main coil and in Fig. 6b with a prominent third and fifth harmonic in the main-coil current. The result of an analysis of these oscillograms is shown in Fig. 7. The torque curves are shown only for the reaction of the funda-

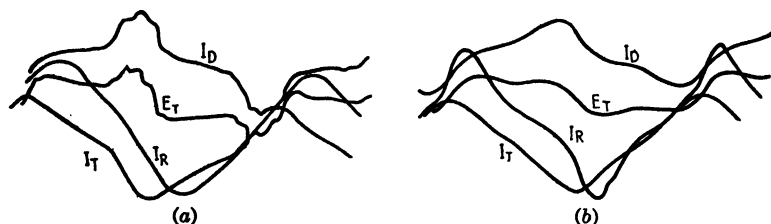


FIG. 6.—Oscillograms showing relations of current and voltage in an induction-type overcurrent relay with different main coil current wave shapes. I_R current in main coil, ten times normal; I_T current in upper coil; I_D current in coil substituted for disc; E_T voltage induced in exploring coil placed on upper pole.

mental of upper pole flux and current induced in the relay disk by the flux from the lower pole. The cross-hatched area is the reverse torque, which is much larger with the distorted wave. An analysis of the third-harmonic relations of flux and current showed considerable torque with the good wave form and practically oscillatory torque with the distorted wave. These results are presented for a qualitative explanation of the torque reduction rather than accurate quantitative results.

The reduction of torque is produced by a shifting of the fundamental and harmonics of current in the disk with respect to the corresponding components of the flux.

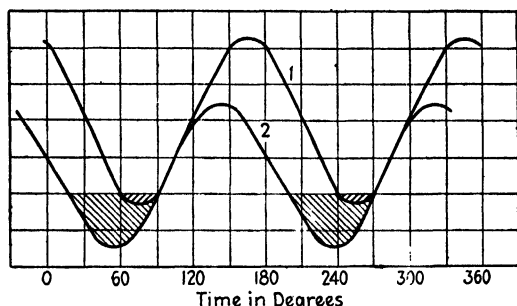


FIG. 7.—Resultant torque due to fundamental current and flux. Curve 1 with wave form of Fig. 6a; curve 2 with wave form of Fig. 6b. The shaded area represents reverse torque.

WAVE FORM IN RELAY TEST CIRCUITS

It is the general practice of most operating companies to test all relays periodically. These tests may be divided into two general classes usually termed *secondary* and *primary* tests. The secondary tests consist of a general inspection of relays and associated circuits and a time-current calibration of the relays. The primary test consists of circulating current at low voltage through the primary of the relay current transformer, thus including current transformer, secondary connections, and relay in an overall test. The primary-type test is less commonly used because more testing equipment is required, and considerable time and labor are involved. Furthermore, it is difficult to handle the current adjustments in calibrating the relays. Generally, some form of high-current testing transformer is used to supply the large current necessary for primary test. It is the practice of some companies to make primary tests only when new equipment is installed or when trouble develops. The simple secondary test is more commonly used for routine tests and calibration of relays.

SECONDARY-TEST CIRCUITS

In order to secure a good wave form in any relay test circuit, it is necessary that the constants of the circuit be such as to offer

high impedance to the flow of harmonics and thus tend to constrain the current to a sine form. To provide good wave form, it is necessary that the ratio of the circuit impedance to relay impedance be high and that the constants of the circuit not be functions of the current.

The inductance of the relay, like all iron magnetic circuits, is a function of the current; consequently no simple relations exist between current and flux. The impedance to the flow of harmonic currents over the normal operating zone of the relay is not directly proportional to the order of the harmonic, due to saturation in the magnetic circuit. If the voltage at the relay terminals is of sine form, the current will be distorted. Conversely, if the current through the relay is constrained to sine form, the voltage across the relay will be distorted. The applica-

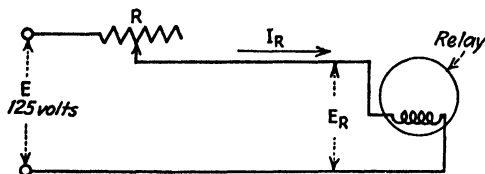


FIG. 8.—Simple test circuit for calibrating relays using variable resistance.

tion of a distorted voltage results in a magnified distortion in the current and consequent reduction in torque. In order to produce maximum torque in the relay it is necessary to constrain the current within practical limits to a sine form. It is obvious that this does not merely necessitate the use of a sine-wave voltage source for relay test.

Constant-voltage Variable-resistance Circuits.—One of the simplest and most effective types of test circuit is shown in Fig. 8. The test current is adjusted by means of the variable resistance. In Fig. 9 is shown an oscillogram of the wave form of the test circuit. The main-coil current I_R is slightly distorted, due to the fact that in order to obtain 50 amp. the circuit resistance is reduced to such an extent that the ratio of circuit impedance to relay impedance is low. The wave form could be improved by using a higher voltage source, thus requiring a greater external resistance to obtain the same current. The wave form can be improved without using a higher voltage by inserting an air-core inductance coil in series with resistor and

relay. The reactance of such a coil to the flow of any harmonic is proportional to the order of the harmonic.

Variable-voltage Test Circuits.—Many test circuits are in use of fundamentally the same type as shown in Fig. 10. The relay current is adjusted by changing the voltage applied to the

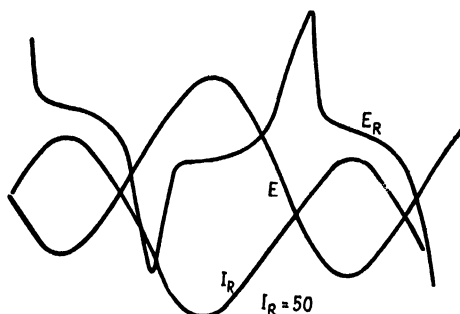


FIG. 9.—Oscillograms showing the effect of resistance and inductance on wave form in test-circuit of Fig. 8, using definite-time-overcurrent relay (5 amp. tap.), current ten times normal.

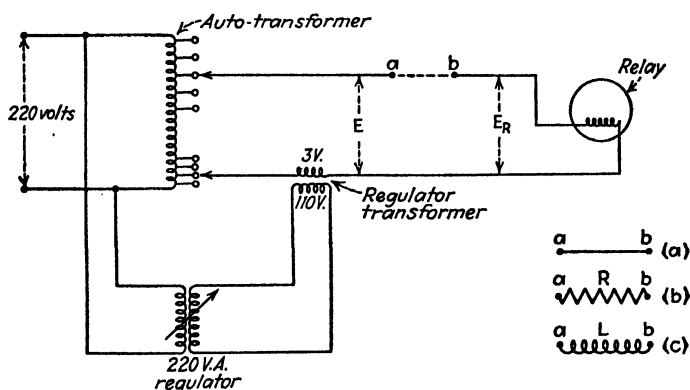


FIG. 10.—Relay-test circuits using variable voltage.

relay terminals by means of a tapped autotransformer and voltage regulator. This general type of test circuit is frequently used, because more compact and lighter equipment can be used than with other types of circuits. The circuit is exceptionally easy to operate but has the disadvantage of distorting the test-current wave.

In Fig. 11a is shown an oscillogram for the test circuit of Fig. 10a. The cause of the distortion in the current wave is

the autotransformer. The relay requires certain harmonics in the magnetizing current. Any harmonics appearing in the

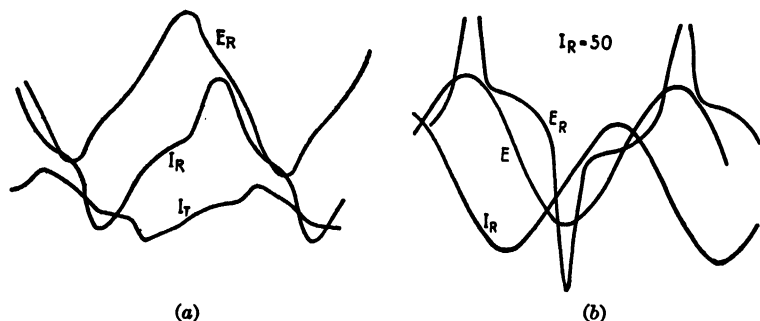


FIG. 11.—Effect of resistance and reactance on wave form in test circuit Fig. 10 using definite-time overcurrent relay. Main-coil current I_R is ten times normal. a, Circuit of Fig. 10a, $R = 0$, $I_R = 50$, 5 amp. tap.; b, circuit of Fig. 10c, $R = .226$, $L = .00592$ henry, $I_R = 50$.

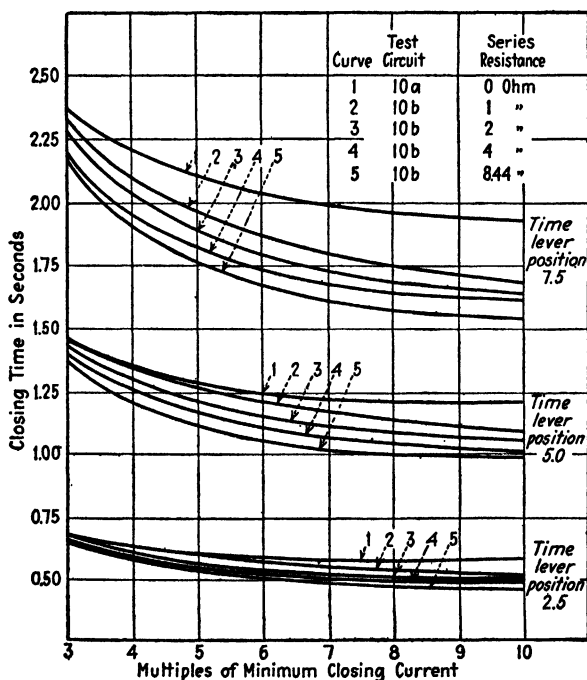


FIG. 12.—Curves showing effect of resistance on closing time of definite-time overcurrent relay.

autotransformer voltage will amplify the distortion in the relay current. This, in turn, alters the flux relations in the auto-

transformer, still further increasing the distortion. The distortion is present irrespective of the wave form of the source voltage.

The addition of impedance in series with the relay stabilizes the circuit and smooths out the wave form of the current. A resistance of 8.44 ohms completely corrects distortion, but the voltage required is too high for safety to the testers.

The same results can be obtained by using an inductance coil in series with the relay (Fig. 10c). The wave form of Fig. 11b is as good as that obtained with 8.44 ohms resistance.

Time-current characteristics of a definite-time induction-type overcurrent relay using the test circuit of Fig. 10 is shown in Fig. 12. As the circuit resistance is increased, the wave form improves and the relay operates in a shorter time. The curve for the highest value of resistance approaches closely the operating time which would be expected under actual fault conditions for equivalent current values in a high-reactance system. This is, of course, assuming a sine wave of fault current and no secondary wave distortion due to current-transformer characteristics. A very slight change in wave form causes an appreciable change in operating time. All induction-type relays are similarly effected by distorted wave forms.

PRIMARY-TEST CIRCUITS

Primary-test circuits are susceptible to wave distortion in the same manner as secondary-test circuits. In Fig. 13 is shown

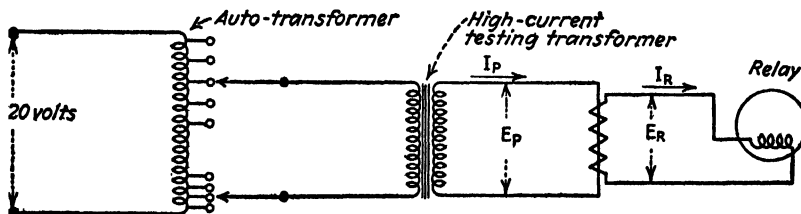


FIG. 13.—Test circuit used in applying primary test to current transformer and relay.

a type of primary-test circuit frequently used in high-current testing. The autotransformer taps are used for adjusting the current. The high-current testing transformer used in the test discussed here is rated at 2000 amp. and 8.5 volts on the low-voltage side. In Fig. 14 is shown an oscillogram for this test

circuit. The distortion of this current wave is produced by the autotransformer. This can be corrected by connecting an air-core inductance coil between the autotransformer and the high-current testing transformer. The same result is obtained by eliminating the autotransformer and inductance coil from the test circuit and simply using a variable resistance for adjusting the current.

It is necessary to use great care in the selection of equipment for use in primary-test circuits. Any one piece of apparatus in the circuit may cause excessive distortion. The high-current testing transformer should be liberally designed, giving particular attention to the saturation characteristics.

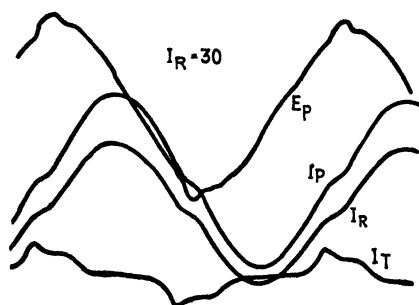


FIG. 14.—Primary test on 20:1 ratio-wound-type current transformers with two relays (50 volt-amp.) burden. Test circuit of Fig. 13. Relay current 30 amp.

In any relay test circuit it is not desirable to use a loading impedance having resistance or inductance that is a function of the current, such as lamp banks and iron-core reactors. The electrical proportions of the test circuit should be such that the time constant L/R is not excessive, or the current wave may be offset sufficiently, when the circuit is energized, to cause error in the operation of the relay due to the magnetization in the iron circuit produced by the direct-current component. This is particularly true of circuits for testing high-speed relays.

It is recommended that, if any unusual test circuits are to be used, a thorough investigation be made of the wave-form characteristics. Such an investigation can be made with an oscillograph or by simply comparing the calibration curves for relays on the particular circuit with those obtained from a test circuit of known good wave form, such as a high-voltage, high-impedance circuit without interposing transformers. If the

error is of sufficient magnitude to justify correction, it may be possible to eliminate the distortion with small air-core inductance coils or resistance. Autotransformers, phantom loads, etc., used as testing devices, should be carefully examined.

WAVE FORM IN CURRENT-TRANSFORMER SECONDARIES

The incorrect application of current transformers for use with protective relays frequently results in the production of highly distorted secondary-current wave forms even with sine-form primary current. Usually this is due either to using current transformers of low ratio, which saturate during heavy faults, or to the overburdening of the secondaries. With a certain primary current and secondary burden, a voltage must be induced in the secondary which will produce a current of such magnitude that the secondary ampere turns will balance the primary ampere turns due to the load component of the primary current. The flux will adjust itself to maintain this voltage and current. If the burden or primary current is increased beyond the limits for which the current transformer is designed, the ratio and phase angle will change and the secondary-current wave form will be distorted.

The characteristics of current transformers with their burden consisting of relays, instruments, etc., under fault conditions bear stressing because generally there is more distortion produced in the secondary-current wave due to saturation than is produced in the primary-current wave due to the characteristics of the machines and associated circuits. This is particularly true of the bushing-type current transformers commonly used in high-voltage systems for the operation of relays. Bushing-type current transformers inherently have poor characteristics for low ratios. The ideal current transformer for use with relays is one which will reproduce exactly the magnitude and wave form of the primary current. In most cases a little care in the application of current transformers will assure a practical approach to this ideal.

It has been found that very appreciable errors may exist in current transformers during transient and steady-state conditions. The maximum error during transient conditions is produced by oversaturation of the transformer core, due to the presence of the direct-current component in the primary fault

current. The presence of the asymmetrical current is uncertain since it depends upon the cyclic time of occurrence of the fault, but on a three-phase system there will invariably be a direct-current component in one or more of the phases. The saturation under such conditions is further increased, if the secondary burden is excessive. The error also depends upon the residual magnetism in the transformer core and its polarity with respect to the direct-current component. Residual magnetism would not normally be present for the initial short circuit on a system because of demagnetization of the load current, but it could be present after the initial fault if the circuit is interrupted before the direct-current transient disappears, and be effective when the circuit breaker is reclosed against the fault. Residual flux would also be present if the circuit is interrupted at any instant when the flux wave is other than zero.

The errors in the current transformers under short-circuit conditions generally appear in the following ways:

1. Ratio errors decrease the crest value of secondary current.
2. Distortion of secondary-current wave.
3. Shift in phase relations of secondary current with respect to primary.

In some cases the effect of oversaturation may be so great as to cause an error of 50 per cent or more in the value of the secondary current for the first few cycles. The operation of high-speed relays under such conditions would be very unsatisfactory.

Current transformers operating with a certain burden may be entirely satisfactory for steady-state conditions and very unsatisfactory with asymmetrical transient currents.

Steady-state Characteristics.—The principal requirement for accuracy under symmetrical short-circuit conditions is that the secondary burden be sufficiently low for the current transformer to operate on the straight part of the saturation curve. The permissible burden can readily be determined. The first step is to determine the saturation curve by applying voltage to the secondary with the primary circuit open and make current and voltage measurements to find the full range of the curve. The resistance of the secondary should be measured and this value combined with the impedance of the burden which is to be used in the circuit. Knowing the maximum value of symmetrical short-circuit current referred

to the secondary circuit, the secondary voltage necessary to force this current through the burden circuit is easily calculated. If this voltage is below the knee of the saturation curve, the current transformer will be satisfactory for symmetrical conditions.

The effect of burden under steady-state conditions is illustrated in Fig. 15*a* and *b*, in which measurements were made with a constant primary wave in a 100:5 ratio bushing current transformer. Figure 15*a* shows the results with no burden other than the oscillograph shunt and Fig. 15*b* with constant burden of 0.426 ohm resistance and 2.23 ohms reactance. For the latter case the secondary wave is distorted, the phase angle between primary and secondary changes, and the magnitude is reduced to 62 per cent of the value with no burden.

The burden imposed upon current transformers during line-to-ground faults, when ground relays are used, is greater than for line-to-line faults. Phase-angle errors of large magnitude may occur in bushing-type current transformers with excessive saturation during fault conditions. In Fig. 16 are shown the phase-angle characteristics of a 100:5 ratio bushing-type current transformer as effected by burden and various values of current. The conditions shown in this figure are somewhat extreme but serve to illustrate the necessity for a thorough understanding of the characteristics under all conditions when used for the operation of relays. Such abnormal errors do not generally appear with higher ratios.

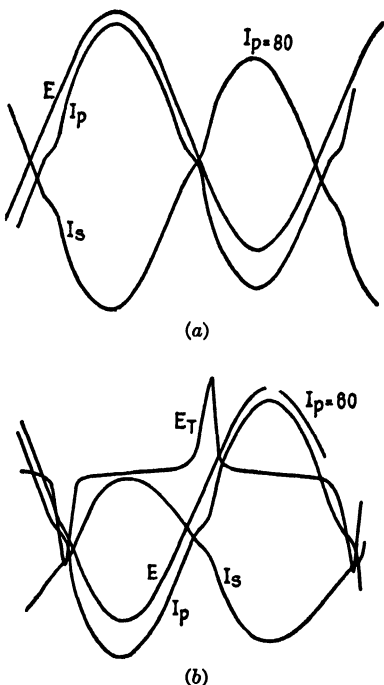


FIG. 15.—Oscillograms showing effect of burden on secondary wave. *a*, Burden 0.1 ohm on 20:1 bushing current transformer; *b*, burden $R = 0.426$ and $x = 2.23$.

Transient Characteristics with Asymmetrical Current Wave.—

The accuracy of current transformers in correctly reproducing in their secondary circuits any wave form of current appearing in the primaries depends upon the flux in the magnetic circuit.¹

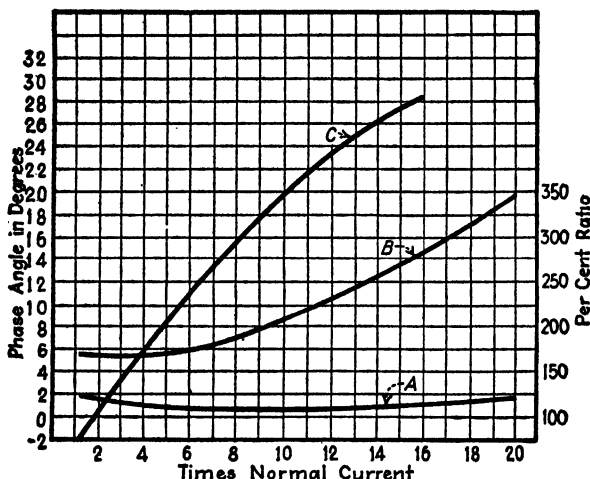


FIG. 16.—Twenty-to-one ratio bushing current-transformer characteristics. a, Ratio 12 volt-amp. burden; b, ratio 45 volt-amp. burden; c, phase-angle error 45 volt-amp. burden.

The maximum flux depends upon the current magnitude, transient duration, secondary burden, and the design of the transformer. The permeability of the current-transformer iron is a complex function of the current and flux during conditions

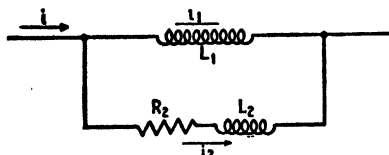


FIG. 17.—Equivalent circuit for current transformers.

which approach or produce saturation. This problem involves the consideration, not only of the fundamental and harmonics of current, but also direct-current components that may exist in the fault current.

It is of interest to consider the case of an asymmetrical non-sinusoidal primary-current wave and derive a general expression for the secondary current. The assumption is made that the magnetization curve is a straight line, and hysteresis and eddy current effects are neglected. Assuming the wave of primary

¹ MARSHALL, D. E., and P. O. LANGGUTH, Current Transformer Excitation Under Transient Conditions, *Trans. A.I.E.E.*, vol. 48, 1929, p. 1464.

current is known, the primary inductance of the current transformer can be neglected and the simple equivalent circuit of Fig. 17 obtained.

Symbols:

L_1 = magnetizing shunt inductance.

$L_2 = L_s + L_L$ = secondary and burden inductance.

$R_2 = R_s + R_L$ = secondary and burden resistance.

$T = L/R$ = time constant of total primary circuit

i = primary current, instantaneous value.

i_1 = magnetizing current, instantaneous value.

i_2 = secondary current, instantaneous value.

I = maximum value of current (subscripts denote the order of the harmonic).

To solve for the secondary current i_2 , from Fig. 17, the following equations may be written:

$$L_1 \frac{di_1}{dt} = R_2 i_2 + L_2 \frac{di_2}{dt} \quad (8)$$

$$i_1 = i - i_2 \quad (9)$$

Substituting for i_1 in Eq. (8) and rearranging terms:

$$\frac{di_2}{dt} + \frac{R_2}{L_1 + L_2} i_2 = \frac{L_1}{L_1 + L_2} \frac{di}{dt} \quad (10)$$

The general solution of Eq. (10) is

$$i_2 = \frac{L_1}{L_1 + L_2} e^{\frac{-R_2 t}{L_1 + L_2}} \int \left(e^{\frac{R_2 t}{L_1 + L_2}} \frac{di}{dt} \right) dt + C e^{\frac{-R_2 t}{L_1 + L_2}} \quad (11)$$

Assuming the primary current is of some form,

$$i = I_1 \left[\sin (\omega t + \alpha_1) - \sin \alpha_1 e^{\frac{-t}{T}} \right] + \\ I_3 \left[\sin (3\omega t + \alpha_3) - \sin \alpha_3 e^{\frac{-t}{T}} \right] + \dots \\ I_n \left[\sin (3\omega t + \alpha_n) - \sin \alpha_n e^{\frac{-t}{T}} \right] \quad (12)$$

where I_n = maximum value of n th harmonic for steady state and α_n = phase angle which fixes the value of instantaneous volts when $t = 0$.

Differentiating Eq. (12) with respect to t , substituting the result in Eq. (11) and evaluating the integral:

$$\begin{aligned}
i_2 = & \frac{K\omega I_1 \cos(\omega t + \alpha_1 - \theta_1)}{\sqrt{\frac{1}{T_1^2} + \omega^2}} + \frac{KI_1 \sin \alpha_1 T_1 \epsilon^{\frac{-t}{T}}}{T - T_1} \\
& + \frac{3K\omega I_3 \cos(3\omega t + \alpha_3 - \theta_3)}{\sqrt{\frac{1}{T_1^2} + 9\omega^2}} + \frac{KI_3 \sin \alpha_3 T_1 \epsilon^{\frac{-t}{T}}}{T - T_1} + \dots \\
& + C\epsilon^{\frac{-t}{T_1}} \quad (13)
\end{aligned}$$

where $\theta_n = \tan^{-1} n\omega T$.

$$\begin{aligned}
T_1 &= \frac{L_1 + L_2}{R_2} \\
K &= \frac{L_1}{L_1 + L_2}
\end{aligned}$$

C can be determined by choosing $t = 0$, when $i_2 = 0$.

The complete equation for i_2 consists of a group of cosine terms and a group of exponential terms and may be written

$$\begin{aligned}
i_2 = \sum_n I_n \left\{ \frac{nK\omega}{\sqrt{\frac{1}{T_1^2} + (n\omega)^2}} \left[\sin \left(n\omega t + \alpha_n - \theta_n + \frac{\pi}{2} \right) - \sin \right. \right. \\
\left. \left. \left(\alpha_n - \theta_n + \frac{\pi}{2} \right) \epsilon^{\frac{-t}{T_1}} \right] + \frac{KT_1}{T - T_1} \sin \alpha_n \left(\epsilon^{\frac{-t}{T}} - \epsilon^{\frac{-t}{T_1}} \right) \right\} \quad (14)
\end{aligned}$$

where $n = 1, 3, 5, \dots$, corresponding to the harmonics present.

This equation shows that each harmonic component consists of three parts during the transient period, the ordinary steady-state sine wave, the unsymmetrical transient term, and a third term, the function of which is to make the secondary current zero at zero time.

For steady-state condition the last two terms become zero and the equation for i_2 reduces to

$$i_2 = \sum_n K' I_n \sin \left(n\omega t + \alpha_n - \theta_n + \frac{\pi}{2} \right) \quad (15)$$

where

$$K' = \frac{nK\omega}{\sqrt{\frac{1}{T_1^2} + (n\omega)^2}}$$

Substituting for K and T_1 in the expression for K' ,

$$K' = \frac{n\omega L_1}{\sqrt{R_2^2 + (L_1 + L_2)^2(n\omega)^2}}$$

In general R_2^2 is negligible compared to $(L_1 + L_2)^2(n\omega)^2$, so an approximation may be written

$$K' = \frac{n\omega L_1}{(L_1 + L_2)n\omega} = \frac{L_1}{L_1 + L_2} = K$$

Thus K' is practically independent of the order of the harmonic, and the fundamental and harmonics of the secondary current bear a constant ratio K to those in the primary current.

Since $\tan \theta_n = n\omega T$ is usually large, θ_n is nearly 90° and therefore cancels out with the $\pi/2$ of Eq. (15).

Hence, on the basis of the original assumptions and for steady-state conditions, the secondary current, for a non-sinusoidal primary current, is given by the approximate equation.

$$i_2 = \sum_n K I_n \sin (n\omega t + \alpha_n) = K i \quad (16)$$

This equation shows that for steady-state conditions the secondary current will have the same wave form as the primary current, for all practical considerations.

The following test data and oscillograph records are given to demonstrate some very common wave phenomena appearing in relay circuits during actual fault conditions due to transient characteristics of current transformers. It is not generally appreciated that such phenomena are of frequent occurrence in properly designed protective systems but this has been found true by scores of staged short circuits at full voltage on an actual typical system by the authors.

In Fig. 18a are shown 20:1 ratio and 100:1 ratio wound-type current transformers balanced for the differential protection of a 12,000/2300-volt, three-phase, 2500-kva. transformer. Figure 18b is the record for a three-phase through short circuit on the low-voltage side of the transformer in which the current waves are very nearly symmetrical. The difference current is of sine form and is also symmetrical. Figure 18c is the record for a test, identical in every detail, except that the current waves contain an asymmetrical component of large magnitude. The

maximum difference current is 3.5 times greater than in the preceding case, and the angular relation between this current and the sum currents has changed by approximately 90° . It will be noted that the distorted difference current gradually decreases

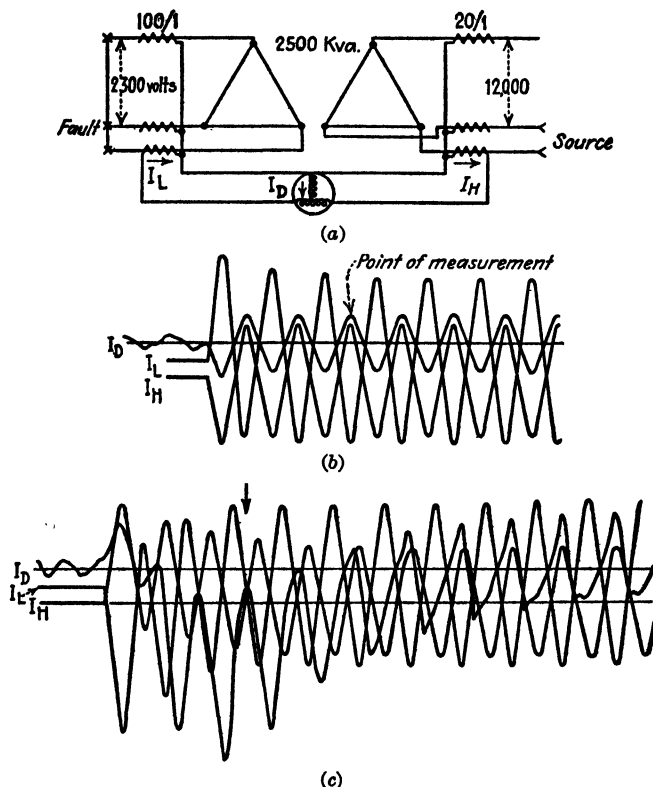


FIG. 18.—*a*, Through short circuit on transformer differential; *b*, symmetrical wave $I_H = 25.2$, $I_L = 26.1$, $I_D = .867$; *c*, asymmetrical wave $I_H = 23.9$, $I_L = 24.6$, $I_D = 3.07$ max.

and, as the circuit becomes demagnetized, gradually assumes the same shape and magnitude as in Fig. 18*b*.

In Fig. 19*b* are shown the results of a single-line-to-ground short circuit on a 25,000-kva., 12,000-volt, three-phase generator. The current transformers are 300:1 ratio through type. A large direct-current component is present in the fault current, but the reproduction is quite good, the unbalance being 3 per cent.

In Fig. 20 is shown a case of simultaneous saturation with a three-phase fault on a three-phase, 66,000/2300-volt, 5000 kva.

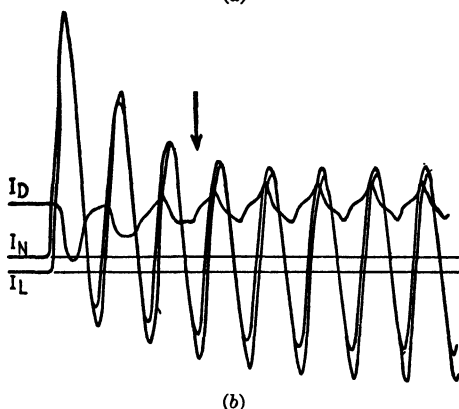
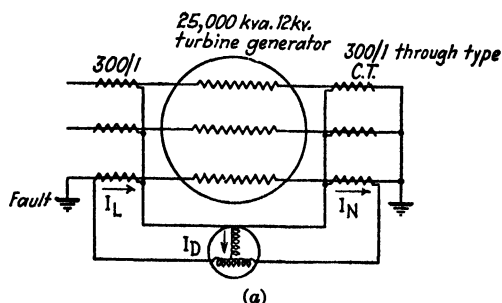


FIG. 19.—a, Single phase through fault on generator; b, asymmetrical wave
 $I_L = 18.5$, $I_N = 21.8$, $I_D = .66$.

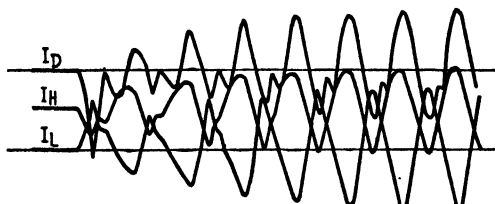


FIG. 20.—Oscillograph record for a three-phase through fault on a 5000-kva.
 66,000/2300-volt transformer showing case of simultaneous saturation.

transformer. The current transformers are 40:1 ratio bushing type and 600:1 ratio wound type. It will be observed that all transients are of the inverse type.

A fairly true reproduction of the primary current and the actual secondary current appearing in the transmission-line relay circuit is shown in Fig. 21. The current transformers are 80:1 ratio bushing type, supplying a burden of 50 volt-amp. The primary-current wave is almost completely offset, and an extreme case of saturation results in the relay current transformer. The secondary-current wave is practically suppressed for 1 cycle, then gradually builds up to a maximum in 13 cycles as the direct-current magnetization disappears. The symmetrical secondary current then decays along with the primary current. This inverse transient and the distortion are due entirely to the saturation produced by direct current as can be seen by

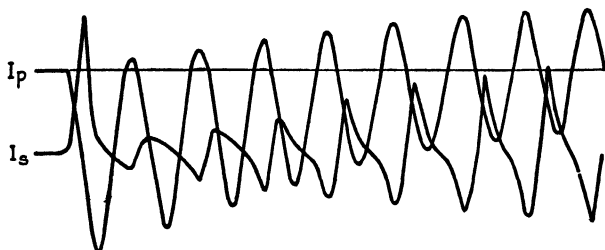


FIG. 21.—Oscillograph record showing inverse decrement resulting from saturation due to direct-current component for single line-to-ground short circuit on 33-kv. transmission line. I_P primary current, I_S secondary current. Eighty-to-one ratio bushing current transformers, 50 volt-amp. burden at 5 amp.

comparison with the true wave. The actual measured minimum relay current expressed as a percentage of the actual current is 20 per cent. A high-speed overcurrent ground relay, normally operating in 2 cycles, did not operate for 10 cycles on this test, due to the low initial secondary current. The characteristics of the current transformer are entirely satisfactory after the effects of the direct-current component disappear.

It is evident from a consideration of these typical illustrations that balanced- and distance-type relays may operate incorrectly or fail to operate, due to the initial transient characteristics of the current transformers. The steady-state performance is not necessarily a criterion of the accuracy during transient conditions.

An approximate method¹ of determining the ability of a current transformer to reproduce transients has been developed. The first step is to determine an average impedance Z_1 by dividing

¹ MARSHALL, D. E., and P. O. LANGGUTH, *op. cit.*, p. 1464.

values of voltage by the corresponding current up to the knee of the saturation curve for the current transformer. The method of determining the saturation curve has previously been given. The time constant T_1 is then calculated from the equation $T_1 = Z_1/2\pi fR_2$. The time constant of the total primary circuit is calculated. The ratio of maximum transient exciting current to maximum secondary steady-state current can be calculated

from the equation $I_m = I \left(\frac{T_1}{T} \right)^{\frac{-T_1}{T_1 - T}}$ where I is maximum value of steady-state primary current. The r.m.a. value of steady-state secondary current which the transformer can carry without excessive saturation under the given transient conditions is found by dividing the r.m.s. exciting current just below the knee of the saturation curve by this ratio. The ratio between the maximum transient and steady-state exciting current is equal to $\omega T_1 \left(\frac{T}{T_1} \right)^{\frac{T_1}{T_1 - T}}$.

EFFECT OF DIRECT-CURRENT COMPONENT ON RELAY OPERATION

The direct-current component appearing in the secondary of current transformers tends to reduce the speed of operation of induction-type relays. The presence of the direct-current component may cause saturation in the magnetic circuit with consequent wave-form distortion and torque reduction. The most pronounced effect, however, is due to the direct-current flux retarding the disk rotation due to eddy currents, induced in the disk in exactly the same manner as the action of the drag magnet.

The effect of the direct-current component on the operation of balanced-beam-type relays such as high-speed distance and differential relays depends upon the particular arrangement of windings. In general, with a simple balanced beam type, the pull of the current element will be increased due to the direct-current component. With a distance-type relay, this would tend to move the balance point farther away from the relay.

CHAPTER XV

RELAY TESTING AND MAINTENANCE

The successful operation of a protective relay system depends upon a correct application of relays and proper testing and maintenance after installation. Relays should be tested and adjusted upon installation and at intervals thereafter. In general, simple overcurrent relays should be tested every 6 months and the delicate and more sensitive relays more frequently. It is advisable in most cases to maintain two inspection schedules, one, a complete calibration and general inspection, the other, an intermediate inspection without test or calibration. The intermediate inspection should consist of a visual examination of the relays and associated equipment and the tripping of the oil circuit breaker with each relay where possible.

On large systems, a highly skilled group of relay testers is required, if the full benefit of relay protection is to be obtained. The testers should be selected on the basis of their knowledge of electricity, mechanical ability, care, personal responsibility, and familiarity with relay and control circuits. In general, it is best to have one engineer to supervise all relay work, make short-circuit calculations, and determine relay settings. The engineer should also check all relay-test record sheets to see that the specified settings are put into effect and also that the relays are properly adjusted.

Investigation of New Relays.—When making applications of new types of relays, the relays should be subjected to a thorough investigation. Work of this nature can best be done in a laboratory. Complete characteristic curves for the relays should be obtained from test to determine the suitability of the relays to the particular system. The mechanical details should be examined carefully and the effects of adjustments noted. Time variation due to temperature effects should be investigated. If the relays are of the directional type, the directional element should be tested to determine the angles within which the contacts open

and close. The sensitivity of the directional element should be determined and the operating time with various voltages, currents, and phase angles.

Initial Tests.—When relays are first installed, it is necessary to trace and check all instrument and control wiring. The tracing of wiring is simplified where a definite wire-color scheme is carried out during the installation. Two telephone test sets and a dry-cell battery can be used effectively in tracing wiring. Two men are each supplied with a transmitter and headset connected in series, and one of them is furnished with dry-cell batteries. By connecting the battery and test set between ground and one end of the wire to be traced and the other test set between the wire to be traced and ground, the two men are able to trace the wire and at the same time carry on the necessary conversation. Insulation tests should be made on all equipment and wiring.

After all circuits have been checked, the relays should be tested and calibrated in accordance with the specified settings. If the relays are of directional type, it is necessary to determine the proper connections and phase relations for the directional elements. On new installations it is advisable to make a final primary test to check the overall operation. The circuit breaker should be tripped by each relay.

Testing Induction-type Relays.—During the periodic inspection and test, each relay should be tested for pick-up current and time calibration. The relay adjustments should be checked for several points on the calibration curve. The tests should be made at both high and low values of current. This is particularly important where relays having an inverse time characteristic are used. If the same procedure is carried out for all relays, it is obvious that all will have approximately the same characteristics. This is necessary where successive relays must operate selectively. It is good practice on relay installations to test the relays for the range of currents that may occur during fault conditions. A simple test set-up for calibrating relays is shown in Fig. 1.

For the relays used to protect important transmission circuits, it is desirable to check the time of operation for the calculated value of current with a fault adjacent to the circuit breaker and one at the extreme end of the section of the line protected and also for a fault at the end of the next section. As an example, in

Fig. 2, assume that all relays are of the directional type, and further that fault currents can flow from both ends of the circuit. Relay *g* is tested for a fault at *h* and *g*. Relay *e* is tested for a fault at *h*, *g*, and *e*. The difference in time between relays *g* and *e* for a fault at *h* and between these same relays for a fault at *g* will show whether the relays are properly timed to secure selectivity. Generally, this difference in time between the two relays should not be less than 0.4 sec. If this procedure is carried out for each relay, there will be no possibility of incorrect operation unless due to mechanical failure. It is sometimes desirable to prepare a time curve for each relay after complete tests have been made.

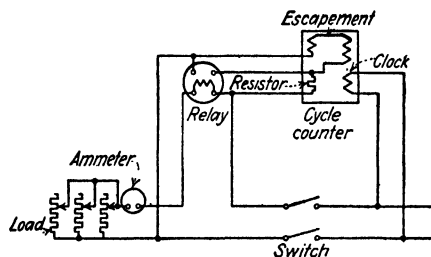


FIG. 1.—Test connections for calibrating relays.

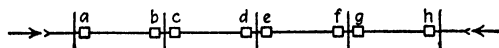


FIG. 2.—Typical sectionalized transmission line.

In some cases it is advisable to give the relays and oil circuit breaker an overall time test. This is particularly necessary where breakers, having different opening speeds, are tripped by relays which must act selectively. If all circuit breakers are of the same type and are properly maintained, this test is not essential. It must be remembered, however, that inselectivity may occur, if the circuit-breaker time varies.

In order that all relay testers follow the same procedure in testing relays, it is necessary to have standard test specifications to cover the types of relays encountered on a particular system. As a typical example, test specifications for testing overcurrent relays are given. Tests 1, 2, and 3, as applied to, say, relay *e* of Fig. 2, might be the application of the calculated values of short-circuit current for faults at *g*, *e*, and *h*, respectively.

SECTION 1. As Found Tests for Overcurrent Relays: These tests are to be made without removing cover from relay.

- A. Determine current necessary to just close relay contacts.
- B. Adjust current for value calculated for test 1. Allow relay to return to its normal position. Wait 2 min. and then test using this value of current. Make three tests on the relay, checking current during each test and allowing 2 min. between tests.
- C. Adjust current for value calculated for test 2. Follow procedure as in B.
- D. Adjust current for value calculated for test 3. Follow procedure as in B.

General:

Each relay is to be tested with at least three values of current in addition to pick-up.

All relays are to be adjusted and timed using secondary tests. Primary tests are to be used for checking current-transformer ratios, secondary wiring, and obtaining overall time of relays and circuit breakers.

SEC. 2. Inspection of Relay: Remove cover and make visual inspection of relay.

- A. Check contacts for corrosion.
- B. Check magnets for cleanliness.
- C. Check for broken terminals.
- D. Check coils to ground.
- E. Check contactor switch for corrosion, freedom of action, and operation.
- F. Check for loose connections.
- G. Check screws in relay.

SEC. 3. As Left Test for Relays: Repeat tests as specified under A, B, C, and D of Section 1.

Space will not permit a detailed description of the adjustments and test specifications applicable to all types of relays. The manufacturers' instruction books and the N.E.L.A. "Relay Handbook and Supplement" cover this thoroughly.

Directional Test.—There are several methods in more or less general use for determining the connections for proper directional operation. The most satisfactory method makes use of the phase-angle meter.

It is necessary to have a complete knowledge of the vector relations of the currents and voltages involved when making directional tests.¹ The portable phase indicator is so constructed that its readings show the angle by which the current through the current coil lags the voltage across the potential coil when the correct polarities are used at the terminals of the instrument. The correct method of connecting this instrument to read the angle of lag between current I_1 and voltage E_{1-0} is shown in Fig. 3. It measures the angle of lag of I_1 with respect to voltage

¹ Refer to Chap. VIII for these vector relations.

E_{1-0} . The polarity must be as indicated in Fig. 3. At 100 per cent power factor this angle is zero. The delta system of vectors

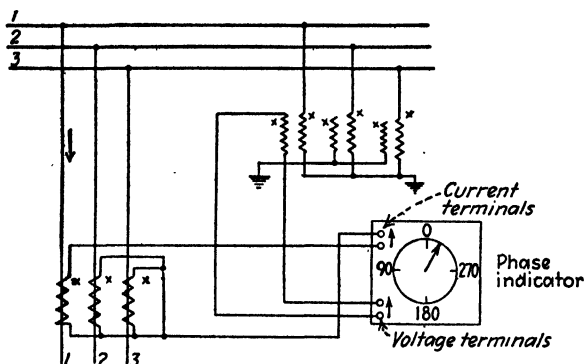


FIG. 3.—Phase-indicator connections for testing relay circuits.

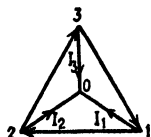


FIG. 4.

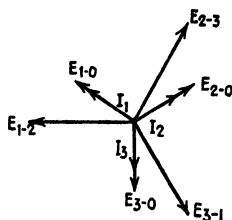


FIG. 5.

FIGS. 4 and 5.—Vector relations of three-phase circuit.

is shown in Fig. 4 and the open system in Fig. 5. Counter-clockwise rotation is used, which gives the phase rotation E_{1-2} , E_{2-3} , and E_{3-1} .

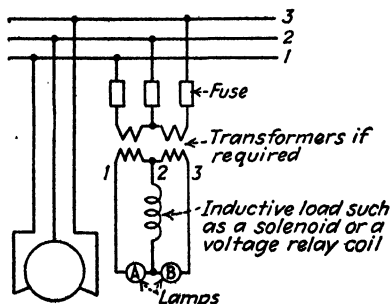


FIG. 6.—Connections for checking phase rotation.

The first step in making directional connections is to determine the phase rotation and to confirm the polarity marking on the potential and current wires. There are several methods which can be used to check phase rotation, such as lamps and a reactance coil, lamps and a condenser, or a small induction motor. The

lamp-and-reactance scheme is illustrated in Fig. 6. When connections are made as shown in Fig. 6, one of the lamps will

be brighter than the other. If lamp *A* is brighter than lamp *B*, phase rotation is counterclockwise, *i.e.*, 1-2-3. Conversely, if lamp *B* is brighter than lamp *A*, the phase rotation is clockwise, *i.e.*, 3-2-1.

In addition to these methods, it is possible to use the phase meter.

The portable phase indicator operates on the principle that a suitable pivoted iron vane, when placed in a rotating field and magnetized by alternating current, will assume a position depending on the phase difference between the current magnetizing the vane and the voltage producing the rotating field. The rotating field is produced by a split phase winding connected to the voltage circuit. In this field is a movable iron vane magnetized by a stationary coil which is part of the current circuit. The iron vane, to which the pointer is attached, is attracted or repelled by the rotating field, and the vane will take up a position so that, when the current wave reaches its maximum, the axis of the iron vane will coincide with the axis of the rotating field at that particular instant. The vane will assume a definite position for any phase relation between current and voltage. The circular scale is divided into 360 electrical degrees.

Consider the connections shown in Fig. 3. Current I_1 lags voltage E_{1-0} by 0° at 100 per cent power factor, provided the instantaneous polarities are as shown in Fig. 3. If the current connections are reversed, this reading will be 180° . Voltage E_{2-0} should then be applied to the voltage terminal. It is exceedingly important to have the same relative polarity for E_{2-0} as was formerly used for E_{1-0} . In other words, the connection to the neutral lead should not be disturbed and the change be made merely by connecting the voltage leads to conductor 2 in place of 1. As shown in Fig. 7, the lag of current I_1 behind voltage E_{2-0} will be 240° . If voltage E_{3-0} is applied to the voltage terminals (in all the test, I_1 connections are left undisturbed) the angle between E_{3-0} and I_1 should read 120° . A further check can be made by using phase-to-phase voltages in place of phase-to-neutral voltages. These angles are all indicated in Fig. 8. The use of phase-to-phase voltages gives a very definite check on the readings taken by using phase-to-neutral voltages. Of course, there will be in all cases a 30° difference

in the angle, as is clearly indicated by a comparison between Figs. 7 and 8. The phase rotation is thus established as counterclockwise.

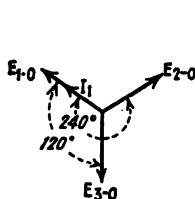


FIG. 7.

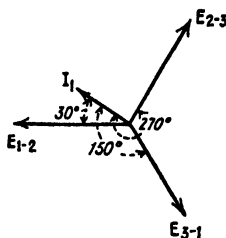


FIG. 8.

Figs. 7 and 8.—Vector relations of phase-rotation test using phase indicator.

In Figs. 9 and 10 is shown the same condition at 30° lagging power factor. Figures 11 and 12 cover the same condition at a 30° leading power factor. In most cases, it will be found that the

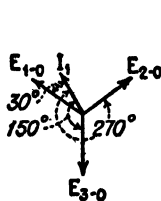


FIG. 9.

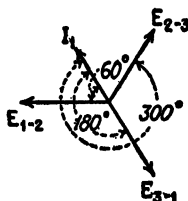


FIG. 10.

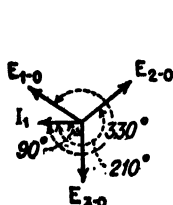


FIG. 11.

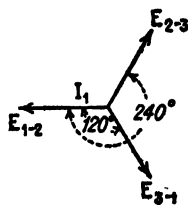


FIG. 12.

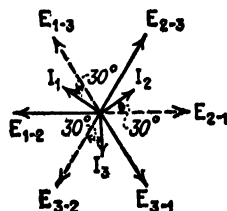


FIG. 13.

Figs. 9 and 10.—Vector relations assuming 30° lagging power factor.

Figs. 11 and 12.—Vector relations assuming 30° leading power factor.

Fig. 13.—Fundamental vector system for 30° relay connection.

power factor of the circuit will be somewhere between these two limits. In Table I the readings for 100 per cent power factor, 30° lagging power factor, and 30° leading power factor are all summarized.

TABLE I.—SUMMARY OF POWER-FACTOR ANGLES IN FIGS. 7 TO 12 INCLUSIVE

	100 %	30° lagging	30° leading
I_1 lags E_{1-0}	Zero	30°	330°
I_1 lags E_{1-2}	30°	60°	Zero
I_1 lags E_{2-0}	240°	270°	210°
I_1 lags E_{2-1}	270°	300°	240°
I_1 lags E_{2-3}	120°	150°	90°
I_1 lags E_{3-1}	150°	180°	120°

The normal-system vector relation, using E_{1-2} , E_{2-3} , and E_{3-1} , should always be determined first and set up in a vector diagram, as shown in Fig. 13. Assuming a 30° connection, since the current leads the voltage applied to the directional element by 30° (330° lag) at unity power factor,¹ the reversed-voltage vectors should be added to the vector diagram as shown. Current I_1 is used with voltage E_{1-3} , current I_2 is used with voltage E_{2-1} , and current I_3 with voltage E_{3-2} . A summary of the angles between these reversed voltages and their respective currents is given in Table II for unity power factor, 30° leading, and 30° lagging power factor. If the line-to-neutral voltages are not available, these voltages can easily be established by connecting a bank of incandescent lamps in Y across the secondaries of the potential transformers.

TABLE II.—SUMMARY OF POWER-FACTOR ANGLES CORRESPONDING TO FIG. 13

	100 %	30° lagging	30° leading
I_1 lags E_{1-3}	330°	Zero (360°)	300°
I_2 lags E_{2-1}	330°	Zero (360°)	300°
I_3 lags E_{3-2}	330°	Zero (360°)	300°

If there is any doubt as to the current-transformer connections, it is desirable to check the polarity by some such method as shown in Fig. 14. The position of the polarity marks on the instrument potential transformers and also the potential coil connections should be as indicated. Adjust the loading rheostats to give 2 or 3 amp. through the rotating standard and phase-angle meter and observe the direction of rotation of the rotating

¹ See Chap. VIII for a complete explanation of the 30° relay connection.

standard. If it rotates in a counterclockwise direction, and if the phase meter indicates an angle between 0° and 90° or between 270° and 360° , the current is feeding in polarity on the primary of the standard instrument current transformers and out polarity on the secondary side. Referring to the diagram, it may be seen that the current is flowing through the oil circuit breakers from *A* to *B* which is equivalent to current flowing from bus to line. The secondaries of the line-current transformer can then be substituted for the standard instrument transformer and the polarity determined from the direction of rotation of the rotating standard.

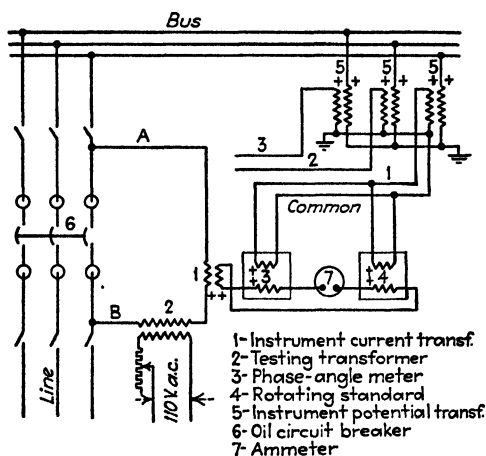


FIG. 14.—Test connections for overall relay and instrument transformer test.

It is now necessary to find the angle between this current and the voltage applied to the directional element of the relay. As formerly explained, current I_1 and potential E_{1-2} are used. Line-to-neutral voltages are used in determining the direction of flow of current I_1 . All that is now necessary is to move the potential wire which goes to no polarity mark of the phase-angle meter and rotating standard to potential wire 3. If the phase-angle meter shows an angle between 0° and 90° or 270° and 360° , the standard should continue to rotate in a counterclockwise direction and the directional-element contacts of the relay should close. If they do not, reverse the secondary wires on the current transformers in the oil circuit breaker, and test again. This test should be repeated for each phase.

The directional overcurrent residual relay using the inside-delta potential connections for the directional element is difficult to connect properly. A simple test may be performed as follows: With all test connections as previously described for the line relays (Fig. 14), disconnect potential transformer 1. Bring the lead that connects to potential wire 1 to the switchboard, and find which terminal it is connected to on the line relay that has just been tested (back view of the relay). Assume that it is connected to the left-hand terminal ground. Then place lead 1 of the auxiliary potential transformer on the left-hand potential terminal of the ground relay, bring the other wire in and place it on the other terminal, and read the phase-angle meter. If the directional contacts close, the angle should be within 10° of the angle obtained during the line-relay test. If the directional contacts do not close, reverse the potential leads coming from the auxiliary potential transformer, and test again. This test should be repeated for each phase.

Some companies use tapped transformers to obtain variable voltages and currents at various phase angles for directional-element tests. Where two- or three-phase voltage is available, two tapped transformers can be connected to different phases of the supply and test voltage obtained from the end tap of one transformer and various taps of the other transformer. A small portable induction regulator can also be used to obtain the same result. The induction regulator has the advantage of finer adjustment of angle.

When the transmission line is loaded and the direction of power flow is known, an alternate method of making directional tests is available. A single-phase 360° power-factor meter is connected in series with each relay, and the power factor of the load is measured. If the relay is properly connected, the power-factor meter will indicate a current leading the indicated load power factor by 30° when the relay voltage is applied to the meter. A phase-angle meter may also be used for this test.

The methods described for determining the proper connections for 30° relations are applicable for any type of connections. In the 60° connections the relay current leads the relay voltage by 60° at unity power factor; and in the 90° connections by 90° at unity power factor.

When testing polyphase directional relays, a separate test should be made on each element in the same manner as for the single-phase relays. A load test should also be made when the relay is put into service, to see that the directional contacts close with power flow away from the bus with one phase connected at a time.

The calibration of the current element of directional relays is made in the same manner as non-directional relays. Some types of directional relays are equipped with two sets of tripping contacts in series. In the directional-control-type relay the overcurrent element does not operate unless the directional contacts are closed. When calibrating the overcurrent element of this type of relay, the directional contacts should be blocked closed. The time of operation of the directional element is variable, depending upon the magnitude and phase relations of the current and voltage existing at the relay under test or fault conditions. The directional element should be tested, using low voltage applied to the potential coil. A test sometimes used is 1 per cent normal voltage with 60-amp. current. The directional elements should be tested for some definite conditions in order to insure uniform speed and sensitivity of operation.

Power relays, used to operate on a predetermined value of power, are tested by applying current and voltage and adjusting the power to the value at which the relay should operate. The time curve is obtained by using different values of current and voltage.

In making the initial tests on balance-relay schemes, such as cross-connected directional overcurrent, current-balance or balance-ground protective schemes, the test applied should be such as to simulate the limiting short-circuit conditions which may occur during operating. The complete circuit should be tested for both through-fault conditions and the worst condition for faults within the section protected by the particular group of relays. It is always advisable to make a complete study of the short-circuit currents which may occur and apply these values of current to the relays when testing. The relays should be calibrated in the usual manner.

Residual-ground directional relays can be tested as shown in Fig. 15. It is not necessary to circulate a great deal of current to check the directional connections. If a high-current testing

transformer is not available, a wound-type current transformer between 100:5 and 500:5 will generally serve as a testing transformer with a 110-volt source connected to the 5-amp. side. As in all directional relays with instantaneous currents in the same direction, the relay has opening torque.

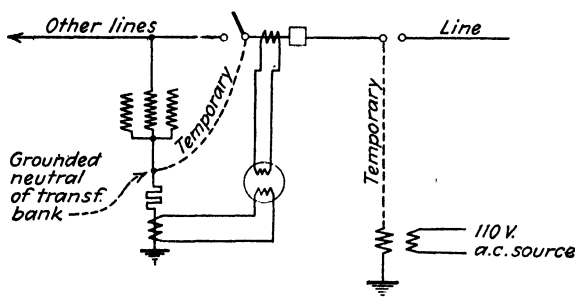


FIG. 15.—Test connections for residual-ground directional relays.

Differential Relays.—In modern relay practice, differential relays are generally applied for the protection of transformers and rotating apparatus; however, there are many installations where simple overcurrent relays are used for apparatus protection and bus-differential protection. In such cases, if the current transformers are not all of the same ratio, autobalance transformers are used for correcting the ratio difference. In testing differential schemes, a complete wiring trace is made. The proper connections and phase relations are then determined using a phase-angle meter. The voltage terminals of the phase-angle meter are connected to some arbitrary voltage source as a reference. The current terminals are connected, in turn, to corresponding phases of the secondaries of the current transformers and readings taken. If the relay is properly connected, corresponding phases should show a difference angle of 180° between the currents. If autobalance transformers are used, they should be adjusted to secure the minimum unbalance current through the relay. The balance test should be made with the maximum possible current passing through the equipment. The relays should be set sufficiently high so that they will not operate on the maximum unbalance current that may occur on a through fault.

The schematic diagram for the differential protection of a delta-Y transformer bank is given in Fig. 12, Chap. X. To

determine the tap setting to be used requires calculations based on the kilovolt-ampere rating of the transformer bank, the current-transformer ratio, and the transformation ratio. It is usually impossible in the application of differential protection to select current transformers which will give a condition of complete balance in the differential circuit. There is usually some unbalance, and the differential relay must be given a tap setting to correct this unbalance. As an example, consider a 25,000-kva., 12,000-volt delta, 66,000-volt Y transformer, having 1500:5 current transformers Y-connected on the low side and 300:5 current transformers delta-connected on the high side.

$$I_{12kv.} = \frac{25,000,000}{\sqrt{3} \times 12,000} = 1200 \text{ amp.}$$

$$I_{66kv.} = 1200 \times \frac{1}{66} = 218 \text{ amp.}$$

$$\text{Secondary } I_{12kv.} = \frac{1200}{1500} \times 5 = 4 \text{ amp. to relay}$$

$$\text{Secondary } I_{66kv.} = \frac{218}{300} \times 5 \times \sqrt{3} = 6.28 \text{ amp. to relay}$$

$$\frac{6.28}{4} = 1.57$$

The proper tap setting to correct this unbalance is readily determined from the characteristic curves for the particular relay, as supplied by the manufacturer. If the current transformers on the high-voltage side of the transformer are of multi-ratio type, a better balance can be obtained by using the 400:5 ratio. The secondary current with 400:5 current transformer is 4.73 instead of 6.28. The tap actually used will be selected by giving consideration to the breakdown characteristic of the current transformers for through faults.

After the relays have been installed and the tap settings determined, it is necessary to check the differential for balance under load conditions. To make this check, the transformer should be loaded to full rating, either by an actual load or by short-circuiting one side of the transformer and applying low voltage to the other side.

One phase of the relay installation (400:5 current-transformer ratio) is shown in Fig. 16, with current links so placed that the current flow in each of the coils of the relay may be measured. The current in coil 2 is equal to the current in coil 3 plus that in coil 1. This figure shows the current values to be

expected for a completely balanced differential. Figure 17 shows the readings to be expected if the current transformers on one side of the differential are reversed; and Fig. 18 shows the readings to be expected if the two sets of current transformers are rotated with respect to each other. This condition might be corrected by reversing the polarity of the Y-connected transformers and then rotating 60° . It may also be corrected by rotating 120° . The readings for each phase are identical for this condition of incorrect connection.

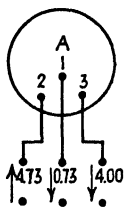


FIG. 16.

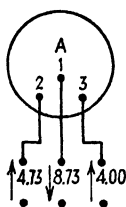


FIG. 17.

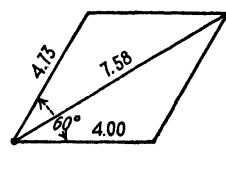
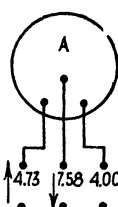


FIG. 18.

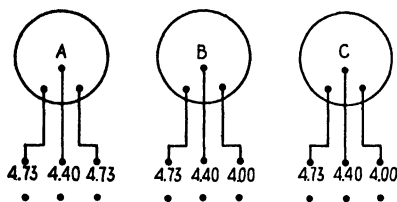


FIG. 19.

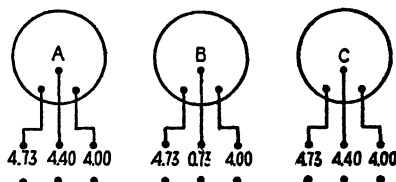


FIG. 20.

Figs. 16, 17, 18, 19 and 20.—Current relations in differential relay circuit.

In Fig. 19 are shown the readings to be expected if the two sets of current transformers are rotated in the opposite direction from that indicated in Fig. 10. The method of correction is the same in this case as before. Figure 20 shows the readings to be expected if the *B* phase connection is correct, but the *A* and *B* connections are interchanged. In this case the *A* and *C* phase readings are similar, but the readings for phase *B* are similar to those in Fig. 16. The obvious means of correcting this condition is to interchange leads *A* and *C*.

In one type of differential relay, taps are available within the relay so that closer balance can be obtained than indicated in

these illustrations. In order to illustrate the current relations, this large unbalance is indicated.

This discussion has been given for the transformer-type relay. The same relations must hold between the currents that pass through the relay when used for generator protection. In the case of generator differentials however the current transformers are usually identical.

High-speed Relays.—The technique in testing high-speed relays, such as distance and balanced-current types, differs somewhat from simpler types, principally due to the high sensitivity, speed, and greater complexity. The time of operation is so short that it is impossible to follow the motion of the various parts of the relay or read the indicating test instruments. It is therefore necessary to use timing devices faster and more accurate than a cycle counter or to use an oscillograph. It is impossible to determine the accuracy of the component elements of a complex high-speed relay without using some accurate timing device capable of recording short intervals.

The performance of most high-speed relays is a complex function of current, voltage, and phase angle. It is advisable to test each element of the relay separately and finally to make an overall test of all elements operating as a unit. In general the tests for distance relays should include determination of amperes pick-up with variable voltage and constant phase angle, and curves of current against phase angle with constant voltage. In all cases it is essential that the electrical quantities applied to the relay be determined by system study and calculations for the limiting fault conditions. This should include all types of faults, both at the far and near end of the particular circuit, and also for conditions of back-up protection when the fault is on other sections. The relays should be tested and adjusted for these limiting conditions. The distance or ohm element must, of course, be tested for the proper balance point. Generally the distance element is set to protect 80 to 90 per cent of the line section.

The same principles should be applied when testing high-speed balanced-current-type relays. The relays should be tested for the limiting fault conditions, particular attention being given to the effects of phase angle, line-capacity effects, and mutual coupling between circuits during faults.

Staged Short-circuit Tests.—Irrespective of the amount of care exercised in testing relays, the adequacy of the system is unknown until actual faults occur. If incorrect operations result, a careful and tedious trouble investigation must be made. Many competent engineers are of the opinion that it is sound practice to make staged short-circuit tests when new relay systems are put into operation. With careful planning, such tests can generally be made at very small cost and without causing service interruptions. The advantage of staged tests is that a complete overall check is obtained of current and potential devices, connections, polarity, directional-element operation, and timing. In addition, the current and voltage magnitudes and phase relations are determined as a check on calculations. The proper operation and adjustment of the oil circuit breakers are also indicated. Frequently unanticipated phenomena occur which can be detected and remedied as a result of such tests. It is essential that an oscillograph be used in order that the maximum benefit be derived.

Testing Equipment.—In order that the cost of testing and inspection be kept to a reasonable minimum, the tests must be performed quickly and easily. One of the chief factors in the cost of relay testing is that of making connections to the relays as they are found in service. A proper layout of the switchboard at the time of installation does not materially increase the cost of the installation and will produce a great saving in test and maintenance costs. Free use of test links or test switches makes it possible to take the relays out of service quickly and with safety to system operation.

Some typical switchboard-wiring schemes using test links for isolating the relays are shown in Figs. 21 and 22. Figure 21 shows the connections for overcurrent relays. Four test links are used in the current-transformer leads. Test links are also used in the control leads. In order completely to isolate this set of relays from the system, it is only necessary to short-circuit the current terminals and remove the links from the current-transformer leads and from the control circuit. All connections for testing the relays can be made at the terminals on the board.

In Fig. 22 is shown a very satisfactory arrangement of test links for use with differential relays. The scheme shown provides complete isolation of the relays and allows insertion of the

necessary ammeters at the time of testing the differential for balance. The buses and test links make it possible to measure the current from each circuit to the differential. In cases where only a few circuits feed into the differential, it is possible to eliminate the buses and carry the leads to the test terminals. Current measurements taken at links H_1 , OA , and L_1 give sufficient data to determine the amount of unbalance and the

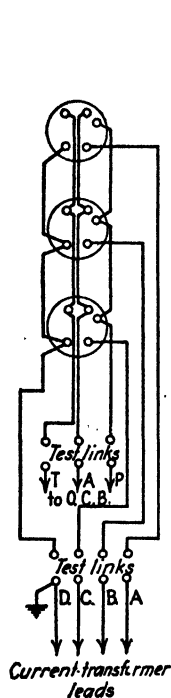


FIG. 21.

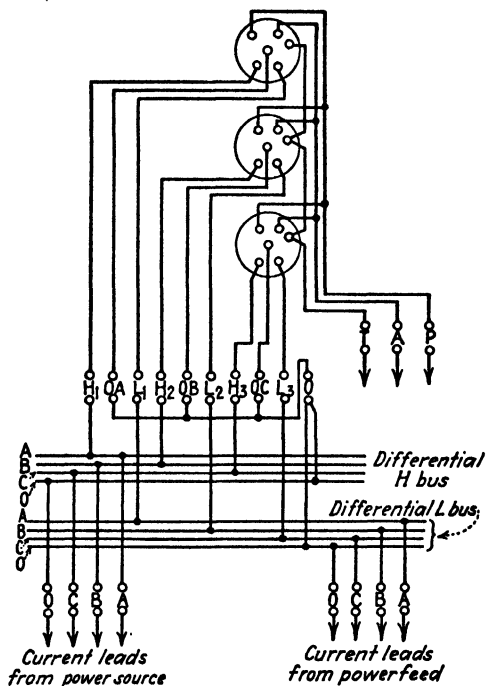


FIG. 22.

Figs. 21 and 22.—Typical switchboard relay connections using test links.

cause for any unbalance that may exist in one phase of the differential. If care is taken to connect the current transformers with correct polarities, much time can be saved in testing the differential for balance.

In most cases the use of test switches is preferable to test links. The test switch consists of base and plug. The connections to the testing equipment are made to the plug. When a test is to be made, the plug is inserted in the base. The

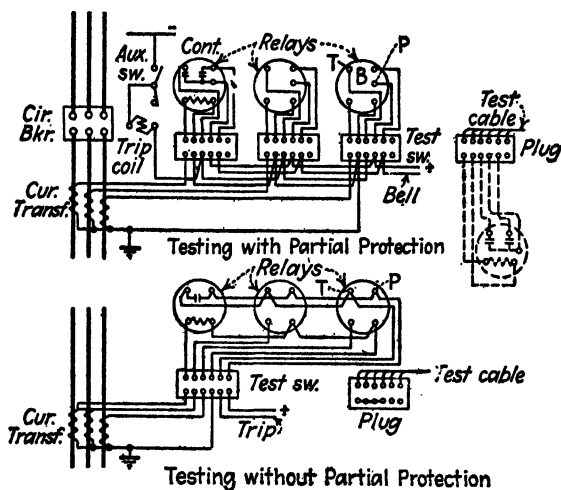


FIG. 23.

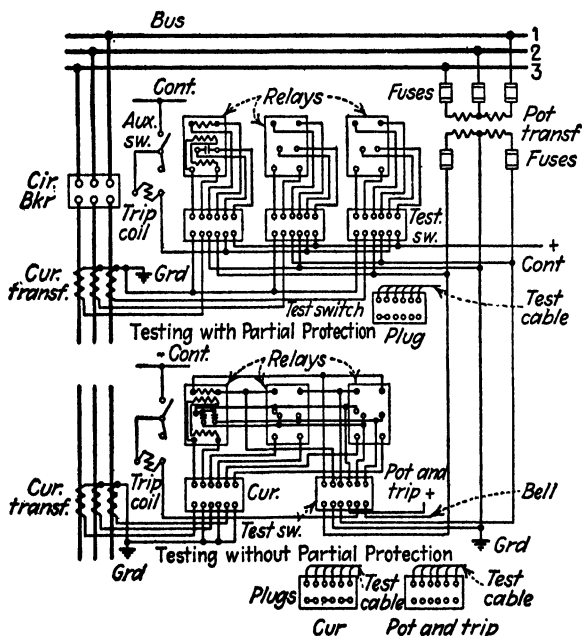


FIG. 24.

FIGS. 23 and 24.—Typical switchboard relay connections using test switches.

current-transformer secondaries are short-circuited, and the normal circuits of the instruments being tested are opened. Relays may be tested much more rapidly by means of test switches, as only one plug is necessary for testing all relays for a particular type. The layout can be made so that the relays on important circuits can be tested one phase at a time without removing protection on the two other phases. Typical connection diagrams are shown in Figs. 23 and 24.

Proper testing equipment in the form of instruments, portable test boards, test tables, leads, and connections will materially reduce the cost of testing. The instruments required depend upon the size and importance of the system and the standard of accuracy that is to be maintained.

Timing Devices.—A cycle counter or synchronous timer can be used for timing slow-speed relays but is unsatisfactory for

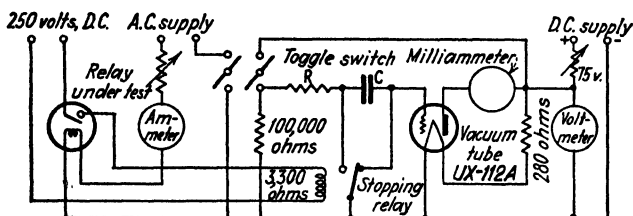


FIG. 25.—Circuit connections for testing high-speed relays.

high-speed types, due to the fact that it is not accurate for short time intervals. A simple arrangement¹ for measuring short time intervals is shown in Fig. 25 which consists essentially of charging a condenser in series with a resistance from a direct-current voltage source for the interval of time to be measured and reading the resultant charge on the condenser with a vacuum-tube voltmeter. The voltage built up across the condenser in time t is

$$V_c = E(1 - e^{-\frac{T}{RC}}) \text{ volts}$$

where C is capacity in microfarads, R is charging resistance in megohms, and E is the charging potential. Knowing the values of R , C , and E , a curve may be calculated showing the relation between volts across condenser and duration of charge. A curve showing the relation between grid voltage and plate

¹ NEHER, J. H., Timing High-Speed Relays, *Elec. World*, Dec. 2, 1933.

current for the vacuum tube may be obtained by calibration. The unknown voltage to be measured is connected between the negative side of the filament and the grid, as shown in Fig. 25. The plate current can be read and the corresponding value of grid voltage taken from the calibration curve. After finding this voltage, the interval of time can be determined from the condenser-charging curve.

The device is operated by adjusting the voltage applied to the tube circuit to the proper value and then closing the toggle switch, which simultaneously starts operation of the relay and charging of the condenser. When the contacts of the relay under test close, the high-speed telephone-type stopping relay is energized. The operation of this relay stops the charging of the condenser and applies the condenser voltage between this grid and filament of the tube. The tube plate current will drop and come to rest momentarily at a point corresponding to the condenser voltage. The time interval corresponding to the minimum plate-current reading can be readily taken from the calibration and condenser-charging curves. The stopping relay time must be deducted from the total time interval to obtain the net relay time.

The range of the timing device can be varied by changing the value of R .

Direct reading stroboscopic timing devices have recently been developed by manufacturers.

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